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Analysis of coupled vibration modes in piezoelectric disks

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Introduction

Piezoelements of disk shape are widely used in simple ultrasonic transducers. Despite simplicity of shape, vibration of a piezoelectric disk is a complex process.

There are many different types of vibration excited in a piezodisk. These types of vibration are called modes. Each mode has unique distribution of displacements. Modes are classified into pure and coupled. This classification is based on one-dimensional approach of vibration.

The pure modes are widely used due the simplicity of piezoelements and good theoretical background of piezoelements' design [1]. Resonant frequencies and efficiency of such transducers are well predicted using transmission line theory [2]. On the other hand, some pure modes, e.g., edge mode [3], are not useable for transducers purposes, but have significant influence to the other modes, e.g., radial.

The coupled mode is type of vibration, which is the product of interaction of a few pure modes. Some coupled modes in rectangular piezoelements have a larger electromechanical coupling factor, comparing to the pure modes. Therefore, efficiency of vibrations can be higher and these modes are widely used in modern piezotransducers.

Pure and coupled modes in piezoelectric disks can be analyzed using analytical [2, 4, 5] or computational [6]-[8] models and also experimentally [4, 9]. Therefore, behaviour of piezoelectric disks is well analysed. On the other hand, most of existing analyses are performed with only one type of piezoceramic or they cover only electrical aspects of vibrations. There is a lack of studies where electrical and mechanical characteristics of coupled modes in a piezodisk are discussed. The use of the coupled modes for increasing of electromechanical coupling require a more detailed analysis.

Finite elements model of piezoelectric disk

Investigations of coupled modes using analytical models are complicated due the complexity of such methods [10] or due the limited accuracy of results [11, 12]. The computational methods are more versatile and more accurate.

Finite elements method (FEM) is one of the most popular computational methods for investigation of piezoelectric bodies. This method allows analysis of coupled modes with a better accuracy than the analytical methods. Therefore, in some studies (e.g., [13]) results of analytical methods were verified by comparison with the FEM results.

The finite elements method is based on segmentation of a piezoelectric body using elements of a simple shape. General FEM model of piezoelectric body is a system of equations [14]:

$$\begin{bmatrix}[M] & [0]\\ [0] & [0]\end{bmatrix}\begin{Bmatrix}\{\ddot{u}\}\\ \{\ddot{\Phi}\}\end{Bmatrix}+\begin{bmatrix}[C] & [0]\\ [0] & [0]\end{bmatrix}\begin{Bmatrix}\{\dot{u}\}\\ \dot{\Phi}\end{Bmatrix}+ \\ +\begin{bmatrix}[K] & \left[K^{z}\right]\\ \left[K^{z}\right]^{T} & \left[K^{d}\right]\end{bmatrix}\begin{Bmatrix}\{u\}\\ \{\Phi\}\end{Bmatrix}=\begin{Bmatrix}\{F\}\\ \{Q\}\end{Bmatrix}, \qquad (1)$$

where $\{u\}$ is the vector of nodal displacement, $\{\Phi\}$ is the vector of electrical potentials on nodes, $\{F\}$ are the mechanical forces, $\{Q\}$ are the nodal charges, $[M]$ is the mass, $[C]$ is the damping, $[K]$ is the stiffness, $[K^d]$ is the dielectric conductivity, $[K^z]$ are the piezoelectric coupling matrixes.

In this study piezoelectric disks were analyzed using the FEM program ANSYS [14]. Axisymmetric two-dimensional FEM model was build for reasonable computing time. However, all unsymmetric modes were neglected. On the other hand, these modes are not piezoelectric active in piezodisks with equal electrodes (Fig. 1). The additional method to save the computing time was assumption of symmetric boundary conditions in the plane of half thickness of piezoelement. Therefore, the piezoelectric disk with the diameter D and the thickness t was simulated using the 2D model of one quarter of the cross-section (Fig. 1).

