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Wave propagation in elastic cylindrical tubes with viscous and heat-conducting fluid

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Introduction

The propagation of sound waves in rigid cylindrical tubes and viscous, heat conducting fluid is investigated by Kirchhoff and some other researchers. In [1] simplifications of the fluid equations and solution for the rigid tube are presented. The purpose of this study is to investigate the viscous heat-conducting fluid in deformable pipes, therefore the interaction of the fluid with the elastic pipe is a principal problem in this paper: the acoustics within the tube depend on the vibration of the tube walls, the vibration of the tube walls depends on the pressure driving the walls.

Waves in elastic circular tube

When longitudinal bending is neglected, symmetrical tube deformations can be described by equations [2]

$$E'h\left(\frac{\partial^2 \eta_v}{\partial z^2}+\frac{\mu}{r_0}\frac{\partial \xi_v}{\partial z}\right)=\rho_h h\frac{\partial^2 \eta_v}{\partial t^2}-\mu_0\frac{\partial \dot{\eta}_v(r_0)}{\partial r},$$

$$\frac{E'h}{r_0}\left(\mu\frac{\partial \eta_v}{\partial z}+\frac{\xi_v}{r_0}\right)=-\rho_h h\frac{\partial^2 \xi_v}{\partial t^2}+P-2\mu_0\frac{\partial \dot{\xi}_v(r_0)}{\partial r},$$

where z, r are the cylindrical coordinates, ξ_V, η_V are radial and longitudinal displacements of the tube, $E'=E/\left(1-\mu^2\right)$, E is Young's modulus, μ is the Poisson ratio, μ_0 is absolute fluid viscosity, h is the tube thickness, ρ_h is the density of the tube, r_0 - tube radius. If $\dot{\xi}_v=q(r_0)e^{i(\omega t-kz)}$, $\dot{\eta}_v=u(r_0)e^{i(\omega t-kz)}$, where ω - frequency, k – wave number, then equations of the tube are

$$\begin{cases} E'h\left(k^2u_0+\dfrac{\mu}{r_0}kiq_0\right)=m_0\omega^2u_0+\mu_0\omega i\dfrac{\partial u(r_0)}{\partial r}, \\ \left(\dfrac{E'h}{r_0^2}-m_0\omega^2\right)q_0-\mu ki\dfrac{E'h}{r_0}u_o=i\omega p_0, \end{cases} \tag{1}$$

where $q_0=q(r_0)$, $u_0=u(r_0)$, $m_0=\rho_h h$ and the term $2\mu_0\frac{\partial \dot{\xi}_v(r_0)}{\partial r}$ is neglected in the second equation. The solution of this system of algebraic equations is

$$u_0=i\mu_r q_0\,,\ q_0=\frac{p_0}{R(\omega)}, \tag{2}$$

where

$$\mu_r=\frac{\dfrac{\mu}{kr_0}+b_\tau w_1}{\dfrac{m_0c^2}{E'h}-1-b_\tau w_2},$$

$$i\omega R(\omega)=\frac{E'h}{r_0^2}\left(1+\frac{\mu^2}{\dfrac{m_0c^2}{E'h}-1}\right)-m_0\omega^2,$$

$$w_1=kh\frac{\rho_h}{\rho_0}\left(\frac{E'h}{m_0r_0^2\omega^2}-1\right),$$

$$w_2=1+\mu\frac{E'h}{\rho_0c^2r_0}.$$

Non-dimensional parameter

$$b_\tau=\frac{\mu_0\omega}{E'hk^2}\alpha_1 i\frac{J_1(\alpha_1r_0)}{J_0(\alpha_1r_0)}\approx(1-i)\frac{\rho_0c^2}{E'h}\sqrt{\frac{\nu}{2\omega}}.$$

The approximate value of b_τ is obtained by applying an asymptotic formula as in [1], and α_j are determined as in Eq. 12, [1].

