

Investigation of bending vibrations of a circular disk

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Introduction

The problem of bending vibrations is common in different engineering and physical applications. Bending vibrations of centrally clamped rotating circular disks play crucial role in the functionality of hard disk drives [1, 2]. Lots of efforts are spent for dynamic stabilisation, control and measurement of bending vibrations in such micro-mechanical systems [3].

Nevertheless, measurement of microscopic deflections from the state of equilibrium is a challenging problem. Different optical measurement techniques are developed for experimental investigation of bending vibrations [4]. Reflection moiré is one of the popular methods for experimental analysis of bending vibrations of structures.

Unfortunately, interpretation of experimental measurement results is a nontrivial inverse engineering problem often having non-unique solutions [5]. This is especially relevant for time average reflection moiré applied for circular structures. Interpretation of fringes generated by time average techniques is already a challenging problem. The problem gets only more complex if the grating lines are circular.

Therefore there exists a definite need for hybrid numerical – experimental techniques [5, 6] that could help to interpret the measurement results. Such techniques usually comprise a numerical model of the system coupled with optical and geometrical parameters of the measurement set-up. Then the predicted response of the experimental optical measurement system can be mimicked in a virtual numerical environment when the dynamical parameters of the analysed object are pre-defined.

The principle of reflection moiré analysis

The principle of the reflection moiré analysis [4] is presented in Fig. 1 where x , y and z denote the axes of the Cartesian frame (y axis is not shown for simplicity). The clamped circular disk in the status of equilibrium is in the plane $x - y$. It is assumed that an ideal mirror film covers the surface of the disk. Moiré grating and the photographic plate are parallel to the plane $x - y$. The distance between the photographic plate and the analysed disk is d . The deflection of the plate is w . It is assumed that the analysed vibrations are small and that $d \gg w$. The subscript denotes partial derivative; N is the normal vector to the surface of the disk; u and v denote x - and y - shifts of the reflected moiré grating with respect to the reflected moiré grating in the status of equilibrium (shift v is not shown in

Fig. 1). This is a schematic representation of an optical set-up for reflection moiré analysis. Real experimental implementation would require a semi-silvered mirror to be introduced in order to assure that the moiré grating and the photographic plate would not overlap each other [4]. Nevertheless, this would not alter the physical processes taking place in the optical set-up.

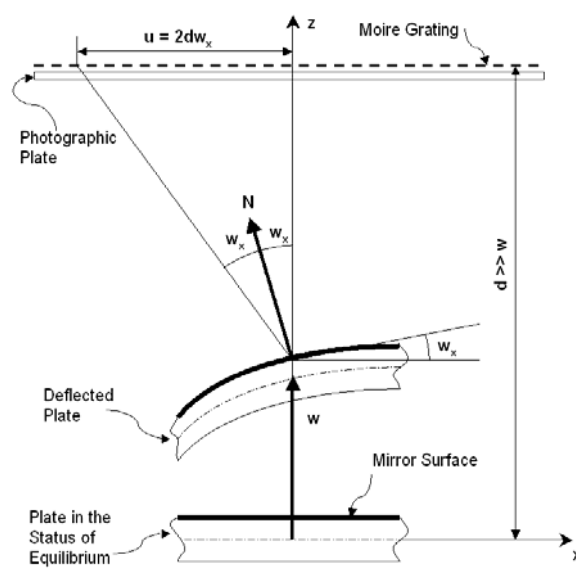


Fig. 1. Schematic diagram of reflection moiré measurement set-up.

Analysis of a centrally clamped disk

Finite element techniques [7] are used to construct the numerical model of a centrally clamped circular disk. Plate bending element with the independent interpolation of the displacement w and the rotations about the appropriate axes θ_x and θ_y is used [7]. Formation of digital reflection moiré images requires nodal components of the derivatives of the deflection w . Therefore nodal derivatives of the transverse displacement are calculated by using the conjugate approximation with smoothing.

