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Wave propagation in rigid cylindrical tubes with viscous and heat-conducting fluid

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Introduction

The propagation of sound waves in cylindrical tubes with compressible fluid is a fundamental and classical problem. Famous names - Helmholtz, Kirchhoff, Rayleigh - are connected with the first studies [1]. The full Kirchhoff solution of a viscous and heat-conducting fluid in rigid circular tubes was later developed in two directions: analytical approximations of a very complicated transcendental equations for various regions of "wide", "narrow", "wide-narrow", "very wide" tubes and an extension of the theory for non-circular tubes or higher modes in circular tubes [1, 2]. Rodarte et al. [3] presents a polynomial approximation of Kirchhoff solution and includes some experimental investigations. Simplifications of the initial equations, describing a viscous and thermal gas, were made by Cummings [4]. In this paper an approximate solution for circular tubes was investigated and generalized for the tubes of arbitrary cross-sectional shape.

Another development of investigations considered deformations of the pipe. Interaction of the compressible ideal fluid and elastic shell is the principle aim of these investigations. One of the first studies of the axisymmetric waves was published by Lin and Morgan [5], where the so-called cut-off frequency is studied. More complicated cases of the modal shapes of a shell and corresponding dispersion equations with complex wave numbers were investigated by Fuller and Fahy [6]. Acoustic energy flow in deformable pipes was investigated by Pavic [7] and Feng [8]. Bansevicius and Kargaudas [9, 10] presented the results of new studies involving multichannel deformable pipes. In all those investigations the fluids are considered as ideal and the thermal conductivity in fluid is neglected. Sound transmission through the deformable ducts and wave attenuation is described by Cummings [11, 12], but, here, transmission loss in these papers is attributed to an external acoustic radiation from the duct walls with rectangular, circular or flat-oval cross-section shapes.

Propagation of elastic waves in a fluid-saturated porous solids and an interaction between the solid and the fluid was studied by Biot [13].

As the Kirchhoff solution for a rigid circular pipe is highly complicated, simplifications of fluid equations are practically inevitable. Simplifications, used in this paper, coincide with the simplifications made by Stinson [14] and applicable to the broad range of frequencies f and radii r_o encompassed by $r_0 f^{3/2} < 10^6$ cm s$^{-3/2}$ and $r_0 > 10^{-3}$ cm, but the assumption "the amplitude of the acoustic pressure p is constant in the cross-section of a pipe" has to be restricted. The assumption p is constant in this paper is applied only for the longitudinal fluid motion equation and the thermal equation. The exact Kirchhoff solution proves pressure p is not a constant, so the problem is: where can p be assumed a constant and where the variation of the pressure p is essential? The Stinson's investigation in [14], based on comparison between the Kirchhoff exact solution and the approximate solution of the longitudinal motion equation where the pressure p is constant , shows that this assumption is acceptable for this equation. However, if elasticity of the pipe is to be considered, assumption that p is constant is completely unacceptable, because the pressure variation in the cross-section is the reason or, in some cases, the consequence of the fluid motion in this section. If equations, describing the fluid motion in the cross-section, have to be solved, then the problem is the way of approximate presentation of a pressure.

In the deformable pipes with viscous and thermal fluid, the attenuation of the propagating pressure waves depend on the viscosity, the thermal conductivity of the fluid and also on damping in the shell material. So, the whole problem of wave attenuation in the pipe can not be solved unless the thermal and viscous fluid equations are solved simultaneously with the shell equations. Influence of the fluid surrounding the tube and the external acoustic radiation from the tube is neglected in this paper, but there are no principal obstacles to include this factor [11, 12].

The Kirchhoff solution and the Stinson's approximation are for the viscous and thermal fluid in a rigid tube. The approximate solution, presented in the third section of this paper, is also for the viscous and thermal fluid in a rigid tube, however the method of solution differs from Stinson's approximation: the equation of fluid motion in the cross-section is applied. The approximate solution, obtained in this paper, after some elementary simplifications, is identical with Stinson's approximation. But this approximation is insufficient if dynamics of the pipe or shell are important. More accurate approximation, presented in the fourth section of this paper for both the rigid and the elastic tubes, includes also the true value of the fluid added mass, i.e. the influence of the fluid dynamics in the cross-section of the pipe. If the viscosity and thermal conductivity in the second approximation approaches zero then a limit of the solution for the ideal fluid and the elastic tube is obtained [9]. The values of added mass coincide in both solutions. The added mass of the first approximation in this paper and in Stinson's approximation is negative, therefore is inconsistent with the true value.