Equations of viscous and heat-conducting fluid for an elastic tube are the same as for a rigid tube, but boundary conditions are different. The solution of the longitudinal fluid velocity Eq. 11 in [1] with condition Eq. 2 is

$$u(r)=\frac{kp_0}{\rho_0\omega}\left[1-\frac{J_0(\alpha_1r)}{J_0(\alpha_1r_0)}\right]+\mu_r i\frac{J_0(\alpha_1r)}{J_0(\alpha_1r_0)}q_0\,. \tag{3}$$

A new, more precise solution can be found if values of τ' and u' in Eq. 15 are substituted from Eq. 12 in [1] and Eq. 3, where variation of p is taken into account:

$$u'=\frac{kp'}{\rho_0\omega}f_1(r)+\frac{kp}{\rho_0\omega}f_1'(r), \tag{4}$$

$$f_j(r)=1-a_\tau\frac{J_0(\alpha_jr)}{J_0(\alpha_jr_0)},\ \mathrm{j}=1,2 \tag{5}$$

$$a_\tau=1-\frac{\mu_r i}{R(\omega)}\rho_0c\ \text{ for }\ j=1\,,\ a_\tau=1\ \text{ for }\ j=2\,.$$

The solution of the differential Eq. 15 in [1] with u' from Eq. 4 is very complicated. The exact value of the derivative Eq. 4 can be replaced by approximate

$$u'=\frac{k\Theta_1}{\rho_0\omega}p'+\frac{kp_0}{\rho_0\omega}f_1'(r), \tag{6}$$

where Θ_1 is some constant, for example, average over cross- sectional area: $\Theta_1 = \langle f_1(r)\rangle$. It can be evaluated from Eq. 3

$$\Theta_1 = 1 - a_\tau \frac{2}{r_0}\sqrt{\frac{\nu i}{\omega}} .$$

The differences between Eq. 4 and Eq.6 are: 1) replacement of the pressure $p = p(r)$ by $p_0 = \text{const}$ and 2) replacement of the function $f_1(r)$ by the constant Θ_1. The first approximation of the solution presented in Eqs. 16–20 in [1] can now be obtained by assuming $\Theta_1 = 0$ in Eq. 6. The derivative of the excess temperature

$$\tau' = \frac{\Theta_2}{\rho_0 C_P} p' + \frac{p_0}{\rho_0 C_P} f_2{}'(r), \qquad (7)$$

where $f_2(r)$ is given by Eq. 5 when $j = 2$ and

$$\Theta_2 = \langle f_2(r)\rangle = 1 - \frac{2}{r_0}\sqrt{\frac{\kappa i}{\omega\rho_0 C_P}} .$$

When Eqs. 6 and 7 are used in Eq. 14 in [1], a new equation of the radial velocity can be deduced

$$\left[1 + i\frac{\varepsilon_0}{\gamma}\left(\Theta_3 - \Theta_1\frac{c_0^2}{c^2}\right)\right]\left(q'' + \frac{q'}{r}\right) - \frac{q}{r^2} - e_1 q$$

$$= p_0 \frac{i\omega}{\rho_0 c_0^2}\left[\frac{c_0^2}{c^2} f_1{}'(r) + (\gamma - 1) f_2{}'(r)\right], \qquad (8)$$

where $\Theta_3 = \gamma(1 - \Theta_2) + \Theta_2 = \text{const}$,

$$e_1 = \frac{\omega}{c_0}\sqrt{\Theta_1\frac{c_0^2}{c^2} - \Theta_3} . \qquad (9)$$

The solution of Eq. 8 with $\varepsilon_0 = 0$, satisfying boundary condition Eq. 2, is

$$q(r) = p_0 D_1 F_1^*(r) + p_0 D_2 F_2^*(r) + \frac{p_0}{R(\omega)}\frac{I_1(e_1 r)}{I_1(e_1 r_0)}, \qquad (10)$$

where

$$\begin{cases} D_1 = \dfrac{\omega}{\rho_0 c^2}\dfrac{i\alpha_1}{\alpha_1^2 + e_1^2}\left(1 - i\mu_r\dfrac{\rho_0 c}{R(\omega)}\right), \\ D_2 = (\gamma - 1)\dfrac{\omega}{\rho_0 c_0^2}\dfrac{i\alpha_2}{\alpha_2^2 + e_1^2}, \end{cases} \qquad (11)$$

$$F_j^*(r) = \frac{J_1(\alpha_j r_0)}{J_0(\alpha_j r_0)}\frac{I_1(e_1 r)}{I_1(e_1 r_0)} - \frac{J_1(\alpha_j r)}{J_0(\alpha_j r_0)},\ j = 1, 2, \qquad (12)$$

$I_1(z)$ is the modified Bessel function.