Digital formation of the time average reflection moiré images is similar to the problems of time average geometric moiré, which are described in detail in [8]. The formation of fringes in reflection moiré is sensitive to deflection derivatives [4]. Thus the difference between the reflection moiré and geometric moiré [4] is that instead of dynamic displacements u and v the following values are used in the analysis using reflection moiré.

$$\begin{aligned} u &= 2dw_x, \\ v &= 2dw_y. \end{aligned} \quad (1)$$

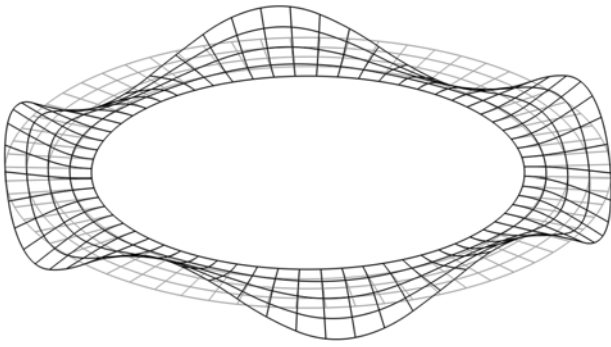


Fig. 2. The tenth eigenmode of the centrally clamped disk

The finite element mesh and the tenth eigenmode of a centrally clamped disk are shown in Fig. 2. The mesh in the status of equilibrium is grey and deflected according to the eigenmode is black. Isolines of the partial derivatives of the dynamic displacement in radial and angular directions are presented in Fig. 3a and Fig. 3b. Smoothed isolines of the partial derivatives of the dynamic displacement are presented in Fig. 4a and Fig. 4b accordingly. It can be noted that the application of smoothing enables to avoid unphysical breakings of the isolines.

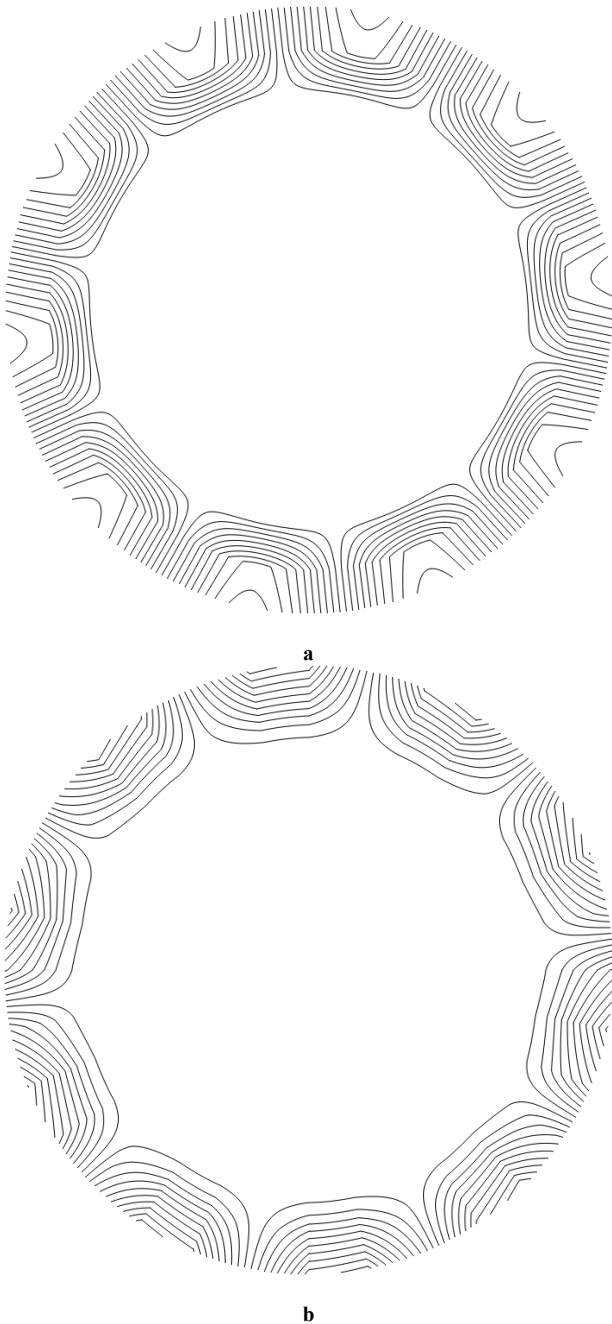


Fig. 3. Unsmoothed isolines of the derivatives of the deflection (a – in the radial direction; b – in the angular direction)