Fluid equations

A viscous and heat-conducting fluid in the cylindrical shell of arbitrary cross-section is investigated. If fluid velocity $\boldsymbol{V}$ is small, then the linearized Navier-Stokes equation is

$$\rho_0 \frac{\partial \boldsymbol{V}}{\partial t} + gradP = \mu_0 \Delta \boldsymbol{V} + \frac{\mu_0}{3} graddiv\boldsymbol{V} , \qquad (1)$$

where ρ_0 is undisturbed fluid density, P - fluid pressure, μ_0 - absolute fluid viscosity. The mass continuity equation and the equation of state for the ideal gas are

$$\frac{\partial \rho_v}{\partial t} = -\rho_0 div\boldsymbol{V} , \qquad (2)$$

$$\frac{1}{P_0}\frac{\partial P}{\partial t} = \frac{1}{\rho_0}\frac{\partial \rho_v}{\partial t} + \frac{1}{T_0}\frac{\partial T}{\partial t}, \qquad (3)$$

where ρ_v and T are the acoustical density and temperature, T_0 and P_0 are the equilibrium density and the equilibrium pressure respectively. Equation, describing thermal conduction in the fluid is

$$\kappa \Delta T = \rho_0 T_0 \frac{\partial S}{\partial t}, \qquad (4)$$

where κ is the thermal conductivity, S – entropy. Equation

$$\frac{\partial S}{\partial t} = \frac{C_V}{P_0}\frac{\partial P}{\partial t} - \frac{C_P}{\rho_0}\frac{\partial \rho_v}{\partial t} \qquad (5)$$

concludes the complete system of equations. The ratio of specific heats $\gamma = C_P / C_V$ is one of the fundamental non-dimensional constants of the solution.

If the wave propagation is considered to be adiabatic, then S is constant and from Eq. 5

$$\frac{\partial P}{\partial \rho_v} = \frac{P_0}{\rho_0}\frac{C_P}{C_V} = c_0^2 \text{ and } \rho_0 c_0^2 = \gamma P_0,$$

where c_0 is the adiabatic undisturbed velocity of sound. In this case Eq. 3 gives distribution of temperature T in the wave. If entropy S is not constant, the distribution of temperature is to be calculated from the boundary value problem.

Stinson [14] discussed sound propagation in both narrow and wide tubes and derived approximation from the exact Kirchhoff solution for rigid circular tubes. In his generalization for tubes of arbitrary cross-section shape Stinson offered three simplifications of the solution: 1) the sound pressure p can be assumed as being constant in the cross-section of the tube, 2) the excess density ρ and sound pressure p are of comparable magnitude $\rho/\rho_0 \approx p/P_0$, 3) the second axial derivative of the axial velocity and temperature is negligible.

Eq. 1 can be presented as

$$\mu_0 \Delta \boldsymbol{V} - \rho_0 \frac{\partial \boldsymbol{V}}{\partial t} = grad\left(P - \frac{\mu_0}{3} div\boldsymbol{V} \right),$$

and then with respect to Eq. 2 and the second Stinson assumption it can be proved that $\left|\mu_0 div\boldsymbol{V}\right| << |P|$.

The complex quantities p, ρ, ρ and $\boldsymbol{v}$ can be introduced through

$$\begin{cases} P = P_0 + \mathrm{Re}\left[pe^{i(\omega t - kz)}\right] \\ \rho_v = \rho_0 + \mathrm{Re}\left[\rho e^{i(\omega t - kz)}\right] \\ T = T_0 + \mathrm{Re}\left[\tau e^{i(\omega t - kz)}\right] \\ \boldsymbol{V} = \mathrm{Re}\left[\boldsymbol{v} e^{i(\omega t - kz)}\right] \end{cases} \qquad (6)$$

where p, ρ , ρ and $\boldsymbol{v}$ are the functions of the cross-section coordinates only, the z axis coincides with the direction of wave propagation (Fig. 1), ω - frequency, k – wave number. Now, Eq. 1 gives

$$\nu \Delta_c v_z = i\omega v_z - \frac{ik}{\rho_0} p \qquad (7)$$

and

$$\begin{cases} \nu \Delta_c v_x = i\omega v_x + \dfrac{1}{\rho_0}\dfrac{\partial p}{\partial x} \\ \nu \Delta_c v_y = i\omega v_y + \dfrac{1}{\rho_0}\dfrac{\partial p}{\partial y} \end{cases} \qquad (8)$$

where $\Delta_c = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is Laplace operator in cross-section plane (Stinson assumption 3), $\nu = \mu_0/\rho_0$.

