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# Application of holographic interferometry methods for analysis of piezodrives with ring actuators

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## Introduction

An important feature of mechanisms containing links made from active materials (e.g. piezoceramics) is that very often various functions be performed by the same transducer, enabling the development of methods for designing adjustable or adaptive mechanisms, so that multiple tasks can be performed by the same mechanism, including the ability to accommodate manufacturing and part-positioning errors in order to widen applicability.

The paper deals with the selection of maximal physical parameters for piezodrives having ring actuators, their theoretical investigation and various design solutions.

The type of generated waves depends on the position of electromechanical actuators with respect to an elastic body. The analysis of wave processes in ring actuators has indicated the design solution to open the way for employing their entire operational area. In many cases it has proved to be more reasonable than the use of linear (rod) actuators. Piezodrives with ring actuators are reliably connected to a rotor, thus ensuring the accurate automatic control and increasing the torque.

The experimental investigation of piezosystems by means of holographic interferometry enables one to obtain appreciably larger amounts of information about the vibrating surface in comparison with traditional methods. This investigation deals with consideration of methods for determination of the vibrational characteristics of piezodrives with ring actuators from holographic interferograms of linked analysis of these characteristics carried out by using numerical techniques based on the theories of piezosystem vibration and holographic interferometry.

## Analysis

The section of a ring small curvature is taken for theoretical analysis. Tangential and radial undulating processes are expressed by differential equations of a ring motion:

$$\frac{\partial^6 w}{\partial y^6}+2\frac{\partial^4 w}{\partial y^4}+\frac{\partial^2 w}{\partial y^2}+h\left(\frac{\partial^4 w}{\partial t^2 \partial y^2}-\frac{\partial^2 w}{\partial t^2}\right)=\frac{R^3 \partial P}{EJ \partial y} \quad (1)$$

$$\frac{\partial^2 F}{\partial y^2}+\frac{\partial T}{\partial y}=\frac{\gamma AR}{g}\frac{\partial^3 U}{\partial t^2 \partial y}, \quad (2)$$

where $h=\dfrac{A\gamma R^4}{gEJ}$ $y$ – angular coordinate corresponding to the counterclockwise turning; $t$ – time; $w$ – tangential coordinate ( of displacement); $A$ – cross-section area of a ring; $E$ – modulus of elasticity; $g$ – aceeleration of gravity; $I$ – moment of inertia; $R$ – radius; $\gamma$ - density; $U$ – radial coordinate; $T$ – tangential coordinate, Fig.1.

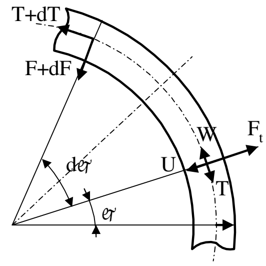


Fig.1. **Forces designed in a ring actuator: *U*-radial coordinate, *T*-tangential coordinate**

The excitation force $P(Y,t)=f(y)\sin\omega t$, where:

$$f(y)=A_0+\sum_{k=1}^{\infty}\left(A_0\cos ky+B_k\sin ky\right).$$

The solutions of Eqs 1 and 2 are written:

$$w(y,t)=\frac{R^3\sin\omega t}{EJ}\sum_{k=1}^{\infty}\frac{k\left(B_k\cos ky-A_k\sin ky\right)}{h\omega^2\left(k^2+1\right)-k^2\left(k^2-1\right)^2}, \quad (3)$$

$$U(y,t)=-\frac{R^3\sin\omega t}{EJ}\sum_{k=1}^{\infty}\frac{k^2\left(A_k\cos ky+B_k\sin ky\right)}{h\omega^2\left(k^2+1\right)-k^2\left(k^2-1\right)^2}. \quad (4)$$

When the exciting forces are subjecting the points $N$, the following expression is obtained:

$$P(y,t)=\sum_{j=1}^{N}c_j\delta_\varepsilon\left(y-y_j\right)\sin\left(\omega t-\theta_j\right) \quad (5)$$

The case of the $N$ points with amplitude $c_j$ and phases displacement $\theta_j$ is applied to the solution of Eqs 1 and 2:

$$U(y,t)=-\frac{R^3}{EJ\pi}\sum_{k=1}^{\infty}\frac{k_2}{h\omega^2\left(k^2+1\right)-k^2\left(k^2-1\right)^2}$$

$$\sum_{j=1}^{N}c_j\sin\left(\omega t-\theta_j\right)\cos k\left(y-y_j\right),$$

$$V=\frac{R\left(k^2-1\right)}{\sqrt{h\left(k^2+1\right)}}\quad \omega=\frac{k\left(k^2-1\right)}{\sqrt{h\left(k^2+1\right)}};\ \mathrm{k}=2,3... \quad (6)$$

