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Sound insulation specificities of hoods of self-propelled vehicles used for road and earthwork

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Introduction

Self-propelled vehicles, tractors and other prime movers, engines of which radiate most of all low-frequency noise, that are used for road and earthwork are the polluters of the ambient environment. Low-frequency noise is insulated with difficulty by means of traditional constructions; they penetrate through them and propagate far away from the noise source.

Certain requirements are set for the hoods of these vehicles. One of these is the effect on the distribution of temperatures under the hood.

The construction of the hood, its material and thickness have a major effect on normal (permissible) heat exchange under the hood (housing). These properties are related to sound insulation. Very often for reduction of sound permeability sound-absorbing materials and thick-walled housing of increased weight are used. Hoods of these constructions increase sound insulation, but they reduce heat permeability.

Hoods currently applied in the constructions of the afore-mentioned vehicles may be divided into rigid and soft. Rigid ones in their turn may be of different modifications: hermetical, frame-type with ventilation, non-dismountable-modular. Soft hoods are frameless, with natural ventilation, and dismountable. Hoods of these constructions in most cases have the rectangular shape with rounded corners. Sound insulation of hoods of such type is computed on the basis of the plate insulation theory.

The paper presents the theory for evaluation of the sound insulation of cylindrical hoods. On the basis of theoretical, theoretical and laboratory investigations, it is shown that sound insulation of cylindrical housings at the low- and medium-frequency range is considerably higher than that of plates.

This paper presents the computation methods for estimation of sound insulation of cylindrical hoods that may be applied in the evaluation of insulation of housings when constructing tractors and other self-propelled vehicles used in agriculture and earthwork.

Theory of sound insulation of cylindrical hoods (shells)

In the work [1, 2] an analysis is made of the sound insulating properties of infinite cylindrical shells. Results of this research may be used only in the case where the length of the cylinder is significantly larger than its diameter and the effect of the boundary conditions on the sound insulation of a shell may be not taken into consideration.

Nevertheless, the above-mentioned condition is not always practically fulfilled, and the length of the cylinder comes close to its diameter. Therefore it is expedient to clarify the field of application and to make a certain evaluation of the impact of the boundary conditions on the sound insulating properties of such cylindrical shells (housings).

Let's consider a finite cylindrical shell l pivoted along the curvilinear edges.

Thus the boundary conditions will take the form of

$$T_1=M_1=w=v, \qquad (1)$$

at $\alpha = 0$ $\alpha = \frac{l}{r}$.

Equations describing the structure are [3]

$$\frac{1}{Eh}\Delta^2\varphi - r\frac{\vartheta^2 w}{\vartheta\alpha^2} = 0;$$

$$r\frac{\vartheta^2\varphi}{\vartheta\alpha^2} + D\Delta^2 w + mr^4\frac{\vartheta^2 w}{\vartheta r^2} - r^4 p^x = 0,$$

where p^x is the load.

We shall rewrite equations in the form

$$\frac{1}{Ehr^4}\left[\frac{\vartheta^2}{\vartheta\alpha^2} + \frac{\vartheta^2}{\vartheta\beta^2}\right]\varphi - \frac{1}{r^3}\frac{\vartheta^2 w}{\vartheta\alpha^2} = 0$$

$$(2)$$

$$\frac{1}{r^3}\frac{\vartheta^2\varphi}{\vartheta\alpha^2} + \frac{D}{r^4}\left[\frac{\vartheta^2}{\vartheta\alpha^2} + \frac{\vartheta^2}{\vartheta\beta^2}\right]^2 w + m\frac{\vartheta^2 w}{\vartheta t^2} = P_1^1 - P_2$$

Solution of the system (2) we will search in the form

$$\varphi = \sum_{n=1}^{\infty}\Phi(\alpha)\cos n\beta \sin \varpi t;$$

$$(3)$$

$$w = \sum_{n=1}^{\infty}W(\alpha)\cos n\beta \sin \omega t,$$

by approximation of the searched functions $\Phi(\alpha)$ and $W(\alpha)$ in series

$$\Phi(\alpha) = \sum_{q=1}^{\infty}C_{1q}\sin(\lambda_q\alpha) = \sum_{q-1}^{\infty}C_{1q}\Phi_q;$$

$$(4)$$

$$W(\alpha) = \sum_{q=1}^{\infty}C_{2q}\sin(\lambda_q\alpha) = \sum_{q=1}^{\infty}C_{2q}W_q,$$

where $\lambda_q = \dfrac{q\pi r}{l}$.