The dispersion equation may be written in form

$$k^2\frac{c_0^2}{\omega^2}\frac{1 + \mu\zeta^2\Psi}{1 + e_1^2\alpha_1^{-2}}\left[1 - \frac{e_1}{\alpha_1}\frac{G_1(\alpha_1 r_0)}{G_1(e_1 r_0)}\right]$$

$$= \gamma - \frac{\gamma - 1}{1 + e_1^2\alpha_2^{-2}}\left[1 - \frac{e_1}{\alpha_2}\frac{G_1(\alpha_1 r_0)}{G_1(e_1 r_0)}\right]$$

$$+ \left[\mu + \left(\beta_S\zeta^2 - 1\right)\frac{e_1 r_0}{G_1(e_1 r_0)}\right]\Psi , \qquad (13)$$

where $G_1(z) = I_1(z)/I_0(z)$, $\zeta = c/c_0$, $\Psi = \beta\frac{r_0}{h}\left[\mu^2 + \left(1 - \beta_S\Omega^2\right)\left(\beta_S\zeta^2 - 1\right)\right]^{-1}$ and non-dimensional parameters of a shell $\beta = \rho_0 c_0^2/E'$, $\beta_0 = m_0/\rho_0 r_0$ – coupling parameter [2], $\beta_S = \beta\beta_0 r_0/h$. For most practical cases only two terms of $e_1/G_1(e_1 r_0)$ series are significant, so

$$\frac{e_1}{G_1(e_1 r_0)} \approx \frac{2}{r_0}\left(1 + \frac{e_1^2 r_0^2}{8}\right). \qquad (14)$$

If, in addition,

$$\frac{1}{r_0}\sqrt{\frac{\nu}{\omega}} << 1,\ \frac{1}{r_0}\sqrt{\frac{\kappa}{\omega\rho_0 C_P}} << 1 ,$$

then $G(\alpha_j r_0) = i$ can be assumed. Neglecting $e_1^2\alpha_j^{-2}$ dispersion Eq. 13 can be presented

$$\left(1 - \beta_S\Omega^2 + \frac{\mu^2}{\beta_S^2\zeta^2 - 1}\right)\left[(1 - Y_1)\zeta^{-2} - 1 - (\gamma - 1)Y_2\right]$$

$$= 2\beta\frac{r_0}{h}\left(1 + \frac{e_1^2 r_0^2}{8} + \frac{0.5\mu Y_1}{\beta_S\zeta^2 - 1}\right), \qquad (15)$$

where

$$Y_1 = \frac{1 - i}{r_0}\sqrt{\frac{2\nu}{\omega}}\left(1 + \frac{e_1^2 r_0^2}{8}\right) \approx \frac{1 - i}{r_0}\sqrt{\frac{2\nu}{\omega}} ,$$

$$Y_2 = \frac{1 - i}{r_0}\sqrt{\frac{2\kappa}{\omega\rho_0 C_P}}\left(1 + \frac{e_1^2 r_0^2}{8}\right) \approx \frac{1 - i}{r_0}\sqrt{\frac{2\kappa}{\omega\rho_0 C_P}} .$$

The parameter e_1 depends on the constants Θ_1 and Θ_2. If $\Theta_1 = \Theta_2 = 0$, then derivative Eqs. 6 and 7 give the first approximation of the solution in which $\Theta_3 = \gamma$ and $e_1 = i\sqrt{\gamma}\,\omega/c_0 = ie_0$. The solutions (10)–(12) can be transformed to the solutions (17)–(18) in [1], Eq. 14, the second term of the series, can be interpreted as added mass of the fluid [3]. When the value $e_1 = ie_0$ is used, the second term of the series $e_0/G(e_0 r_0)$ in Eq. 19 [1], gives a negative fluid added mass. However, this is not true, and only the first approximation of this expression is valid.

If $\nu \to 0$, $\kappa \to 0$, then $\Theta_1 \to 1$, $\Theta_2 \to 1$, $\Theta_3 \to 1$. When $\Theta_1 = \Theta_3 = 1$, then from Eq.9 $e_1^2 = k^2 - \omega^2/c_0^2 = \lambda$ [3]. When $Y_1 = Y_2 = 0$ is substituted in Eq. 15 the dispersion equation for ideal fluid and elastic shell is obtained [3, 4]. The fluid added mass is the same in both cases.

Eq. 15 depends on fluid viscosity, thermal conductivity and elasticity of the shell. As real values of Θ_j are $0 < |\Theta_j| < 1$, fluid viscosity and thermal conductivity have some influence on the added mass (the term $e_1^2 r_0^2/8$ in Eq. 15). This may be important for narrow tubes and low frequencies.

When the Young's modulus E is replaced by $E(1 + i\psi/2\pi)$, material damping can also be included [5]. The real number ψ is independent of the frequency ω, but may be dependant on a vibration amplitude.