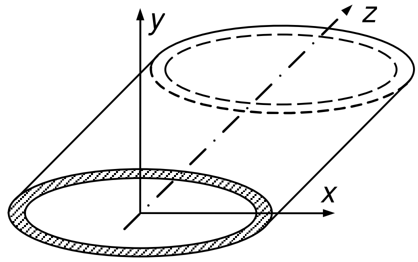

Fig. 1. **A cylindrical tube of arbitrary cross-sectional shape**

The first Stinson's assumption that p is a constant in the cross-section can be applied for the longitudinal motion and the longitudinal heat propagation, but not for the fluid motion in the cross-section of the tube. The pressure derivatives in Eq.8 can be small, but the values of $\rho_0 \omega v_x$, $\rho_0 \omega v_y$ are small too and, thus, can not be ignored.

If $\partial \rho_v / \partial t$ from Eq. 3 is substituted into Eq. 5, then equation

$$\kappa \Delta_c \tau = i\omega \rho_0 C_P \tau - i\omega p \qquad (9)$$

can be deduced from Eqs. 4, 5 and 6. If $p = p_0$ is assumed the constant in Eqs. 7 and 9, then two boundary value problems for $v_z = v_z(x, y)$ and $\tau = \tau(x, y)$ can be solved with $\tau = 0$ and $v_z = v_{z0}$ on the boundary. The tube velocity $v_{z0} = 0$ if tube longitudinal displacements are neglected.

If $\partial \rho_v / \partial t$ from Eq. 3 is substituted into Eq. 2 and Eq.6 are used then

$$\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}=ikv_z+i\omega\left(\frac{\tau}{T_0}-\frac{p}{P_0}\right). \qquad (10)$$

There are two equations in system (8), and Eq. 10 with unknown functions $v_x=v_x(x,y)$, $v_y=v_y(x,y)$, $p=p(x,y)$. If terms $\nu\Delta_c v_x$, $\nu\Delta_c v_y$ are ignored in Eq. 8, then the values of v_x, v_y from Eq. 8 can be substituted to Eq. 10. The non-homogenous Helmholtz equation for one unknown function $p(x,y)$ is obtained, but the boundary conditions require v_x, v_y functions to be solved. If p is substituted from Eq. 10 to Eq. 8 a system of two partial derivative equations is deduced. Solution of these equations depends on boundary conditions, and boundary conditions depend on the solution of the shell dynamic equations. However, general solution of the shell with arbitrary cross-section is complicated, and only numerical analysis can be suggested in the present state of investigation.

Waves in rigid circular tube

In the case of a circular tube of radius r_0 Eq. 7 and Eq. 9 can be given by

$$\begin{cases}\nu\left(u''+\dfrac{u'}{r}\right)=i\omega u-\dfrac{ik}{\rho_0}p,\\ \kappa\left(\tau''+\dfrac{\tau'}{r}\right)=i\omega\rho_0 C_P\tau-i\omega p,\end{cases} \qquad (11)$$

where the longitudinal velocity magnitude $u=u(r)$ and the excess temperature $\tau=\tau(r)$ are functions of the radial distance r only. If $p=p_0$ is the constant, then solutions, satisfying zero boundary conditions at the $r=r_0$, are

$$\begin{cases}u(r)=\dfrac{p_0 k}{\rho_0\omega}\left[1-\dfrac{J_0(\alpha_1 r)}{J_0(\alpha_1 r_0)}\right],\\ \tau(r)=\dfrac{p_0}{\rho_0 C_P}\left[1-\dfrac{J_0(\alpha_2 r)}{J_0(\alpha_2 r_0)}\right],\end{cases} \qquad (12)$$

where $J_0(z)$ is the Bessel function of the zero order, $\alpha_1=\sqrt{\dfrac{\omega}{\nu i}}=(i-1)\sqrt{\dfrac{\omega}{2\nu}}$, $\alpha_2=\sqrt{\dfrac{\omega\rho_0 C_P}{\kappa i}}=(i-1)\sqrt{\dfrac{\omega\rho_0 C_P}{2\kappa}}$.