The obtained expressions of the linear rate and the resonance frequency of a wave have made it possible to choose more precise parameters for developing piezodrives with ring actuators. The vibration produced in the ring actuator is shown in Fig.2. The surface of a ring drive is machined at the same angle as was the rotor. This simple structure is reliable in operation. The maximal and automatic control enables a considerable increase in the torque.

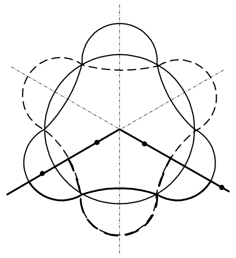

Fig.2. **Mode of vibration in the ring actuator**

As mentioned above, the developed deformations (on the basis of a wave) in a ring actuator causes the rotation of a rotor. We assume *y(t)* as the angular speed of a rotor (at the instant *t*), $v$ – as the linear speed of a wave in a ring actuator. To set up a differential equation for expressing the piezoactuator speed we take some assumptions. The force component $d\tau$ with which the actuator surface element acts at the corresponding rotor element is rectilinearly proportional to the magnitudes of:

– the initial pressure *B* pressing the unit ring drive element in the initial equilibrium state at the corresponding rotor part;

– the contact area $aRd\varphi$, where *a* is the thickness of a ring actuator.

The difference $C - U\ \varphi T$ corresponds to the dynamic clamping of the ring actuator contact surface on the piezodrive, *C* – experimental constant, $C > max\ U\ (t\ \varphi\ )$.

$$\frac{dy}{dt} = \frac{fa}{I}\int_0^{2\pi}(v - Ry)(c - u)d\varphi\ . \tag{7}$$

The angular speed of the rotor is:

$$y(t) = \frac{v}{R}\left(1 - e^{-Pt}\right). \tag{8}$$

The coefficient of proportionality *f* depends on the rotor mass:

$$\tau = faR\int_0^{2\pi}(v - Ry)(c - u)d\varphi \tag{9}$$

and expressing the force $\tau$ by a linear acceleration of the rotor surface $R\frac{dy}{dt}$, the expression becomes:

$$IR\frac{dy}{dt} = faR\int_0^{2\pi}(v - Ry)(c - u)d\varphi\ , \tag{10}$$

$$\frac{dy}{dt} = \frac{fa}{I}\int_0^{2\pi}(v - Ry)(c - u)d\varphi\ . \tag{11}$$

Eq. 9 yields where *I* is the coefficient of proportionality.

We assume that the force of a viscous friction is proportional to the rotor speed *y(t).* The viscous friction force between the rotor and the ring drive proportional to the difference of the wave *v(t)* and the rotor *Ry(t)* is obtained by equalizing to the force which creates the rotor acceleration $\frac{dy}{dt}$.

Then we obtain:

$$\frac{dy}{dt} + \eta y + \eta_1(v - Ry) = k\int_0^{2\pi}(v - Ry)(c - u)d\varphi\ , \tag{12}$$

where $\eta$ and $\eta_1$ are coefficients of friction and $b < 2\pi ck$, in this way:

$$\frac{dy}{dt} = (2\pi ck - \eta)\left[v - \left(R + \frac{\eta_1}{2\pi ck - \eta}\right)y\right], \tag{13}$$

where *I* is the coefficient of proportionality. The final expression is:

$$y = \frac{v}{1 + \dfrac{\eta_1}{R(2\pi ck - \eta)}}\ . \tag{14}$$

The rotational speed can be said depends on the undulating deformation of a driving link. The illustrated phase displacements of the undulating deformation indicate friction dependability in a cinematic pair on the force of tension. It should be noted that reducing the friction force the wave processes tend to recur [1].

The deformation of a ring drive occuring in piezoactuators is regulated and depends on the high frequency voltage of the feeding source, Fig.3. Depending on the regulated speeds (linear or angular) in a ring drive, the respective amplitudinal and angular modulation has been used. The generator has been developed and it works by applying a division frequency signal. It allows to regulate the magnitude of the applied force, to change the number and location of excited points.