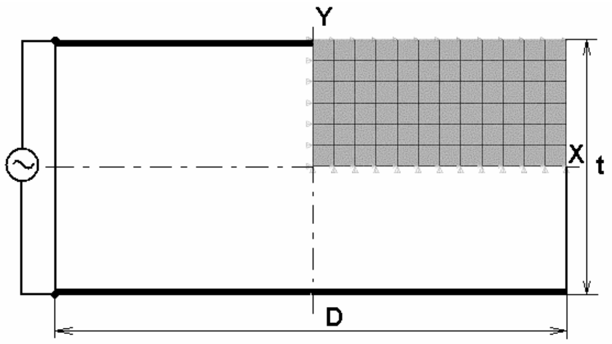

Fig.1. **Axisymmetric finite elements model of piezoelectric disk with symmetric boundary conditions in y=0 plane**

Reasonable accuracy of results requires a fine FEM grid size. The finite element size was chosen less than 1/40 wavelength of the thickness extensional mode frequency for the simulation error less than 0,1%.

In piezoelectric disks with a different aspect ratio *D/t* level of modal coupling is different. In thin disks radial and thickness modes are almost uncoupled. In thick disks there is strong coupling between radial and thickness modes. FEM calculations should be performed keeping one of the mode frequency constant in order to have comparable results. Usually it is the thickness extensional mode frequency. Therefore thickness of piezoelectric disks was constant (t=1,6 mm, f_t≈1,1 MHz). The aspect radio was variable (*D/t*=0.1...25).

Electrodes of piezoelements were simulated as ideal conductive and having zero mass. All nodes on electrodes were coupled by electrical degree of a freedom.

Impedance characteristics analysis

The usual way for investigation of modes is by measurements or calculations of impedance characteristics of a piezoelement. The modes and harmonics are represented as local minimums (resonances) and maximums (antiresonances) of the impedance magnitude characteristic. The difference between radial and thickness frequencies of the piezodisk require analysis in a wide frequency range. Therefore, the harmonic FEM analysis was performed for the frequencies 0…2000 kHz with the step 0,1 kHz.

Solution of Eq. 1 in the frequency domain gives the vector of the nodal charge on electrodes $\{Q_i\}$ at the frequency ω. The admittance Y of the piezoelement then can be calculated as [6]

$$Y = \omega \int_A Q_i dA = \omega \sum_n Q_i \, , \qquad (2)$$

The results of this analysis are impedance characteristics

$$Z(\omega) = 1/Y(\omega) \qquad (3)$$

These characteristics for the disks of CTS-19 piezoceramic are presented in Fig. 2 as 3D surface. The x axis is the aspect ratio *D/t*, y axis – the frequency, normalized by thickness of the piezoelement (kHz·mm), and z axis is the magnitude of the impedance.

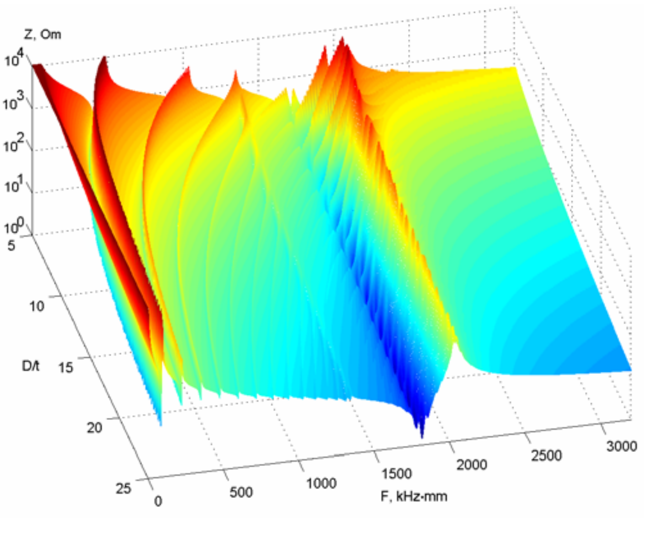

Fig.2. **Impedance magnitude of a piezoelectric disk with the aspect ratios D/t = 5…25**

The more suitable for analysis is representation of impedance characteristics using the frequencies of resonances and antiresonances (Fig. 3). Changes of mode frequencies are more obvious in this plot.