Inserting Eq.4 into the system of Eq.2, we will get (solution is effected by a method [4]:

$$\frac{1}{Ehr^4}\sum_{q=1}^{\infty}\sum_{n=1}^{\infty} C_{1q}\left(\lambda_q^4 + 2n^2\lambda_q^2 + n^4\right)\sin(\lambda_q\alpha) + \frac{1}{r^3}\sum_{q=1}^{\infty} C_{2q}\lambda_q^2 \sin(\lambda_q\alpha) = 0$$

$$\frac{1}{r^3}\sum_{q=1}^{\infty} C_{1q}\lambda_q^2 \sin(\lambda_q\alpha) + \frac{D}{r^4}\sum_{q=1}^{\infty}\sum_{n=1}^{\infty} C_{2q}\left(\lambda_q^4 + 2n^2q^2 + n^4\right)\sin(\lambda_q\alpha) - m\omega^2\sum_{q=1}^{\infty} C_{2q}\sin(\lambda_q\alpha) = \left(P_{10}^1 - P_{20}\right)\sin(\lambda_q\alpha). \quad (5)$$

However, the result of system solution (5) in view of its complexity is not subject to a simple analysis and, consequently, is of no practical value. Therefore we shall solve a problem in the first approximation, i.e. consider that displacement of the wall of the shell at a sufficiently narrow frequency band shall be determined by a displacement at the allowed frequency, included into that band. We will neglect the impact of the remaining allowed frequencies.

Taking into account this circumstance as well as requiring orthogonality of the system (5) and function Φ_q, we will get

$$\frac{1}{Ehr^4}\int_o^{l/r} C_{1q}\left(\lambda_q^4 + 2n^2\lambda_q^2 + n^4\right)\sin^2\left(\lambda_q\alpha\right) + \frac{1}{r^3}\int_o^{l/r} C_{2q}\lambda_q^2 \sin^2\left(\lambda_q\alpha\right)d\alpha = 0;$$

$$\frac{1}{r^3}\int_o^{l/r} C_{1q}\lambda_q^2 \sin^2\left(\lambda_q\alpha\right)d\alpha + \frac{D}{r^4}\int_o^{l/r} C_{2q}\left(\lambda_q^4 + 2n^2\lambda_q^2 + n^4\right) = 0, \quad (6)$$

or

$$\sin^2\left(\lambda_q\alpha\right)d\alpha - m\omega^2\int_o^{l/r} C_{2q}\sin^2\left(\lambda_q\alpha\right)d\alpha = \left(P_{10}^1 - P_{20}\right)\int_o^{l/r}\sin^2\left(\lambda_q\alpha\right)d\alpha. \quad (7)$$

$$\left(C_{1q}/Ehr^4\right)\left(\lambda_q^4 + 2n^2\lambda_q^2 + n^4\right) + \left(C_{2q}\lambda_q^2/r^3\right) = 0;$$

$$\left(C_{1q}/r^3\right) + \left(D/r^4\right)C_{2q}\left(\lambda_q^4 + 2n^2\lambda_q^2 + \lambda_q^4\right) - m\omega^2 C_{2q} = P_{10}^1 - P_{20}.$$

Let's exclude C_{1q} from Eq.7 and then divide the obtained equation into $i\omega C_{2q}$, then we will get

$$\frac{\lambda_q^4 Eh}{r^2\left(\lambda_q^4 + 2\lambda_q^2 n^2 + n^4\right)} + \frac{D}{r^4\omega}\left(\lambda_q^4 + 2\lambda_q^2 n^2 + n^4\right) - m\omega = \frac{P_{10}^1 - P_{20}}{i\omega C_{2q}}. \quad (8)$$

The left part of the Eq. 8 is the impedance of the cylinder:

$$Z = i\left[\omega m - \frac{D}{r^4\omega}\left(\lambda_q^4 + 2n^2q^2 + n^4\right) - \frac{\lambda_q^4 Eh}{\omega r^2\left(\lambda_q^4 + 2n^2q^2 + q^4\right)}\right]. \quad (9)$$