Eq. 8 are replaced by one

$$\nu\frac{1}{r}(rq')'=i\omega q+\frac{1}{\rho_0}\frac{dp}{dr}, \qquad (13)$$

where $q=q(r)$ is the radial velocity of the fluid. The equation of continuity (10) is now

$$\frac{\gamma}{\rho_0}p=C_P(\gamma-1)\tau+\frac{c_0^2}{c}u-\frac{c_0^2}{i\omega}\frac{(rq)'}{r}, \qquad (14)$$

where relation, valid for ideal gas, $C_P-C_V=P_0/(\rho_0 T_0)$ is used. The principal equation for the radial velocity is obtained, when pressure p from Eq. 14 is substituted into Eq. 13

$$(1+i\varepsilon_0)\left(q''+\frac{q'}{r}\right)+e_0^2 q-\frac{q}{r^2}=i\frac{\omega}{c}u'+iC_P\omega\frac{\gamma-1}{c_0^2}\tau', \qquad (15)$$

where $\varepsilon_0=\gamma\nu\omega/c_0^2$, $e_0=\sqrt{\gamma}\,\omega/c_0$ and u', τ' are derivatives from Eq. 12. The non-dimensional constant $\varepsilon_0=\gamma\Omega^2/s^2<<1$ in most practical cases and can be ignored in Eq. 15. Eq. 11 also have small constants ν and κ, but there is a principal difference between Eq. 11 and Eq. 15. Perturbation of Eq. 11 by ν and κ is singular, while perturbation of Eq. 15 is not singular. If constants ν, κ are assumed equal to zero in Eq. 11, then the degrees of the differential equations are reduced and the solutions lose possibilities to satisfy all boundary conditions. Assuming that $\varepsilon_0=0$, the degree of the differential Eq. 15 remains the same. For the same reason, solving the system of Eqs. 8, 10, it can be assumed that $\nu=0$ in Eq. 8, but that is unacceptable in Eq. 7.

A general solution of non-homogenous Eq. 15 when $\varepsilon_0=0$ is

$$q(r)=AJ_1(e_0 r)+BY_1(e_0 r)+p_0C_1\frac{J_1(\alpha_1 r)}{J_0(\alpha_1 r_0)}+p_0C_2\frac{J_1(\alpha_2 r)}{J_0(\alpha_2 r_0)}, \qquad (16)$$

where

$$C_1=\frac{\omega}{\rho_0 c^2}\frac{i\alpha_1}{\alpha_1^2-e_0^2},\quad C_2=(\gamma-1)\frac{\omega}{\rho_0 c_0^2}\frac{i\alpha_2}{\alpha_2^2-e_0^2}. \qquad (17)$$

The limit $\lim\limits_{r\to 0}Y_1(e_0 r)\to\infty$, so B = 0. The constant A is determined by the second boundary condition $q(r_0)=0$, so the solution of Eq. 15, satisfying both boundary conditions, is

$$q(r)=p_0C_1F_1(r)+p_0C_2F_2(r), \qquad (18)$$

where

$$F_j(r)=\frac{J_1(\alpha_j r_0)}{J_0(\alpha_j r_0)}\frac{J_1(e_0 r)}{J_1(e_0 r_0)}-\frac{J_1(\alpha_j r)}{J_0(\alpha_j r_0)},\ j=1,2.$$

The solution (18) has to satisfy Eq. 14. There are $\tau(r_0)=0$, $u(r_0)=0$ and $(rq)'/r=(q/r)+q'$ when $r=r_0$, because $q(r_0)=0$. So, $i\omega\gamma p_0=-\rho_0 c_0^2 q'(r_0)$.

The derivative $q'(r_0)$ can be determined from Eq. 18, therefore

$$\frac{c_0^2}{c^2}\frac{\alpha_1^2}{\alpha_1^2-e_0^2}\left[1-\frac{e_0}{\alpha_1}\frac{G(\alpha_1 r_0)}{G(e_0 r_0)}\right]=\gamma-(\gamma-1)\frac{\alpha_2^2}{\alpha_2^2-e_0^2}\left[1-\frac{e_0}{\alpha_2}\frac{G(\alpha_2 r_0)}{G(e_0 r_0)}\right], \qquad (19)$$

where $G(z)=J_1(z)/J_0(z)$. This equation can be considered a dispersion equation. The product $e_0 r_0=\sqrt{\gamma}\,\omega r_0/c_0=\sqrt{\gamma}\,\Omega$, where the reduced frequency $\Omega=\omega r_0/c_0$ is usually a small number. If Ω^2 can be ignored with respect to 1, then $G(e_0 r_0)\approx e_0 r_0/2$ and all the more $e_0^2<<\alpha_1^2$, $e_0^2<<\alpha_2^2$, so Eq. 19 can be transformed to

$$k^2\frac{c_0^2}{\omega^2}\left[1-\frac{2}{\alpha_1 r_0}G(\alpha_1 r_0)\right]=1+2\frac{\gamma-1}{\alpha_2 r_0}G(\alpha_2 r_0). \qquad (20)$$