Comparison of results of FEM and analytical methods can be interesting. Using the analytical one dimensional approach frequencies of radial harmonics can be found as [15]

$$fr_n = \frac{\xi_n}{2\pi R} \cdot \sqrt{\frac{1}{\rho \cdot s_{11}^E \cdot (1-\sigma^2)}} \, , \qquad (4)$$

where R is the radius of a disk, $s_{11}{}^E$ is the stiffness, σ is the Poisson ratio, ρ is the density of a piezoceramic, and ξ_n are the non-zero roots of equation [15]

$$\xi \cdot J_0(\xi) - (1+\sigma) \cdot J_1(\xi) = 0 \, . \qquad (5)$$

The radial frequencies are presented in Fig. 3 as solid lines.

It can be seen, that the smaller is *D/t*, the more is difference between FEM and analytical results. This difference is due the modal coupling, which is not included into the analytical model.

The other effect is a “terrace” structure of radial harmonics frequencies. There are three frequencies, where radial harmonics are “unaffected” by *D*. This phenomenon is due the interaction between radial (R), edge (E), thickness shear (TS) and thickness extensional (TE) modes [3, 8]. The more deep understanding of this interaction requires identification of modes.

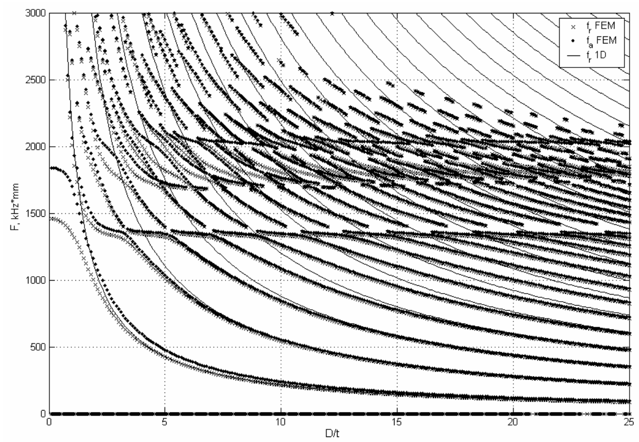

Fig.3. **Frequencies of resonance and antiresonance of piezoelectric disks with aspect ratio *D/t*=0,1…25 calculated using FEM model (dotted lines) and analytical model of radial mode (solid lines).**

Identification of modes

The resonances and antiresonances have a limited information about modes they represent. The more correct identification of modes can be done using visualization of displacements. The FEM calculated pure modes are presented in Fig.4. There is presented one quarter of disk in these plots (see Fig. 1). The brief characterization of these modes is presented below.

The pure radial mode is the type of vibration of a thin piezodisk, when displacements are dominant in a lateral direction. The frequency of this mode is related to the diameter *D*. When *D*<<*t*, the lowest mode in piezoelements is a bar mode.

The edge mode in piezodisks is the type of vibration, related to the bar mode, but its frequency is lower due to the edge mechanical clamping by the center of a piezodisk.