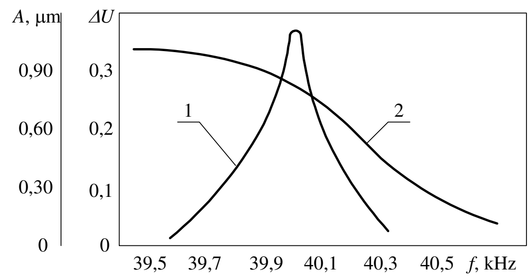


Fig.3. **Dependence of the wavelength (curve 1) and deformation on the voltage (curve 2)**

The experimental investigation has been performed by a holographic interferometric method [2] enabling to watch the picture of deformations taking place in a ring drive which practically come to its conclusions.

The experimental investigation of precision vibrosystems by means of holographic interferometry enables one to obtain appreciably larger amounts of information about the vibrating surface in comparison with traditional methods. The paper deals with the consideration of methods for determination of the vibrational characteristics of precision mechanical systems from the holographic interferograms of linked analysis of these characteristics earned out by using numerical techniques based on the theories of mechanical system vibration and holographic interferometry. A multipurpose device has been developed for storing the holographic interferograms. It allows the application of various methods of holographic interferometry in order to obtain interferograms of excellent quality. When analysing the performance of the wave systems it is necessary to investigate the wave characteristics of the input member, its influence on the other elements of the system. It is very important to calculate the normal and tangential components of the amplitudes of the surface points of the input member. The determination of their values enables to use the traditional laws of the classical mechanical vibration theory, where the initial data for the theoretical calculations are taken from the holographic interferograms.

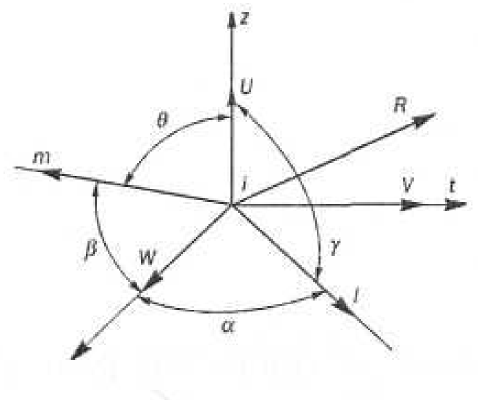


Fig.4 **The scheme of optical measurement**

This approach enables to realize the calculations according to the constructed mathematical model of the system more fully and precisely, because the calculations themselves and the analysis of the systems following from them are based on the deterministic data of this real model.

In the work the method is proposed which enables to calculate the parameters of vibrations of the surface points of rigid bodies at any point of the body from a finite number of data obtained by using the holographic experiment.

For this purpose we will use Fig. 4. The scheme of optical measurement of vibrations of point *i, r, t, z* - the orthogonal system of coordinates; $R$ -the vector of spatial vibrations of the $i$ – th point of the surface; $U,V,W$ - the components of the vector of spatial vibrations of the $i$ - th point in the directions of the coordinate axis $r,t,z$ correspondingly (in the text $\bar{U},\bar{W},\bar{V}$ ); *l*-the unit vector of lightening of the point *i* (in the text *Ki)*; $m$ - the unit vector of observation of the point *i* (in the text *Ko*); $\alpha,\beta$ - the angles of unit vectors of lightening and observation with the coordinate axis $r$ correspondingly (in the text corresponding to $\theta_1^i,\theta_2^i$); $\gamma,\theta$ - the angles between the coordinate axis $z$ and the unit vectors of lightening and observation correspondingly (in the text $\varphi_1^i,\varphi_2^i$ ).

We consider that the spatial vibration of the surface point / of the input member is described by

$$\bar{R}_i(t) = U_i(t)\hat{i} + V_i(t)\hat{j} + W_i(t)\hat{k}$$

The tangential *U, V* and the normal *W* components of the vector $\bar{R}_i(t)$ at the point *i* are expressed

$$U(t) = U_0^i \cos(wt + a_i), V(t) = V_0^i \cos(\ wt + b_i),$$
$$W(t) = W_0^i \cos(wt + g_i),$$

where $U_0^i, V_0^i, W_0^i$ – the amplitudes of forced vibrations at the point *i* in the coordinates $r,t,z$ respectively.