After some transformations Eq. 9 may be written as

$$Z = i\left[\omega m - \frac{D\omega^3}{c^4}\sin^4\vartheta\left(\frac{\pi^2 r^2}{l^2}\sin^2\theta + \cos^2\theta\right)^2 - \frac{\pi^4 r^2}{\omega l^4}Eh\sin^4\theta\Big/\left(\frac{\pi^2 r^2}{l^2}\sin^2\theta + \cos^2\theta\right)^2\right] =$$

$$i\omega m\left[1 - \left(\frac{\omega}{\omega_2}\right)^2\sin^4\vartheta\left(\frac{\pi^2 r^2}{l^2}\sin^2\theta + \cos^2\theta\right)^2 - \left(\frac{\omega_o^1}{\omega}\right)^2\sin^4\theta\Big/\left(\frac{\pi^2 r^2}{l^2}\sin^2\theta + \cos^2\theta\right)^2\right], \quad (10)$$

where $\omega_0^1 = \dfrac{C_n \cdot r\pi^2}{l^2}$.

The main difference of the impedance of an infinite cylindrical shell from the impedance of the finite shell is the presence in the latter of the additional multipliers $\left[\dfrac{\pi^2 r^2}{l^2}\sin^2\theta + \cos^2\theta\right]^{\pm 2}$. These also differ in the expression for determining of the additional boundary frequency ω_o^1.

It is interesting to note that difference in the sound insulating properties of finite and infinite cylindrical shells disappears when the length of the cylinder is equal to the length of its semicircle, i.e. $l = \pi r$.

To obtain the quantitative dependence for determining the value of sound insulation of the finite cylindrical shell, at first we will find the coefficient of acoustic permeability at statistical distribution of angles of incidence of sound waves, i.e.

$$\tau = \frac{2}{\pi}\int\limits_o^1\int\limits_0^{x/r}\frac{d(\sin^2\vartheta)d\theta\cdot}{\left\{1+\frac{\omega m\cos\vartheta}{2\rho c}\eta\left[\left(\frac{\omega}{\omega_2}\right)^2\sin^4\vartheta\left(\frac{\pi^2r^2}{l^2}\sin^2\theta+\cos^2\theta\right)^2+\left(\frac{\omega_o^1}{\omega}\right)\sin^4\theta/\left(\frac{\pi^2r^2}{l^2}\sin^2\theta+\cos^2\theta\right)^2\right]\right\}^2+}$$
$$\frac{d\left(\sin^2\vartheta\right)d\theta}{+\left\{\frac{\omega m\cos\vartheta}{2pc}\left[1-\left(\frac{\omega}{\omega_2}\right)^2\sin^4\vartheta\cdot\left(\frac{\pi^2r^2}{l^2}\sin^2\theta+\cos^2\theta\right)^2-\left(\frac{\omega_o^1}{\omega}\right)^2\sin^4\theta/\left(\frac{\pi^2r^2}{l^2}\sin^2\theta+\cos^2\theta^2\right)^2\right]\right\}^2} \quad (11)$$

No possibility exists for calculation of expression (11) in the closed form. However, the possibility exists to simplify this expression being guided by the following.

With the increase of the length of the cylinder, starting with the length $l=\pi r$, the expression $\frac{\pi^2r^2}{l^2}$ will get reduced. Therefore the first addend may be neglected in the expression $\left(\frac{\pi^2r^2}{l^2}\sin^2\theta+\cos^2\theta\right)^2$ as in the case when it is included in the composition of a member, taking account of the effect of the rigidity of tension.

Then the expression (11) may be written thus:

$$\tau = \frac{2}{\pi}\int\limits_o^1\int\limits_o^{\pi/2}\frac{d\left(\sin^2\vartheta\right)d\theta}{\left\{1+\frac{\omega m\cos\vartheta}{2\varsigma c}\eta\left[\left(\frac{\omega}{\omega_2}\right)^2\sin^4\vartheta\cos^4\theta+\right.\right.}\rightarrow$$
$$\rightarrow\frac{d\left(\sin^2\vartheta\right)d\theta}{\left.\left.\left(\frac{\omega_o^1}{\omega}\right)^2tg^4\theta\right]\right\}^2}+$$
$$\rightarrow\frac{d\left(\sin^2\vartheta\right)d\theta}{\left\{\frac{\omega m\cos\vartheta}{2\rho c}\left[1-\left(\frac{\omega}{\omega_2}\right)^2\sin^4\vartheta\cos^4\theta-\left(\frac{\omega_o^1}{\omega}\right)tg^4\theta\right]\right\}^2}. \quad (12)$$

Let's calculate the expression (12) by the method laid out in [2], considering

$$tg^2\theta_o=\left(\frac{\omega}{\omega_o^1}\right);\ tg^4\theta=(1+\varepsilon)tg^4\theta_o. \quad (13)$$