Solution of the dispersion equation

A very simple approximate formula (21), [1] given by Landau and more complicated formula (20), [1] given in Stinson's investigation [6] are presented for waves in viscous, heat-conducting fluid and a rigid circular tube. This formula can also be deduced from Eq. 19, [1] which is an approximate solution of the differential Eq. 15. Better approximation for rigid tubes can be obtained if derivatives (6) and (8) are used in Eq. 15, [1]:

$$k^2 \frac{c_0^2}{\omega^2} \frac{1}{1+e_1^2\alpha_1^{-2}}\left[1-\frac{e_1}{\alpha_1}\frac{G(\alpha_1 r_0)}{G(e_1 r_0)}\right]$$
$$= \gamma - \frac{\gamma-1}{1+e_1^2\alpha_2^{-2}}\left[1-\frac{e_1}{\alpha_2}\frac{G(\alpha_2 r_0)}{G(e_1 r_0)}\right], \qquad (16)$$

where e_1 is given by Eq. 9. These approximations for attenuation coefficient $\Gamma' = \mathrm{Re}\,\Gamma$ are shown in Fig.1. Approximation of the Kirchhoff solution described in [7] is also plotted. Wave propagation in [7] is expressed by factor $e^{i\omega t - \Gamma\xi}$, where $\xi = \omega z/c_0$, therefore $\Gamma\Omega = ikr_0$, $\Gamma = \Gamma' + i\Gamma''$. All approximations are practically identical when the frequency $f \leq 1000$ Hz. The Stinson's approximation is valid for a tube radius and sound frequency, given by $r_0 > 10^{-3}$ cm and $r_0 f^{3/2} < 10^6$ $\mathrm{cm}\cdot s^{-3/2}$. According to Westons classification [6] this regime includes both narrow and wide tubes. When $r_0 = 1$ cm the upper frequency is 10 kHz and this correlates well with Fig.1. When $f > 10$ kHz some difference can be observed for all approximations, but Landau approximation is still very near Kirchhoff approximation.

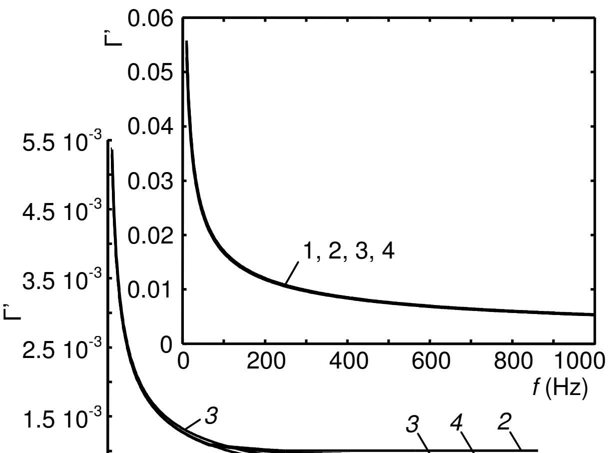

Fig.1. **Dependence of attenuation coefficient Γ' on the wave frequency for air in the rigid circular tube of radius $r_0 = 1$ cm: 1 – Landau approximation [1], 2 – Stinson approximation [6], 3 – Kirchhoff approximation [7], 4 – the second approximation Eq. 16**

Approximation (15) for an elastic tube has two roots for every given frequency $f = \omega/2\pi$. One of the roots strongly depends on the material damping ψ. In Fig. 2 this root is depicted by three approximately horizontal lines. The other three lines for $\psi_1 = 0.5$, $\beta_S\Omega^2 << 1$, $\psi_1 = 0.15$ practically coincide and decrease when frequency increases. These lines present attenuation of the predominantly fluid deflection wave, while the three horizontal lines present the predominantly pipe deflection wave. If fluid is absent, then

$$\Gamma^2 = -\beta_S \frac{1-\beta_S\Omega^2}{1-\mu^2-\beta_S\Omega^2}.$$

If $\beta_S\Omega^2 << 1$ and material damping is taken into account, then

$$\Gamma \approx i\sqrt{\frac{\beta_S}{1-\mu^2}}\left(1-\frac{\psi}{4\pi}i\right).$$

