Experimental - numerical techniques for thermoelastic analysis of structural vibrations

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Introduction

The importance of thermo-elastic analysis of vibrations is described in [1, 2]. A cyclically loaded structure experiences in-phase temperature variations that are proportional to the change of the sum of the normal stresses. Thus resonant vibrations according to the eigenmode are effectively analysed by this method. The change in temperature ΔT for the problem of plane stress is given by:

$$\Delta T = -CT\Delta(\sigma_x + \sigma_y), \qquad (1)$$

where C is the thermo-elastic constant of the material, T is the absolute ambient temperature, $\Delta(\sigma_x + \sigma_y)$ is the change of the sum of the normal stresses. Vibration isolation is unnecessary and the method is usable in industrial environment.

The calculation of isopachics is important in the procedures of hybrid experimental - numerical analysis of vibrations [2, 3]. The photo-detector of a thermo-elastic system focuses on a spot of finite size of the structure. This causes problems at the edges of a specimen: the edge effect takes place because the photo-detector sees a spot that is partly on the stressed specimen and partly on the stress free background. The quality of an edge signal is reduced by the vibrating motion of the structure, which provides different data to the photo-detector from different spatial positions. The lack of reliable thermo-elastic edge data is compensated by the numerical results of the hybrid procedure: if the correlation of the experimental and numerical images of isopachics in the region further from the edges is acceptable, then the numerical image should be preferred in the region near to the edges.

Visualisation techniques of the results from finite element analysis procedures are important due to several reasons. First is the meaningful and accurate representation of processes taking place in the analysed structures. Second is building the ground for hybrid numerical - experimental techniques. A typical example of application of finite elements in developing a hybrid technique is presented in [3].

Conventional finite element analysis techniques are based on the approximation of nodal displacements (not stresses) via the shape functions. Therefore analysis of natural vibrations using thermo-elastic techniques is an attractive methodology evaluating the quality of the structural design in terms of the stress distribution.

Conventional finite elements would require unacceptably dense meshing for producing sufficiently smooth thermo-elastic patterns of isopachics. Therefore there exists a need for the development of a technique for smoothing the generated patterns of isopachics representing the stress distribution and calculated from the displacement distribution. The proposed smoothing technique is similar to conjugate approximation used for the calculation of nodal values of stresses in [4, 5] and enables to obtain the thermo-elastic images of isopachics of acceptable quality on a rather coarse mesh by using the displacement formulation for the calculation of the eigenmodes.

The developed techniques builds the ground for hybrid numerical - experimental thermo-elastic analysis and enables to couple the results of displacement and stress analysis.

Construction of the digital thermoelastic images of isopachics

First of all the eigenmodes for the structure in the state of plane stress are calculated by using the displacement formulation common in the finite element analysis. It is assumed that the structure performs high frequency vibrations according to the eigenmode (the frequency of excitation is about equal to the eigenfrequency of the corresponding eigenmode and the eigenmodes are not multiple). Further σ_x, σ_y denote the first two components of the stresses in the problem of plain stress. In this case in order to obtain the image of isopachics the problem is to calculate the nodal values of $\sigma_x + \sigma_y$ of acceptable quality for the eigenmode (the eigenmode of $\sigma_x + \sigma_y$), which are further used for the calculation of the thermo-elastic image of isopachics.

The values of $\sigma_x + \sigma_y$ at the points of numerical integration of the finite element are calculated in the following way:

$$\sigma_x + \sigma_y = D[B]\{\delta_0\}, \qquad (2)$$

where $\{\delta_0\}$ is the vector of nodal displacements of the eigenmode; $[B]$ is the matrix relating the sum of the first two components of strains ε_x and ε_y with the displacements:

$$[B] = \begin{bmatrix} \dfrac{\partial N_1}{\partial x} & \dfrac{\partial N_1}{\partial y} & \dots \end{bmatrix}, \qquad (3)$$

where N_1, … are the shape functions of the finite element, x and y are orthogonal Cartesian axes of coordinates; D is the elastic constant relating the value of $\sigma_x + \sigma_y$ with the value of the sum of the components of the strains ε_x and ε_y:

$$D = \frac{E}{1-\upsilon}, \qquad (4)$$

where E is the modulus of elasticity and υ is the Poisson's ratio. The displacements are continuous at inter-element boundaries, but the calculated values of $\sigma_x + \sigma_y$ due to the operation of differentiation are discontinuous.