This approximation exactly coincides with Stinson's approximation (43) in [14]. The ways of deduction of Eq. 20 by Stinson and in his paper are different. According to Stinson's procedure the average of the continuity Eq. 10 is

$$\frac{k}{\omega}\langle u\rangle = \frac{p_0}{P_0} - \frac{\langle\tau\rangle}{T_0},$$

where the average of some value $\xi = \xi(x, y)$ is defined as

$$\langle\xi\rangle = A^{-1}\oint \xi dA,$$

A – the cross-section area. From the flow symmetry $\langle v_x\rangle = \langle v_y\rangle = 0$ follows and the value of pressure $p = p_0 =$ const is used. After integration of Eq.12 the same Eq. 20 is found. But this procedure is inapplicable for elastic tubes, because $\langle v_x\rangle = \langle v_y\rangle = 0$ remains true when the deformation of the shell is symmetric. Therefore, the dispersion equation remains the same Eq.20 for deformable circular tubes – an obviously wrong outcome. Why does the average of the continuity Eq. 10 give acceptable approximation of dispersion equation only for rigid pipes? Can this procedure be improved to be acceptable for deformable shells? In any case, constant pressure premise for the longitudinal motion and thermal equations, but not for the cross-sectional movement equation, gives the same approximation as an average procedure for rigid pipes and it also enables to apply elastic boundary conditions.

The Bessel functions of solution (12) can be expressed by Kelvin functions

$$J_j(\alpha_1 r_0) = ber_j\sqrt{\frac{\omega}{\nu}}r_0 + i\cdot bei_j\sqrt{\frac{\omega}{\nu}}r_0,$$

j = 0, 1, and have no limits when $\nu \to 0$. If asymptotic expansion of the Kelvin functions [15]

$$ber_j z = \frac{e^{z/\sqrt{2}}}{\sqrt{2\pi z}}\cos\left(\frac{z}{\sqrt{2}} + \frac{\pi j}{2} - \frac{\pi}{8}\right),$$

$$bei_j z = \frac{e^{z/\sqrt{2}}}{\sqrt{2\pi z}}\sin\left(\frac{z}{\sqrt{2}} + \frac{\pi j}{2} - \frac{\pi}{8}\right)$$

is applied, the asymptotic formula

$$G(\alpha_j r_0) = i,\ j = 1, 2,$$

when $\nu \to 0$, $\kappa \to 0$ can be proved. Thus from Eq. 20 the wave number can be deduced for $|\alpha_j r_0| >> 1$:

$$k = \frac{\omega}{c_0} + \frac{i-1}{c_0 r_0}\sqrt{\frac{\omega}{2}}\left[\sqrt{\nu} + (\gamma - 1)\sqrt{\frac{\kappa}{\rho_0 C_P}}\right]. \quad (21)$$

The value *Imk* describes wave attenuation in the tube and this outcome coincides with the formula given by Landau in [16, §79].

Conclusions

In this study any fluid (i.e. gas or liquid) is appropriate if it is, with some degree of accuracy, represented by the linearized motion equation and the equation of state for the ideal gas. The area of application of the solutions, suggested in this paper, is not less than in Stinson's approximation: to narrow and wide tubes, and to the study of the acoustic properties of porous materials. Possibility to satisfy not only the rigid, but elastic shell boundary condition also, extends the area of application. Propagation of the acoustic waves in pipes is a fundamental problem, and it can be applied in the field of mechatronics, too.

The developed equations, presented in this paper, are valid for an arbitrary shape of the cross-section, but the solution is presented only for a circular cross-section as in the Kirchhoff solution.

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Bangos, sklindančios klampiame ir šilumai laidžiame skystyje standžiame apskritame vamzdyje

Reziumė

Bangų sklidimą klampiame šilumai laidžiame skystyje Kirchofo teorija aprašo tuo atveju, kai skystis yra standžiame apskritame vamzdyje. Kirchofo sprendimas labai sudėtingas, todėl yra pasiūlyti įvairūs praktiniai artiniai. Šiame straipsnyje panaudoti skysčio judesio lygčių suprastinamai jau yra taikyti ankstesniuose tyrimuose, bet viena iš principinių prielaidų apie pastovų slėgį vamzdžio skerspjūvyje netinka lygčiai, aprašančiai skysčio judėjimą vamzdžio skerspjūvio plokštumoje. Gautas standaus apskrito vamzdžio sprendinys, tiriami kai kurie artiniai.

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