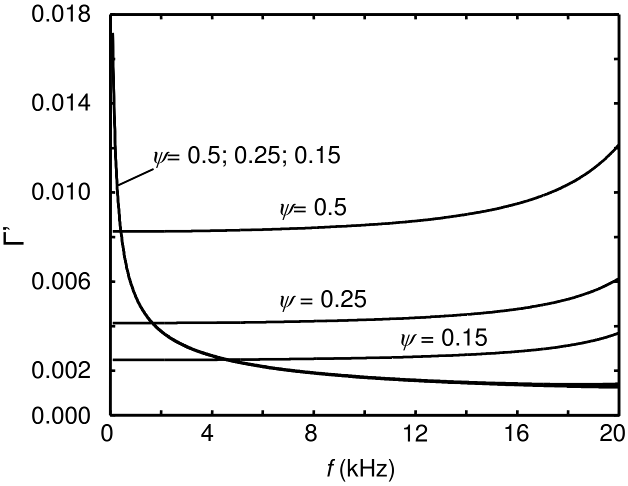

Fig.2. **Dependence of attenuation coefficient Γ' on the wave frequency for air in the elastic tube of radius $r_0 = 1$ cm, *h* = 0.1cm, $E = 3.2\cdot10^5$ N/cm², $\mu = 0.35$ and different material damping ψ. Two different waves can propagate at the same frequency *f*: predominant fluid deflection wave (three lines coincide for different ψ) and predominant pipe deflection wave (three different lines for $\psi = 0.5$; $\psi = 0.25$; $\psi = 0.15$)**

When the fluid is in the tube every longitudinal deflection of the tube is related to the radial deflection of the tube and both deflections are related to the deflection of the fluid. So, the predominantly fluid wave is supplemented by small tube deflections and the predominantly tube wave is supplemented by small fluid deflections. When $f \approx 2.6\cdot10^4$ Hz, both roots of the equation have peaks of the attenuation (Fig. 3). This crisis of the wave propagation is connected with the oscillation amplitudes. The ratio of the tube longitudinal and radial deflection amplitudes in vacuum

$$\xi_{rv} = \left|\frac{u_0}{q_0}\right| = \frac{1}{\mu}\sqrt{\frac{\left|\left(1-\beta_S\Omega^2\right)\left(1-\mu^2-\beta_S\Omega^2\right)\right|}{\beta_S\Omega^2}}.$$

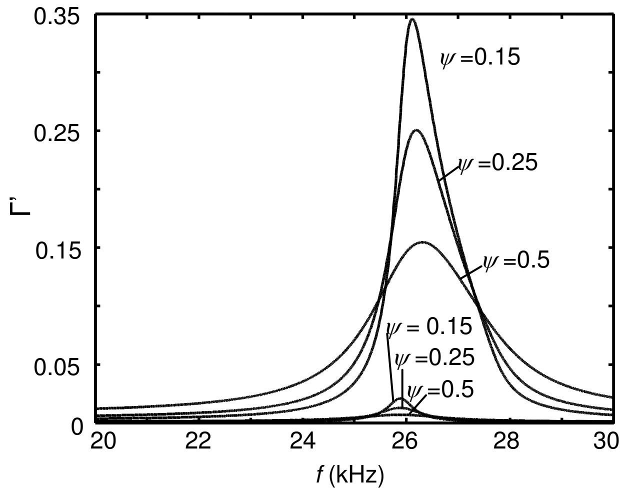

Fig. 3. **Dependence of Γ' on high frequencies *f* for the same tube as in Fig. 2. Increasing of Γ' at some frequency is crisis of the wave propagation**

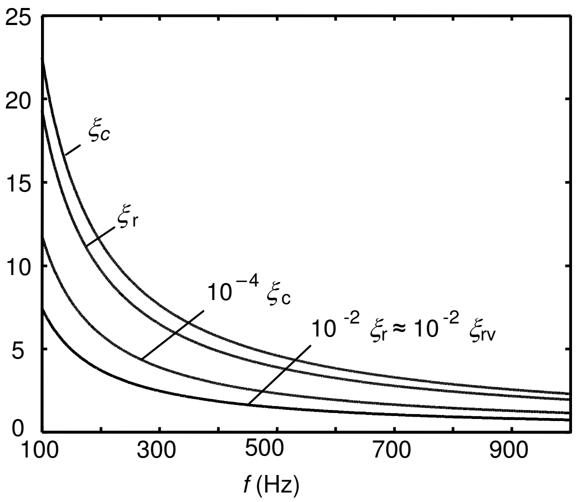

Fig. 4. **Dependence of ratios $\xi_c = |u(0)/q_0|$, $\xi_r = |u_0/q_0|$ for the same elastic tube with air inside as in Fig. 2 and $\xi_{rv} = |u_0/q_0|$ for the elastic tube in vacuum on frequency *f* of the waves. Two different waves and different values of ξ_c, ξ_r can be evaluated for every *f***