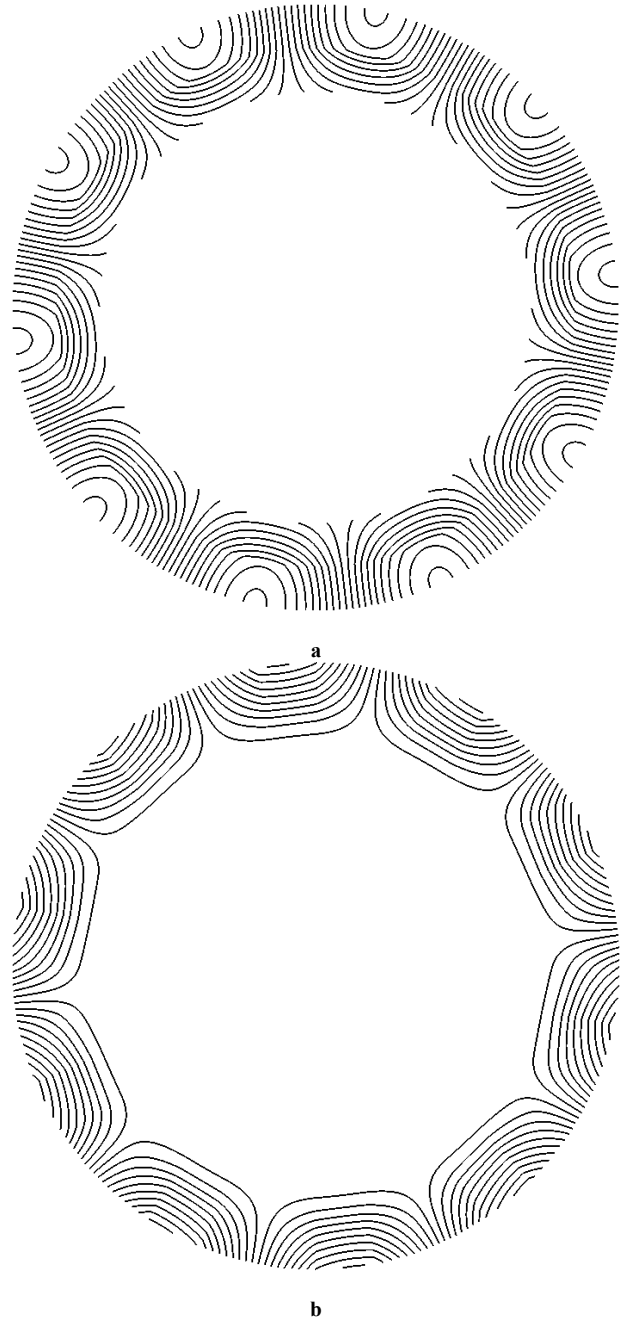


Fig. 4. Smoothed isolines of the derivatives of the deflection (a – in the radial direction; b – in the angular direction)

Time average reflection moiré images are presented in Fig.5, a and Fig.5, b for radial and angular gratings

accordingly. It can be noted that the correspondence between the patterns of fringes and the previously presented isolines is evident.