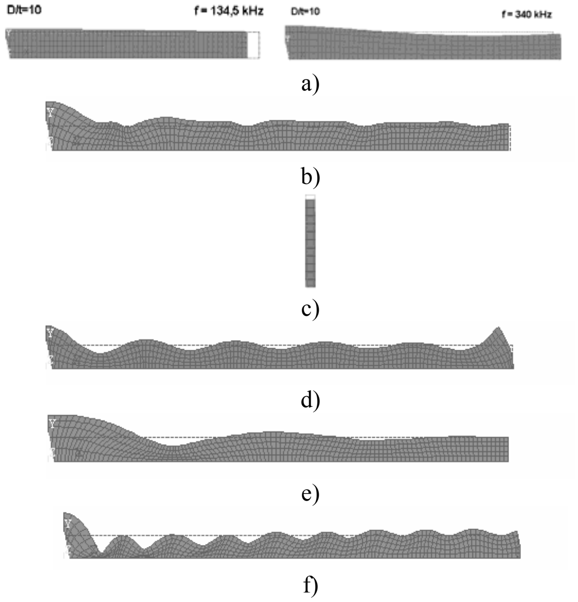

Fig.4. **Modes of vibration in piezoelectric disks *D/t*=20 (except radial and bar modes): a) radial mode, b) thickness mode, c) bar mode, d) edge mode, e) thickness shear mode, f) high frequencies radial (A) mode**

The frequency of E mode is *f*≈1340 kHz·mm for CTS-19 piezoceramic.

The piezoelectric disk has shear displacement in frequencies 1680-1880 kHz·mm (Fig. 4-e). Thickness shear mode is wide used in quartz oscillators, but in piezoelements its use is limited due the low electromechanical coupling.

The most used mode in piezotransducers is the thickness extensional mode (*f*=1790 kHz·mm). It can be seen, the pure TE mode can not be described as a piston-type displacement.

Distribution of displacements at frequencies above TE mode differs from a usual displacement of radial harmonics. It can be seen, that distribution of minimums and maximums of the displacement is composed from two components – high frequency and low frequency. Therefore, these modes are identified as high frequency radial or A modes [16].

The "terrace" structure of radial harmonics near frequency of E mode requires more detailed investigations. There is a obscurity in identification of a radial harmonics number. The first sight into Fig. 3 could indicate, that there are regions of *D/t*, where radial frequencies are "unaffected" by *D*. In order to clarify this phenomenon and to identify these modes, an additional visualization is presented in Fig. 5. Resonance frequencies of different harmonics were marked as "A" to "F". Visualization of displacements for different *D/t* are presented on the right side of Fig. 5.

In can be seen, there is a strong coupling between the radial harmonics and the edge mode near 1340 kHz·mm. Pure distribution of radial harmonics is changed due the increased vibrations of a piezoelements edge (*D/t*=4,5…5 "C" and "D"). The presented shapes of displacements shows, that identification of the radial harmonics number by tracing its resonance frequency is false. It can be demonstrated on the third radial harmonic. Pure distribution of the radial harmonic (*D/t*=6 "C") near the edge mode (*D/t* = 5 "C") remains the same. Tracing the same "line" of the resonance, it can be seen, that the resonance "C" of *D/t*=3,3 and *D/t*=2,5 has distribution of a displacement, which is identical to the second radial harmonic.