If the elasticity modulus E is a complex number, then ξ_{rv} has its minimal value when $\Omega = \sqrt{(1-\mu^2)/\beta_S}$, so $f_c = 2.6 \cdot 10^4$ Hz. All calculations in Fig. 2 – Fig. 5 are for the polymer tube with $r_0 = 1$ cm, h=0.1 cm, $E = 3.2 \cdot 10^5$ N/cm^2, the Poisson ratio $\mu = 0.35$ and air inside the tube. The ratios of the tube longitudinal deflection amplitude u_0 and fluid longitudinal deflection amplitude $u(0)$ at the tube center $r = 0$ to the radial deflection amplitude q_0 at the tube and fluid when $r = r_0$ are depicted in Fig. 4 and Fig. 5. These dependences can be deduced from Eq. 2 and 3 when $r = r_0$ and $r = 0$. When a frequency is not high and far from critical value $f_c = 2.6 \cdot 10^4$ Hz, then all the ratios $\xi_c = |u(0)/q_0|$, $\xi_{rv} = |u_0/q_0|$ decrease monotonically. For the predominantly fluid deflection wave, the longitudinal deflection amplitude of the fluid is significantly greater than the radial deflection amplitude. When frequencies are high, the dependence of the fluid amplitude in the tube center $r = 0$ is complicated (Fig. 5). Sufficiently accurate coincidence $\xi_{rv} \approx \xi_r$, where ξ_r is the predominant tube deflection ratio, can be explained by inequality $\rho_0 << \rho_h$ for air and polymer.

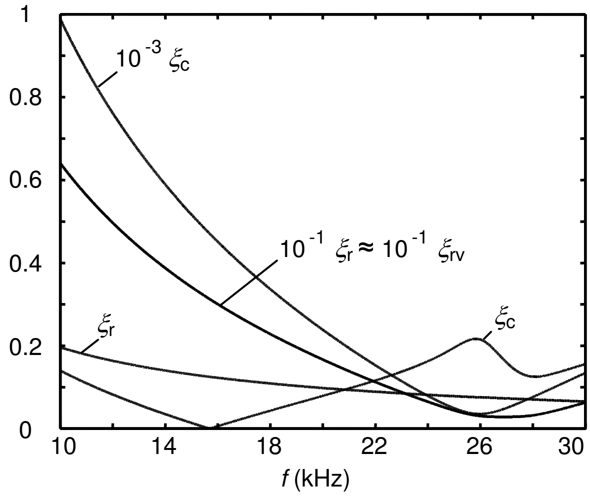

Fig. 5. **Dependence of ratios ξ_c, ξ_r and ξ_{rv} as in Fig. 4 on high frequencies *f* of the waves. As in Fig. 4 predominant pipe deflection ratio ξ_r practically coincides with ratio ξ_{rv} for the elastic tube in vacuum**

The influence of the viscosity, heat conducting and material damping can be evaluated from Eq. 15. For the fluid predominant wave and f = 10 Hz, $\Gamma' = 5.75 \cdot 10^{-2}$ when all resistances are included, $\Gamma' = 1.79 \cdot 10^{-2}$ when fluid viscosity is neglected, $\Gamma' = 3.84 \cdot 10^{-2}$ when heat conducting in the fluid is neglected and $\Gamma' = 0.0011 \cdot 10^{-2}$ when both viscosity and heat-conducting are neglected. When the frequency $f = 1000$ Hz, the values of Γ' are $0.533 \cdot 10^{-2}$, $0.183 \cdot 10^{-2}$, $0.350 \cdot 10^{-2}$ and $0.0011 \cdot 10^{-2}$ correspondingly. So, viscosity of the fluid is more significant than heat conducting and the role of material damping is small for this wave. But for other tubes, the fluid and frequency influence of these factors can be different.

The boundary condition for the excess temperature is assumed $\tau = 0$ when $r = r_0$, but in reality some alteration of the tube temperature takes place. If heat conductivity is important, this alteration has to be considered.

The longitudinal bending of the tube is neglected in this investigation and this is acceptable when the wave length is not very short. When $f = 5 \cdot 10^5$ Hz, the error of the wavelength calculation in the tube described above is

~0.01%. But if $f = 50 \cdot 10^5$ Hz, longitudinal bending is important for this tube and can not be ignored. In this case the term $D\Delta\Delta\xi_v$, $D = E'h^3/12$ must be included in the dynamic equation of the tube, and the algebraic dispersion equation has a higher degree of $kr_0 = -\Gamma\Omega$, but there are no additional principal problems.