Then we will get

$$\tau=\frac{2}{\pi}\frac{tg^4\theta_o\cos^2\theta_o}{3tg^3\theta_o}\int\limits_o^1\int\limits_{-\varepsilon}^{+\varepsilon}\frac{d\left(\sin^2\vartheta\right)d\varepsilon}{\left(1+\frac{\omega m\cos\vartheta}{2\rho c}\eta\right)^2+\left(\frac{\omega m\cos\vartheta}{2\rho c}\right)^2\varepsilon^2} \quad (14)$$

where $tg^3\theta\cong tg^3\theta_o$; $\cos^2\theta\cong\cos^2\theta_o$.

We calculate the integral

$$\tau=\frac{2}{3}\sin\theta_o\cos\theta_o\int\limits_o^1\frac{d\left(\sin^2\vartheta\right)}{\left(\frac{\omega m\cos\vartheta}{2\rho c}\right)\left(1+\frac{\omega m\cos\vartheta}{2\rho c}\eta\right)}. \quad (15)$$

Integration of Eq.15 at an angle ϑ gives

$$\tau=\frac{4}{3}\frac{\sin\theta_o\cos\theta_o}{\omega m/2\rho c}\cdot\frac{\ln(1+\omega mr/2\rho c)}{\omega m\eta/2\rho c}. \quad (16)$$

Expression (16) may be written as follows

$$\tau=\frac{4}{3}\frac{1}{\left(\omega_o^1/\omega\right)^{1/2}+\left(\omega/\omega_o^1\right)^{1/2}}\cdot\frac{1}{\omega m/2\rho c}\cdot\frac{\ln(1+\omega m\eta/2\rho c))}{\omega m\eta/2\rho c} \quad (17)$$

Consequently, sound insulation of the infinite cylindrical shell is determined by the expression

$$R=10\lg\frac{\omega m}{2\rho c}+10\lg\left[\left(\frac{\omega_o^1}{\omega}\right)^{1/2}+\left(\frac{\omega}{\omega_o^1}\right)^{1/2}\right]+$$
$$+10\lg\frac{\eta\frac{\omega m}{2\rho c}}{\ln\left(1+\frac{\omega m}{2\rho c}\eta\right)}. \quad (18)$$

The satisfactory coincidence of the results, obtained according to the formula (18), with the results of numerical integration of the expression (12) with a special programme on a computer may be stated only at the frequencies $\omega<\omega_o^1$ (Fig.1). In the remaining area of frequencies, the coincidence of results is unsatisfactory. This circumstance may be explained as follows.

The accepted method of calculation of the integral (12) satisfies the required precision only in the case of the coefficient of sound permeability being calculated according to the area, bounded by the curve of wave coincidence, approximate by its character to δ – function. This condition is sufficiently precisely maintained for frequencies $\omega<\omega_o^1$. Beyond these frequencies, the form of the curve of wave coincidence gets "spread" and now it is not allowed to neglect the areas lying outside the boundaries $\pm\varepsilon$. Therefore, the sound insulation, calculated according to the formula (18), is augmenting with the increase of the frequency for $\omega>\omega_o^1$.

However, one more method for calculation of similar integrals exists, which is less "sensitive" to the mentioned shortcoming. For that purpose, making use of the known ratio [4] and considering ϑ = 45°, the coefficient of sound permeability at statistical distribution of angles of incidence of sound waves we will write thus

$$\tau \cong \frac{2}{\pi}\int_{-\varepsilon_{01}}^{+\varepsilon_{02}} \frac{\tau_{\max} d\varepsilon_o}{1+\varepsilon_o^2/\delta_o^2} = 2\tau_{\max}\delta_o \,, \qquad (19)$$

where $\varepsilon_o = tg^2\theta_o - tg^2\theta$ is the accepted designation; $tg^2\theta_o = \left(\frac{\omega}{\omega_o^1}\right)$; $\tau_{\max}$ is maximum value of the coefficient of sound permeability; $2\delta_o = 2\left(tg^2\theta_o - tg^2\theta_n\right)$ (θ_n is the angle at which $\tau = \frac{1}{2}\tau_{\max}$).