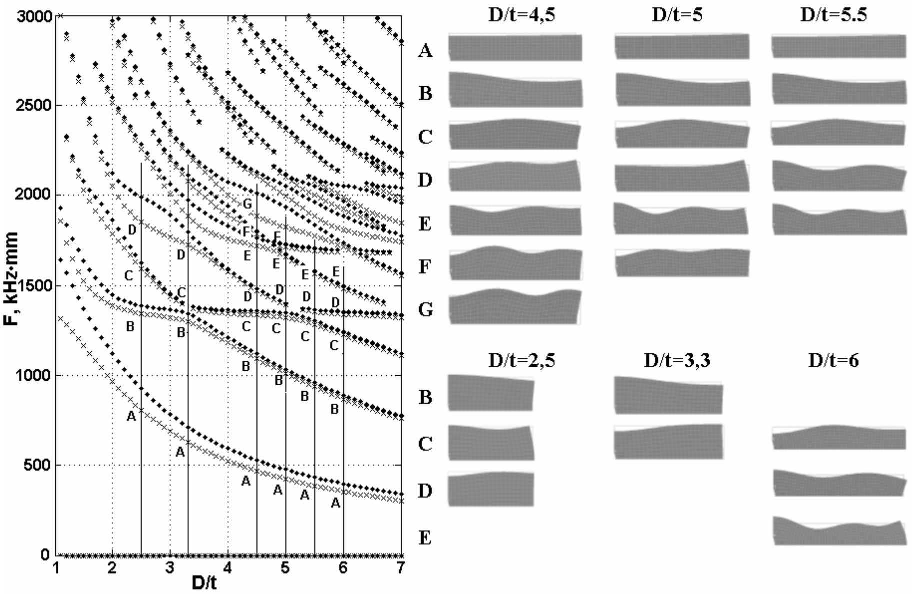

Fig.5. **Coupled radial and edge modes: identification of radial harmonics in the region of the edge mode**

Mechanical characteristics of piezodisk

Another effect of coupled modes is a increased distance between resonance (f_r) and antiresonance (f_a) frequencies in some regions of *D/t*. This phenomenon is related to the effective coefficient of electromechanical coupling [1]

$$k_{eff}^2 = \frac{f_a^2 - f_r^2}{f_a^2}. \quad (6)$$

This coefficient describes efficiency of conversion of electrical energy to mechanical and *vice versa*. Therefore, most efficient aspect ratios of piezodisks could be found using the results of FEM analysis. On the other hand, there is a lack of studies on mechanical characteristics of piezoelements.

Displacements of the surface of a piezodisk have a direct influence to the properties of radiated acoustic field. The FEM analysis of displacements confirmed that in some regions of *D/t* the magnitude of vibrations is higher (Fig. 6. a-b). These regions are coincident with the regions of increased $\Delta f = f_a - f_r$. It can be seen, that high magnitude of displacements is in the regions of the thickness extensional mode and the coupled radial-bar mode (Fig. 6-a). The edge of a disk has a highest magnitude of vibration in bars with the aspect ratio D/t<1 (Fig.6-b). Less intensive vibrations exist at the frequency of the edge mode.

The lateral surface of a piezodisk has highest intensity of vibrations at the radial mode frequencies (Fig. 6. c-d).

On the other hand, the magnitude of a displacement at one point does not display a full information about displacements of the whole surface. Therefore, it is necessary to use integral parameter for evaluation of efficiency, such as the coefficient of electromechanical coupling [6]. Aspect ratios corresponding to the maximal k_{eff} are known for rectangular two dimensional piezoelements [17, 18], but not for disk shape piezoelements.

Intensive FEM simulations were performed using four different piezoceramics (CTS-19, PZT-5A, PZT-5J and PZT-7A) in order to find the most effective mode. It was found, that radial-bar mode has the highest k_{eff}. The results of simulations are presented in Fig. 7.

Analysis of Fig.7 shows, that use of the coupled radial–bar mode gives increase of k_{eff}. This coefficient has a maximum value in bars with the aspect radio *D/t*=0.6…1. Relative increase of k_{eff} depends on a piezoceramic type and usually it is 2…4% comparing to k_{eff} of the bar mode.

However, the results presented in Fig.7 are obtained during simulation of an unloaded piezoelement. Influence of an acoustical load to the k_{eff} is a problem, which is not discussed in available studies. Therefore, the FEM model of a piezoelectric disk radiating into acoustic media was