The amplitudes of the forced vibration are expressed by representing them through the eigenmodes of vibrations:

$$U_0^i = \sum_1^k A_j^u F_{ij}^u;\ V_0^i = \sum_1^k A_j^v F_{ij}^v;$$

$$W_0^i = \sum_1^k A_j^w F_{ij}^w; \qquad j = 1,2,\ldots,k$$

where $F_{ij}$ – the amplitude value of the *j*-th eigenmode of vibration at the point *i*, which is calculated according to the analytical expressions by taking into account the conditions of fastening of the input member, $A_j$ – the influence coefficient of the $j$ - th eigenmode of vibrations, $k$ - the number of given eigenmodes of vibrations of course by taking into account the presented relationships. It is clear that in order to calculate the components of the vector of spatial vibrations it is necessary to determine $F_{ij}^u, F_{ij}^v, F_{ij}^w, A_j^u, A_j^w, \alpha_i, \beta_i, \gamma_i$. $F_{ij}^u, F_{ij}^v$ and $F_{ij}^w$ are calculated according to the known analytical expressions that are used in the theory of vibrations for the calculation of the amplitudes of vibrations of eigenmodes by taking into account the geometry of the body analysed and the boundary conditions of its fastening.

The parameters $A_j^u, A_j^v, A_j^w, \alpha_i, \beta_i, \gamma_i$ are determined on the basis of experimental data. The method of their calculation is given in [2, 3]. We shall briefly review the essential points of the method.

According to the characteristic function of distribution of the interferention bands on the surface the following nonlinear algebraic equation is constructed:

$$\frac{\Omega^i \lambda^2}{4\pi} = \left[\begin{array}{l} \left(\sum_1^k A_j^w F_{ij}^w\right)\cos\lambda_i K_r^i + \left(\sum_1^k A_j^v F_{ij}^v\right)\cos\beta_i K_t^i + \\ + \left(\sum_1^k A_j^{lt} F_{ij}^{lt}\right)\cos\alpha_i K_z^i \end{array}\right]^2 + \left[\begin{array}{l} \left(\sum_1^k A_j^w F_{ij}^w\right)\sin\gamma_i K_r^i + \left(\sum_1^k A_j^v F_{ij}^v\right)\sin\beta_i K_t^i + \\ + \left(\sum_1^k A_j^{lt}\right)\sin\alpha_i K_z^i \end{array}\right]^2 \qquad (15)$$

where $\Omega$ - are calculated from the holographic interferograms at the centers of dark interferentional bands.

$K_r^i, K_t^i, K_z^i$ are the projections of the sensitivity vector that are calculated by taking into account the optical scheme of the holographic measurement.

$F_{ij}^{lt}, F_{ij}^{v}, F_{ij}^{W}$ are calculated according to the known analytical expressions of vibration of eigenmodes.

The nonlinear algebraic Eq. 15 is solved according to the method presented in [3].

Fig. 5 presents the structural diagram of a stand for experimental study of wave processes in the piezodrives ring actuator with the aid of holographic interferometry utilizing the method of time averaging. The stand contains the searching piezodrive 1 and the signal generator with the electronic switch 2. The optical circuit of the stand includes the holographic installation UIG-2G-1 with a helium-neon laser (model LG-38) which serves as a source of coherent radiation.

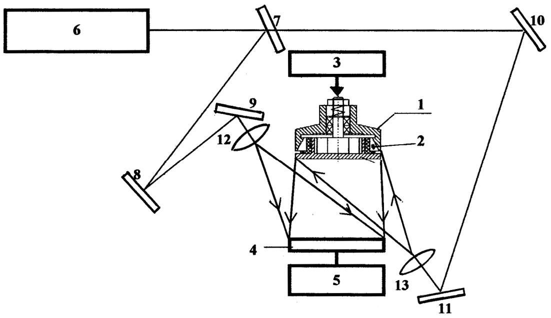


Fig.5. **Structural diagram of the holography stand: 1 - piezodrive, 2 - ring actuator, 3 - signal generator, 4 - photographic plate, 5 - camera or visualize camera, 6 - laser, 7 - through the beam splitter (clear), 8 – 11 mirrors, 12-13 - lenses**