Dispersion Eq. 15 is deduced from Eq. 13 and only two first terms of the Bessel functions series of arguments $e_1 r_0$ are considered. This restriction has its own limits of application, too. So, the general dispersion Eq. 13 with the Bessel functions in it can be applied for any tube radius r_0 and frequency f if $r_0 > 10^{-3}$ cm and $r_0 f^{3/2} < 10^6$ cm s$^{-3/2}$ (regime of both narrow and wide tubes [6]). But the simplified dispersion Eq. 15 is deduced by imposing additional conditions, some of which can be satisfied if wavelength is many times larger than tube radius r_0. Furthermore, the commonly accepted assumptions on the cylindrical shell dynamic Eq. 1 are considered [2].

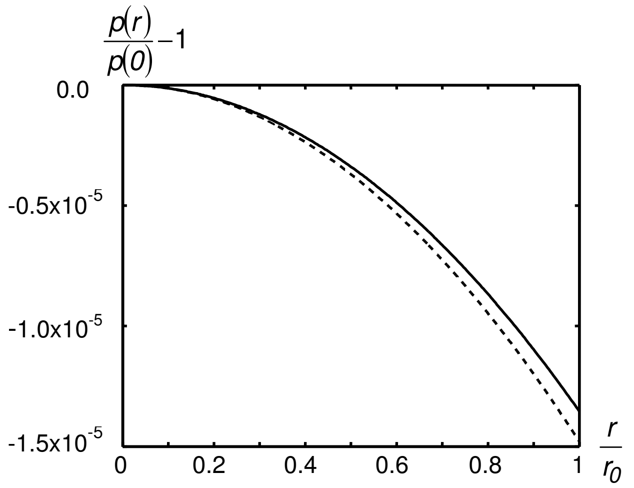

Fig. 6. **Dependence of the fluid pressure function $(p(r)/p(0)) - 1$ on the radial distance r/r_0 when frequency $f = 10000$ Hz and other data as in the Fig. 2. Continuous line for the rigid tube and the elastic tube, when the material damping $\psi = 0$; dotted line for $\psi = 0.5$**

When the dispersion equation is solved the pressure dependence on radial distance $p = p(r)$ can be obtained from Eq. 14 [1]. The values of $u = u(r)$, $\tau = \tau(r)$ and $q = q(r)$ have to be used from [1] or Eq. 10, thereby

$$\gamma \frac{p}{p_0} = \gamma + \frac{c_0^2}{c^2} - 1 - N_c F_c(r) - E_{H1} H_1(r) - E_{H2} H_2(r), \tag{17}$$

where

$$N_c = \left(1 - i\mu_r \frac{\rho_0 c}{R(\omega)}\right) \frac{c_0^2}{c^2} \frac{2}{\alpha_1 r_0} G(\alpha_1 r_0) + (\gamma - 1) \frac{2}{\alpha_2 r_0} G(\alpha_2 r_0) + \frac{2\rho_0 c_0^2}{r_0 i\omega R(\omega)},$$

$$F_c(r) = \frac{e_1 r_0}{2} \frac{I_0(e_1 r)}{I_1(e_1 r_0)} \approx \frac{1 + e_1^2 r^2/4}{1 + e_1^2 r_0^2/8},$$

$$H_j(r) = \frac{J_0(\alpha_j r)}{J_0(\alpha_j r_0)} - \frac{e_1}{\alpha_j} G(\alpha_j r_0) \frac{I_0(e_1 r)}{I_1(e_1 r_0)},$$

$$E_{H1} = \frac{c_0^2}{c^2} \frac{\alpha_1^2}{\alpha_1^2 + e_1^2} \left(1 - i\mu_r \frac{\rho_0 c}{R(\omega)}\right),$$

$$E_{H2} = (\gamma - 1) \frac{\alpha_2^2}{\alpha_2^2 + e_1^2}, \quad j = 1, 2.$$

If $\left|e_1^2/\alpha_j^2\right| << 1$ the two terms $E_{Hj} H_j(r)$ in Eq. 17 can be neglected. Solution for the rigid tube can be deduced if the limit $R(\omega) \to \infty$ is inserted.