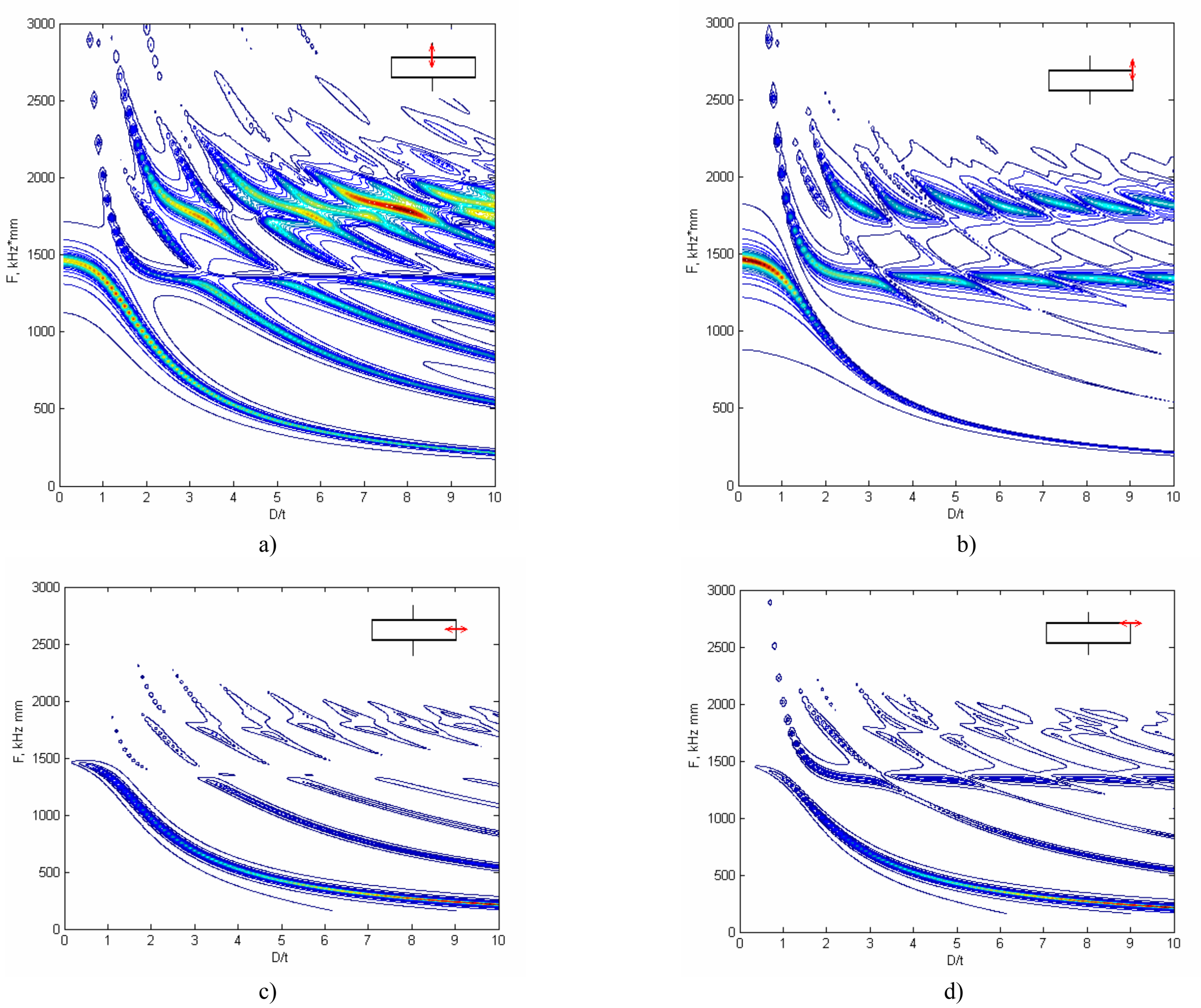

Fig.6. **Displacement of surfaces in piezodisks *D/t*=0…10: a) normal displacement in the center, b) normal displacement on the edge, c) displacement on the side of the disk, d) tangential displacement on the edge of the disk**

built. Thickness of the disk was the same as in the revious simulation. Acoustic media was water, because it is one of the common acoustical loads. Results of simulation of the piezodisk are presented in Fig.8 for the CTS-19 piezoceramic. It is obvious, that the aspect ratio of the piezoelectric bar is not affected by radiation into water.