The appropriate eigenmode of the values of $\sigma_x + \sigma_y$ is obtained by minimising the following error:

$$\frac{1}{2}\iint \left(\begin{array}{l} \left([N]\{\delta\} - (\sigma_x + \sigma_y)\right)^2 + \\ + \lambda \left(\left(\frac{\partial(\sigma_x + \sigma_y)}{\partial x}\right)^2 + \\ + \left(\frac{\partial(\sigma_x + \sigma_y)}{\partial y}\right)^2 \right) \end{array} \right) dxdy =$$

$$= \frac{1}{2}\iint \left(\begin{array}{l} \left([N]\{\delta\} - (\sigma_x + \sigma_y)\right)^2 + \\ + \lambda\{\delta\}^T [B*]^T [B*]\{\delta\} \end{array} \right) dxdy, \qquad (5)$$

where λ is the smoothing parameter; $\{\delta\}$ is the vector of nodal values of $\sigma_x + \sigma_y$; $[N]$ is the row of the shape functions of the finite element; $[B*]$ is the matrix of the derivatives of the shape functions (the first row with respect to x; the second – with respect to y). This leads to the following system of linear algebraic equations for the determination of the values of $\sigma_x + \sigma_y$:

$$\iint \left([N]^T [N] + [B*]^T \lambda [B*]\right) dxdy \cdot \{\delta\} =$$
$$= \iint [N]^T (\sigma_x + \sigma_y) dxdy. \qquad (6)$$

The analysis of the digital thermo-elastic image of isopachics has to be made to define the value of the smoothing parameter. The optimal value of the smoothing parameter is related to the generated image of acceptable quality without the unphysical behaviour produced by the approximation.. The over-smoothed image is generated when the smoothing parameter is too large, and the image is of unacceptable quality when the smoothing parameter is too small.

Numerical results

The rectangular plate in the state of plane stress is analysed. For this test problem it is assumed that the structure is free.

The representation of the temperature by the values of intensity for the fourth eigenmode are shown in Fig. 1 and for the eighth eigenmode are shown in Fig. 2.

The values of $\sigma_x + \sigma_y$ represented as radiuses of circles for a number of points of the structure for the fourth eigenmode are shown in Fig. 3 and for the eighth eigenmode are shown in Fig. 4. Black circles correspond

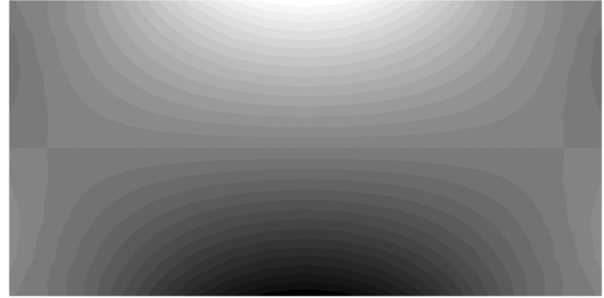to negative values and grey circles correspond to positive values.

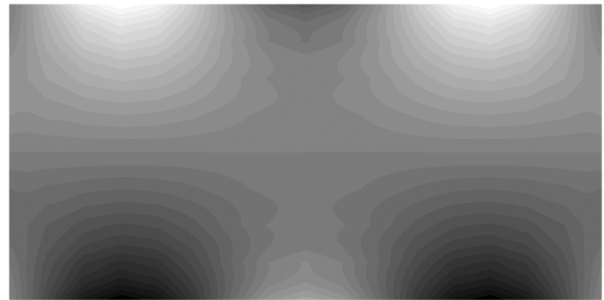Fig. 1. **Surface temperatures for the fourth eigenmode**

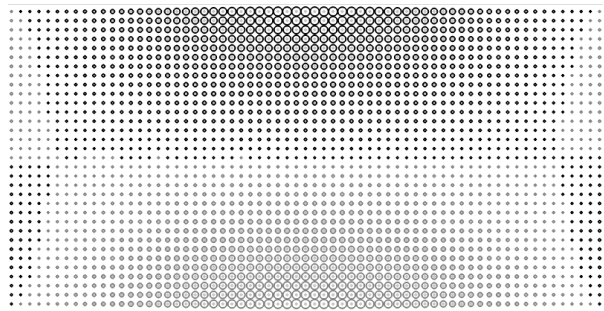Fig. 2. **Surface temperatures for the eighth eigenmode**

Fig. 3. **The values of $\sigma_x + \sigma_y$ represented as radiuses of circles for the fourth eigenmode (black circles correspond to negative values and grey circles correspond to positive values)**
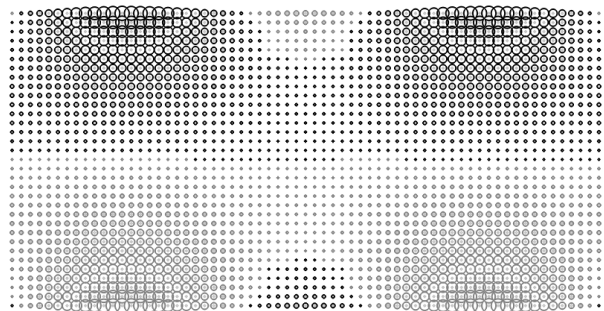