Then "semiwidth" δ_o will equal

$$2\delta_o \cong 2\left(\frac{\omega}{\omega_o^1}\right)/\left(\frac{\omega m \cdot 0{,}7}{2\rho c}\right). \qquad (20)$$

The coefficient of sound permeability may be written as

$$\tau = \frac{2{,}8}{\omega_o^1 m / 2\rho c}. \qquad (21)$$

Expression for determining the value of sound insulation will be defined by the expression

$$R = 10\lg\frac{\omega_o^1 m}{2\rho c} - 4{,}5 \text{ dB}. \qquad (22)$$

Eq. 22 is obtained without taking into account of the effect of losses on the sound insulation of the cylindrical shell. However, analysis of the results of numerical integration shows that this effect is substantial at high frequencies and it may be taken into account by increasing the design quantity of sound insulation according to the Eq. 22 by 2–9 dB.

Thus the Eq. 22 may be written as follows

$$R = 10\lg\frac{\omega_o^1 m}{2\rho c} - 1{,}5 \text{ dB}. \qquad (23)$$

The results, presented in Fig. 1, show that Eq. 23 describes satisfactory the change in the sound insulation of the shell at frequencies $\omega > \omega_o^1$.

Thus, for calculating the sound insulation of the finite shell, pivoted along the curvilinear edges, it is possible to use Eq. 18 for frequencies $\omega < \omega_o^1$ and Eq. 23 for frequencies $\omega > \omega_o^1$.

Conclusions

The obtained theoretical results make it possible to draw a conclusion on the sound insulation of the housing (shell). The calculated sound insulation of the housing (shell) is reflected in Fig.1. It is seen here that sound insulation of such housing at low frequencies is higher than that of plate and increases insignificantly with a frequency. Sound insulation up to the ω_o additional reducing frequency is less than in plates, since the rigidity stretching member at frequencies $\omega < \omega_o$ may become equal to the inertia member due to the certain incidence angle of a sound wave, which is characteristic of each frequency.

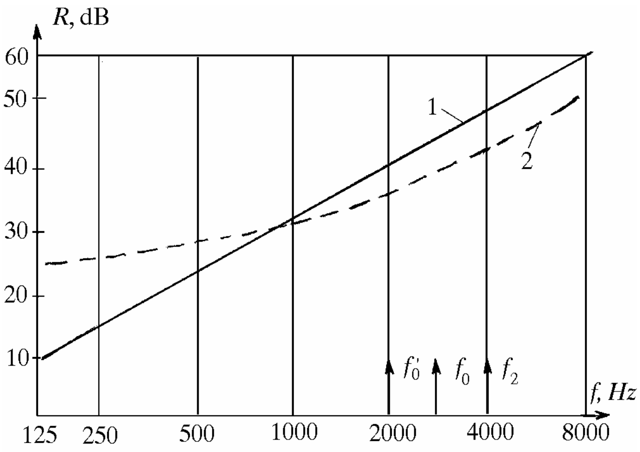

Fig. 1. **Sound insulation: plate (1) and cylindrical shell (2): *n* = 2 mm. 2*r* = 600 mm; *l* = 1000 mm**

Notation

T_1 – normal forces;
M_1 – bending moment;
w – normal displacement;
v – tangential displacement;
$\alpha = \frac{l}{r}$
ω – boundary frequency;
ω_0 – additional boundary frequency;
W – amplitude of oscillations;
$q = \frac{\omega}{c} r \sin\vartheta \sin\Theta$;
E – Young's modulus;
$n = \frac{\omega}{c} r \sin\vartheta \cos\Theta$;
p – sound pressure;
$m = \rho_m h$ – unit mass of the material surface;
h – shell wall thickness;
Θ – angle between the plane of incidence and the axis plane;
ϑ – angle between normal;
ρc – specific acoustic resistance of medium;
τ – coefficient of sound permeability;
R – sound insulation of a barrier.

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Kelių ir žemės darbams naudojamų savieigių mašinų kapotų garso izoliacijos ypatumai

Reziumė

Straipsnyje pateikiama teorija, kaip nustatyti cilindrinių kapotų (kevalų) garso izoliacijos ypatumus, priklausančius nuo kapotų formos.

Mūsų ir kitų autorių [1, 2] atliktais tyrimais nustatyta, kad cilindrinių gaubtų (kevalų) garso izoliacija žemų ir vidutinių dažnių diapazone yra daug aukštesnė negu plokščių paviršių.

Čia pateikiama skaičiavimo metodika cilindrinių kapotų (kevalų) garso izoliacijai įvertinti. Gauti rezultatai gali būti taikomi traktorių ir kitų žemės ūkyje naudojamų savieigių mašinų kapotų izoliacijai įvertinti.

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