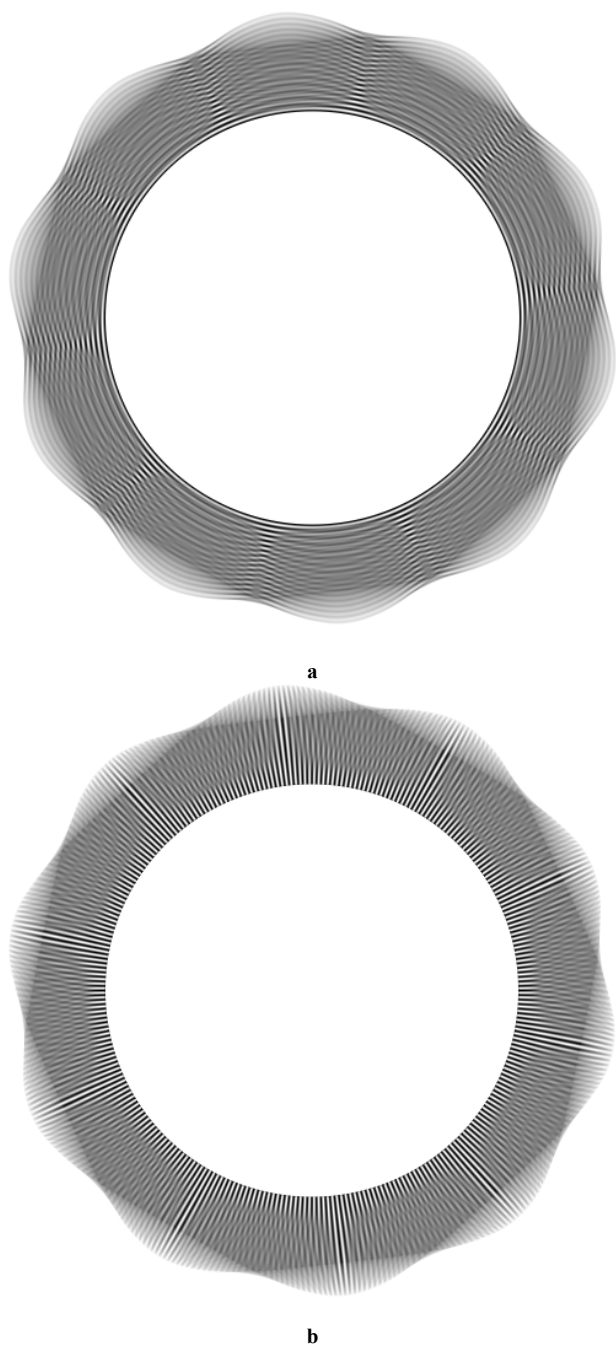


Fig. 5. Time average reflection moiré images (a – radial grating; b – angular grating)

Estimation of dynamic deformations from the results of analysis by using reflection moiré

Time average fringe based experimental methods can be effectively applied for the analysis of dynamic deformations. A typical example when time average geometric moiré is applied for measurement of plane motions is presented in [8].

The explicit relationship among the fringe order n , pitch of the grating λ and dynamic one-dimensional displacement u is [8]:

$$u = \frac{\lambda}{2\pi} b_n, \quad (2)$$

where b_n is the n -th root of the zero order Bessel function of the first kind. The values of the roots of the zero order Bessel function of the first kind are available in classical texts [9].

Time average reflection moiré analysis is sensitive to bending vibrations, not plane motions. But analogous relationship can be constructed taking into account Eq.1:

$$w_x = \frac{\lambda}{4\pi d} b_n. \quad (3)$$

It can be noted that the component w_y can be reconstructed similarly but using a mutually orthogonal grating. In general, all optical moiré techniques can be used to reconstruct physical quantities in the direction orthogonal to the lines of the gratings [4]. If the grating lines are circular in the analysed problem of centrally clamped disk, only the radial derivative of the deflection w_r can be reconstructed using such a grating.

The goal of experimental analysis of bending vibrations of a clamped disk is to determine the field of dynamic deflections, not their derivatives. Keeping in mind that the internal radius of the disk r_0 is fastened, the dynamical deflections can be reconstructed integrating the approximated derivatives:

$$w(r) = \int_{r_0}^r W_r(r) dr, \quad (4)$$

where $W_r(r)$ is an approximated polynomial over the discrete values of w_r at the centres of interference fringes.

The procedure of the reconstruction of dynamic deflections from the pattern of interference fringes is illustrated by the following example. First of all the derivatives of the deflections at the centres of interference fringes must be determined. That requires application of the fringe counting technique [4]; then Eq. 3 can be used to calculate the discrete values of the derivatives of the deflection.

The zoomed part of the pattern of fringes produced by time average mirror moiré for the tenth eigenmode of the internally clamped disk is presented in Fig. 6. Dynamic deflections are reconstructed along an axial line going through the area of maximum deflections. Fringe counting is straightforward due to the fact that the internal radius of the disk is fastened; fringe orders are shown at the appropriate intersections of fringes and the axial line. The clamped internal radius of the disk is 100 mm; the free external radius is 140 mm; the width of the analysed section of the disk is 40 mm. The pitch of the grating $\lambda = 2,5$ mm; the distance between the photographic plate and the clamped disk $d = 100$ mm. For simplicity it is assumed that the parameter r is equal to 0 at the point where the internal radius of the disk is fastened ($r_0 = 0$). Then the analysed range of r is between 0 and 40 mm.