Fig. 4. **The values of $\sigma_x + \sigma_y$ represented as radiuses of circles for the eighth eigenmode (black circles correspond to negative values and grey circles correspond to positive values)**

The unsmoothed isopachics of the fourth eigenmode are shown in Fig. 5 and of the eighth eigenmode are shown in Fig. 6. The oscillations of the isopachics produced by the approximation are evident.
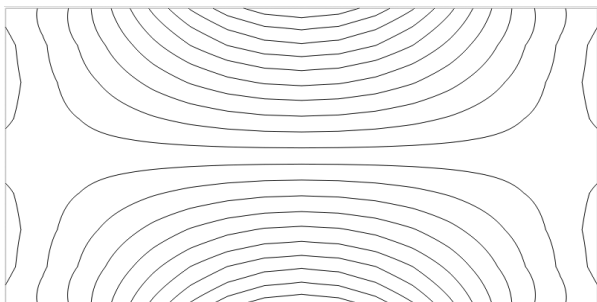
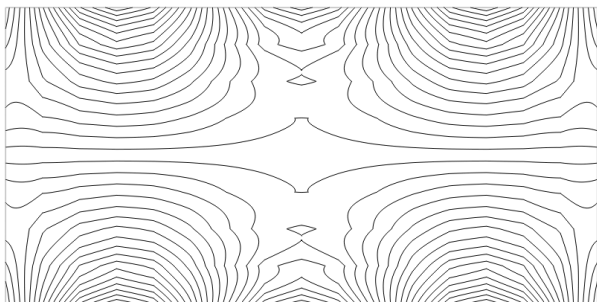Fig. 5. **Unsmoothed isopachics of the fourth eigenmode**

Fig. 6. **Unsmoothed isopachics of the eighth eigenmode**

The smoothed isopachics of the fourth eigenmode are shown in Fig. 7 and of the eighth eigenmode are shown in Fig. 8. The effect of the smoothing procedure in suppressing the unphysical oscillations is evident.
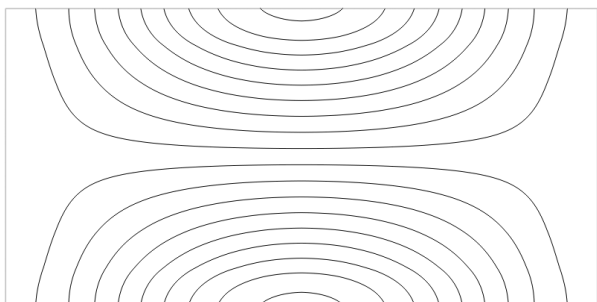

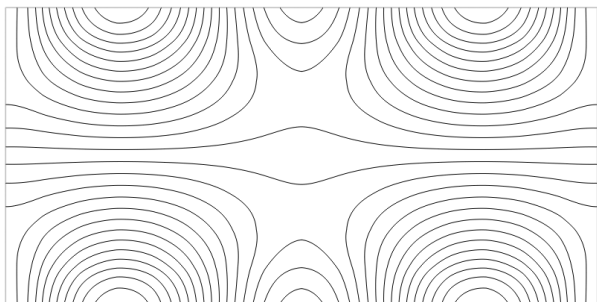Fig. 7. **Smoothed isopachics of the fourth eigenmode**

Fig. 8**. Smoothed isopachics of the eighth eigenmode**.

Conclusions

The construction of digital thermo-elastic images of isopachics builds the ground for hybrid numerical - experimental thermo-elastic analysis and enables to analyse the experimental results with a greater precision. The calculation of the precise field of the values of $\sigma_x + \sigma_y$ from the displacement finite element formulation requires the application of the proposed procedure of conjugate approximation with smoothing for this specific problem.

This smoothing procedure enables the generation of thermo-elastic images of isopachics on rather coarse conventional finite element meshes.

References

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Eksperimentinė–skaitmeninė termotamprių konstrukcijų virpesių analizės metodika

Reziumė

Skaitmeniniai izopachų vaizdai, gaunami taikant hibridines skaitmenines-eksperimentines procedūras, įgalina tiksliau interpretuoti termotampriuoju būdu atlikto eksperimento rezultatus. Nagrinėjami tamprios konstrukcijos plokščioje įtemptoje būsenoje virpesiai pagal savąją formą. Pasiūlyta jungtinės aproksimacijos glotninimu procedūra šiam uždaviniui spręsti ir pateikti gautieji izopachų vaizdai.