The beam from the laser 6 splits into two mutually coherent beams passing through the beam splitter 7. The object beam, reflected from the mirror 10 and mirror 11, is split by the lens 13 and illuminates the surface of the 2 and the piezodrives ring actuator 1, after reflecting from it, impinges on the photographic plate 4, which fortification permissible to make the processes of photographic. It is possible not to change the plates position. That is permissible with the aid of a holographic interferometry utilizing the method of real time. The reference beam, reflected by the mirror 8 and mirror 9, and expanded by the lens 12, illuminates the photographic plate 4, where the interference structure is recorded. The object and the reference beams create interference in the plane of the photographic plate and during exposure a constant in the time three-dimensional interference structure is generated and recorded as a hologram. In order to reconstruct the image, the hologram is illuminated by the reference beam. The reconstructed interference image is photographed by the camera or the visualize camera 5. If you like to minimize the photographic plate 4 exposition time and to improve the contrast of the reconstructed interference image, the piezodrives ring actuator 2 was returning lusterless white color.

On the basis of the developed methodology of analyzing the experimental data derived from a holographic interferometry, and by using the experimental holography stand, we have obtained results making it possible to optimize the design and operation of the piezodrives and the ring actuators. The signal generator 3 can to change and to regulate a frequency signal and location of excited points.

The obtaining of holographic interferograms enables to optimize the design of the working regimes of the mechanisms with ring actuators, to obtain supplementary data with help of which it is possible to develop the design of devices, to select the conditions of transfers vibration of the ring actuator that are least affected by rotational inertia. In the holographic interferogram (Fig.6 a, c, d) the case when the pressure force of a piezoring does not fully ensure the motion of the rotor is presented. In the case Fig.6b the holographic interferogram is presented indicating the normal working regime of the piezodrives.

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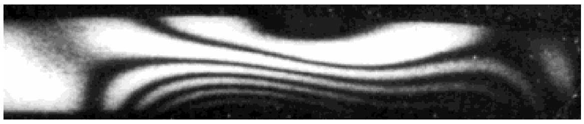
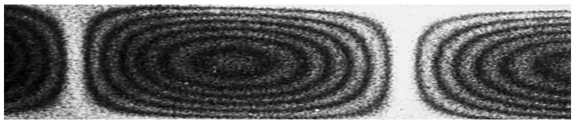
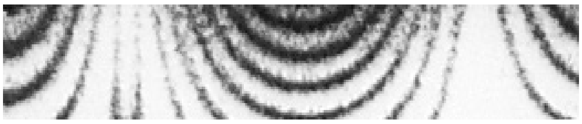


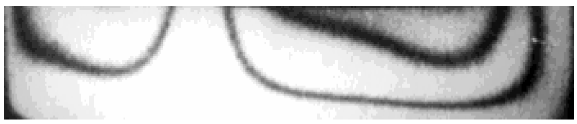
**a) F=39,7kHz**



**b) F=40 kHz**



**c)F=40,5 kHz**



**d)F=41,5 kHz**

Fig.6 **The holographic interferograms of the links of the ring actuator**

## Conclusions

The above presented theoretical and experimental investigation makes it possible to draw the following conclusions. The expressions of differential equations have been obtained for describing dynamic variations in ring drives. Dependability of friction and tension force in a kinematic pair has been analysed and it confirms the fact that reducing the friction force magnitude the wave processes become recurring processes in the ring drive. The expressions of linear and rotational speeds allow to calculate and apply the deformations to the optimal extent in piezoactuators.

Distribution of tangential and radial deformations in piezoactuators with a ring drive plays an important role in designing piezodrives and it has been employed when developing new structures. The generator has been developed and due to it by applying a division frequency signal the excitation force magnitude is regulated at certain points whereas the change of the number and location of the points is also possible. The holographic interferometry method used in the experimental work has validated the expressions of differential equations and was used for getting conclusions of the investigation.

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**Pjezovariklių su žiediniu keitikliu analizė holografinės interferometrijos metodais**

Nagrinėjama pjezovariklių su žiediniu keitikliu dinamika. Žiedinis keitiklis sužadinamas artimųjų dažnių virpesiais įvairiuose jo taškuose. Šių taškų išdėstymas apskaičiuojamas teorinėje dalyje. Be to, sprendžiamos diferencialinės lygtys ir analizuojami bėgančiosios bangos parametrai. Holografinės interferometrijos metodai, palyginti su kitais nekontaktiniais matavimo metodais, leidžia gauti daugiau informacijos apie kietų deformuojamų kūnų paviršiaus virpesius. Aptariamas šių metodų taikymas vibrovarikliams tirti, precizinių deformuojamų kūnų grandžių virpesių parametrams apskaičiuoti.

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