The discrete values of the derivative of the deflection are calculated at appropriate intersection points (Fig. 7), where x -coordinates of the points are calculated from Fig.6; y -coordinates – from Eq.3. The next step is approximation of the continuous deflection derivative function $W_r(r)$ through the discrete values of w_r (Fig. 7). Some scattering of points around the approximated polynomial can be explained by the measurement errors of the centres of interference fringes. Finally, dynamic deflections of the clamped disk can be reconstructed integrating the approximated polynomial (Eq.4). The produced curve (Fig.7) characterises the dynamic behaviour of the disk. Good adequacy between the reconstructed dynamic deflections and the geometrical shape of the disk in its extreme deflection (Fig. 2) validates the presented procedure.

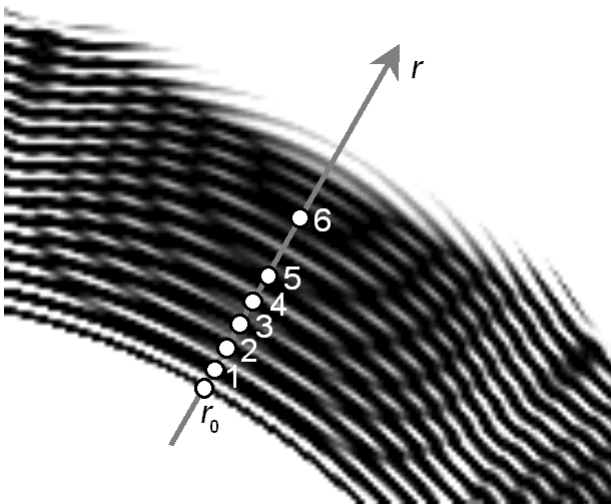


Fig.6. Zoomed time average reflection moiré image with radial grating.

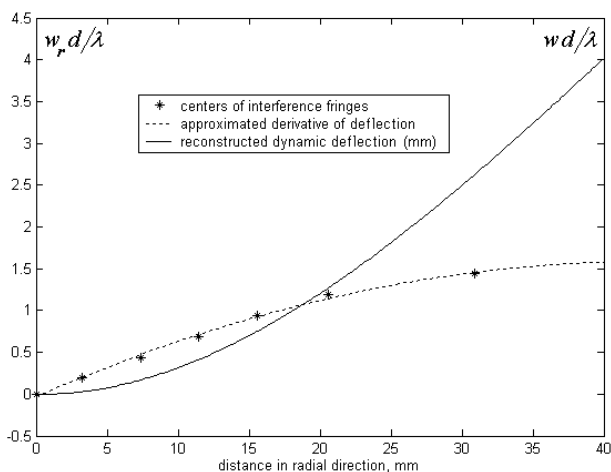


Fig.7. Reconstruction of the dynamic deflection.

Conclusions

Development of digital time average reflection moiré images builds the ground for hybrid numerical – experimental techniques for the analysis of bending vibrations of different engineering structures. Such techniques can help to analyse and interpret optical experimental results with greater precision.

Digital time averaging techniques had to be applied in order to generate fringe patterns corresponding to time average reflection moiré fringes. Finally, techniques for solving the inverse problem of the reconstruction of the deflections from the pattern of fringes had to be developed.

The presented technique for interpretation of fringes is applicable in the hybrid numerical – experimental time average reflection moiré analysis and shows the complexity of the inverse problem of the reconstruction of dynamic displacements.

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Apskritiminio disko lenkimo virpesių tyrimas

Reziumė

Sprendžiamas dinaminių poslinkių atkūrimo taikant laike suvidurkintą atspindžio muaro metodą uždavinys. Tiriamas apskritiminis diskas su įtvirtintu vidiniu skersmeniu. Realizuojama eksperimentinė hibridinė skaitmeninė procedūra ir pateikiami baigtinių elementų skaičiavimų rezultatai.

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