If the frequency $f = 10000$ Hz, $r_0 = 1$ cm and all other data for the polymer tube with air inside are the same, then $\left|e_1^2/\alpha_j^2\right| << 10^{-7}$, but $e_1 r_0$ can not be assumed small. The value of $e_1 r_0$ depends on the constants Θ_j, $j = 1, 2$. If $\Theta_1 = \Theta_2 = 0$, then $e_1 r_0 = e_0 r_0 i = 2.19i$. If $\Theta_1 = \Theta_2 = 1$, then $e_1 r_0 = 0.0485 + 0.1172i$. For the averaged values $\Theta_j = < f_j(r) >$ $e_1 r_0 = 0.1066 + 0.1068i$, so in this case $\mathrm{Re}\, e_1 r_0 \approx \mathrm{Im}\ e_1 r_0$. The function $F_c(r)$ and the pressure $p(r)$ also depend on Θ_j. When average values of Θ_j are applied, the pressure function $p(r)/p(0)$ is almost the same for the rigid and the deformable pipe (Fig. 6), but these functions are different when $\Theta_j = 1$ or $\Theta_j = 0$, $j = 1, 2$. It can be seen from Fig. 6 that the pressure function $p(r)/p(0)$ depends considerably on the tube material damping ψ. If $\left|e_1^2 r_0^2\right| << 1$ dependence of the pressure on r is parabola. Note that the first approximation can not be applied if the value of $e_0 r_0$ approaches 2.405, because the function $J_0(2.405) = I_0(2.405i) = 0$ and $F_c(r) \to \infty$. On the whole, may be that any values of the constants Θ_1, Θ_2 are inappropriate if high approximation for the $p(r)$ is required and therefore next, the third, approximation of the solution has to be investigated.

Conclusions

Averaged values over the cross-section of the tube are applied to deduce Stinson approximation of the wave propagation in viscous and heat conducting fluid [6]. This is acceptable for a rigid tube and can not be implemented when elasticity of a tube is taken into account. Examination of the fluid radial motion equation in non-dimensional variables discloses why wave pressure can't be assumed constant in the tube cross-section in this equation, but it can be assumed constant in longitudinal motion and excess temperature equations with high accuracy. The importance of the pressure gradient in a tube cross-section can be seen from the Helmholtz equation for ideal fluid $\Delta_c \varphi = \lambda \varphi$ where the pressure $P = p_c \varphi e^{i(\omega t - kz)}$, $p_c = \text{const}$, $\lambda = k^2 - \omega / c_0^2$ and Δ_c is the Laplace operator in the cross section plane [3]. If $\varphi(x, y) = \text{const}$ and the pressure is constant in the cross-section plane, then $\lambda = 0$, the wave velocity $c = \omega / k = c_0 = \text{const}$ and propagation of the wave is without dispersion. So, accuracy of the radial fluid motion equation is the key to the exact solution of the whole problem.

Dispersion Eq. 13 or 15 presents a solution for the wave propagation in an elastic cylindrical tube with viscous, compressible, heat-conducting fluid inside. For real values of the wave frequency ω, complex wave numbers k (or non-dimensional product $kr_0 = -i\Gamma\Omega$) can be evaluated. Several roots of the equation correspond to different wave velocities and different ratios of longitudinal and radial displacements of the tube and the fluid. All properties of the fluid and elastic tube are related: imaginary part of k (or real part of Γ) defines attenuation of the wave and depends not only on viscosity, heat-conductivity of the fluid and damping in the tube material, but also on the complex wave velocity as a whole.

Linearized fluid equations and the thermal conductivity equation are solved for a circular tube only when fluid and tube velocities are symmetrical with respect to the central longitudinal axis, but the main ideas and assumptions of the paper can be applied to a cylindrical shell of arbitrary cross-section, too.

Thus, by investigating two levels of approximation excellent coincidence with the published expressions for the rigid tube is found. A comparison with the published solutions for a deformable tube and ideal fluid is also carried out.

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Bangos, sklindančios klampiame ir šilumai laidžiame skystyje tampriame apskritame vamzdyje

Reziumė

Bangų sklidimą klampiame šilumai laidžiame skystyje Kirchofo teorija aprašo tuo atveju, kai skystis yra standžiame apskritame vamzdyje. Šiame straipsnyje teikiamas apytikris klampaus, šilumai laidaus skysčio deformuojamame vamzdyje sprendinys. Jis visiškai sutampa su kai kuriais anksčiau skelbtais apytikriais sprendiniais, jei vamzdžio tamprumo nepaisome, ir parodo klampumo bei šilumos laidumo skystyje ir slopinimo vamzdžio medžiagoje poveikį. Detaliai tiriamas gautasis sprendinys ir jau skelbti tampraus vamzdžio su idealiu skysčiu sprendiniai.

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