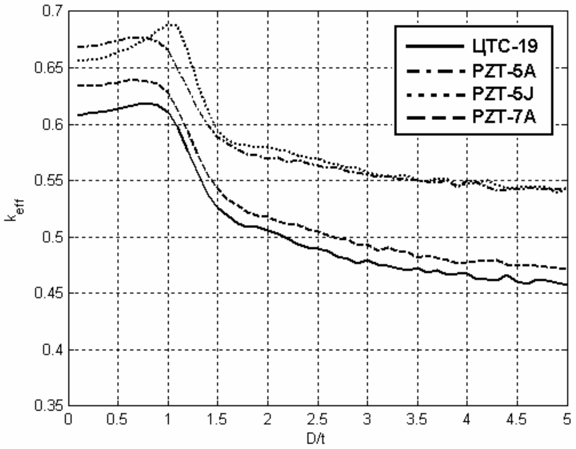

Fig.7 **Electromechanical coupling coefficient k_{eff} for the coupled radial-bar mode in different piezoceramics**

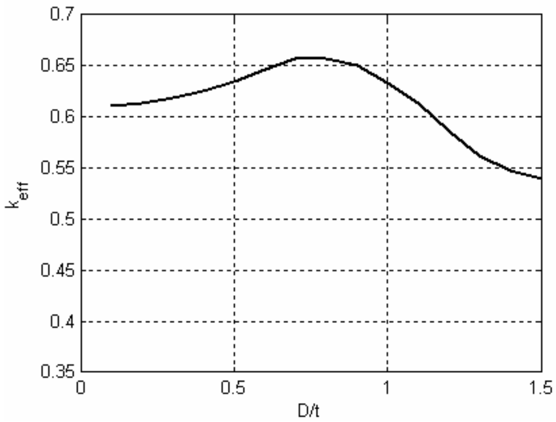

Fig.8 **Electromechanical coupling coefficient k_{eff} of the bar–radial mode in a piezoelement loaded by water (CTS-19 piezoceramic).**

Conclusions and discussion

Analysis of vibration modes in a piezoelectric disk shows that modal coupling is one of the main reasons of a complex impedance characteristic shape near the frequencies of edge, thickness shear and thickness extensional modes. This interaction affects distribution of displacements of radial harmonics. Influence of modal coupling to the frequencies of modes can cause errors in measurement of piezoceramic properties using common resonance – antiresonance methods.

Modal coupling between radial and edge modes can be the reason of wrong identification of the radial harmonics number. The presented analysis shows, that tracing of the radial harmonic resonance is a wrong way for identification of the harmonic. The frequencies of radial modes are sensitive to the disk diameter and the regions of "terrace effect" occurs due the influence of the edge mode.

On the other hand, modal coupling increases the coefficient of electromechanical coupling in piezodisks with the aspect ratio $D/t<1$. Despite of relative low increase in k_{eff}, this effect can increase sensitivity of an ultrasonic transducer. The aspect ratio of maximal k_{eff} is not affected by radiation of ultrasonic waves into water. This effect can be important in design of piezocomposites. Characteristics of piezoelements loaded by materials with a high acoustical impedance on lateral surface require a more detailed analysis.

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Pjezoelektrinių diskų surištųjų virpesių modų analizė

Reziumė

Straipsnyje baigtinių elementų metodu analizuojamos pjezoelektrinių diskų surištosios virpesių modos. Pateikiamos pjezoelementų impedanso dažninių charakteristikų priklausomybės nuo diskų geometrinių proporcijų, identifikuojant skirtingas modas pagal pjezodisko mechaninių virpesių išsidėstymą. Nagrinėjama spindulinių ir briaunos modų sąveikos įtaka pjezoelemento impedanso dažninėms charakteristikoms. Šios charakteristikos lyginamos su pjezodiskų centro, krašto ir šoninio paviršiaus mechaninių virpesių dažninėmis charakteristikomis. Be to, nustatytos pjezodiskų matmenų proporcijos, kurioms esant efektyvusis elektromechaninio ryšio koeficientas yra maksimalus. Parodoma, kad šios proporcijos lieka tokios pačios ir pjezoelementui spinduliuojant į vandenį.

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