Spread spectrum signal performance investigation in a bandlimited channel

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Abstract

The paper investigates performance of spread spectrum signals performance in a bandlimited channel. Available spectrum spreading techniques are discussed. The performance is evaluated as the time of flight (ToF) accuracy estimation using the direct correlation technique. Numerical simulations have been carried out. Ultrasonic transducer was approximated as a bandlimited channel for simulation. The influence of ToF estimation with a constant chirp center frequency spreading and continuously rising chirp center frequency where analyzed. Random errors of ToF and how effective bandwidth of chirp signal is affected by a bandlimiting channel where evaluated. It has been shown that when chirp sweep frequencies are around the constant center frequency then the ToF deviation is insignificant. When the upper chirp sweep frequency is increasing, the random errors are reduced. The results showed that the optimum chirp signal bandwidth can be beyond the channel bandwidth.

Keywords: Spread spectrum signals, time-of-flight estimation, ultrasonic signal processing.

Introduction

Positioning and navigation systems are used in a variety of applications. Location estimation is essential in order to implement autonomous robot navigation [1]. There are several location estimation methods. The location estimation techniques used in navigation can be assigned to triangulation, trilateration and the combination of the two. The trilateration technique is using the time-of-flight (ToF) for lateral distances estimation [2, 3].

From a radar theory the time standard deviation estimation is defined using the Cramer Rao lower bound [4,5]. This relation suggests that there are two possible ways to minimize the uncertainty of the ToF estimation [6]: i) by maximizing the signal energy; ii) by maximizing the effective signal bandwidth. Many authors investigate the case when the ultrasonic signal spectrum is matched to the passband shape of the ultrasonic transducer. The aim of this paper was to investigate the behavior of spread spectrum signals when their spectrum is not matched to the ultrasonic transducer passband shape.

The ToF estimation precision

As suggested in [4] and [5] the ToF estimation can be done by the direct correlation maximization to find and estimate the true position of the signal arrival. The variance of the ToF standard deviation is given by [7]:

$$std_{CRLB}(TOF) \geq \frac{1}{2\pi F_e \sqrt{\frac{2E}{N_0}}}, \tag{1}$$

where E is the signal $s_T(t)$ energy, F_e is the effective bandwidth of the signal. The effective signal bandwidth can be calculated as:

$$F_e^2 = \beta^2 + f_0^2, \tag{2}$$

where β is the envelope bandwidth and f_0 is the center frequency:

$$\beta^2 = \frac{\int_{-\infty}^{\infty} (f - f_0)^2 |S(f)|^2 df}{E}, \quad f_0^2 = \frac{\left[\int_{-\infty}^{\infty} f |S(f)|^2 df\right]^2}{E^2}. \tag{3}$$

Eq. 1 suggests that increasing effective signal bandwidth the ToF variation decreases linearly while increasing the signal energy is influencing the ToF variation by the square root. Therefore an assumption can be made that in a bandlimited channel it is more efficient to increase the effective signal bandwidth despite energy losses.

The spectrum spreading

Reduction of random errors is possible by maximizing the signal energy, reducing the noise level and increasing the effective signal bandwidth as Eq. 1 suggests. Since a noise level has some physical limits it is more feasible to increase only the energy and the bandwidth. The increase of both is possible only if spread spectrum signals are used. The spread spectrum technique allows having a long duration pulse and the desired bandwidth simultaneously.

In order to have the pulse compression at the reception end, the signal must be passed through a matched filter, which corresponds to a direct correlation processing. In addition, the pulse compression allows reducing the non-singularity which is present in long signals. The example in Fig.1 demonstrates the peak area of the correlation function of the continuous wave (CW) burst.

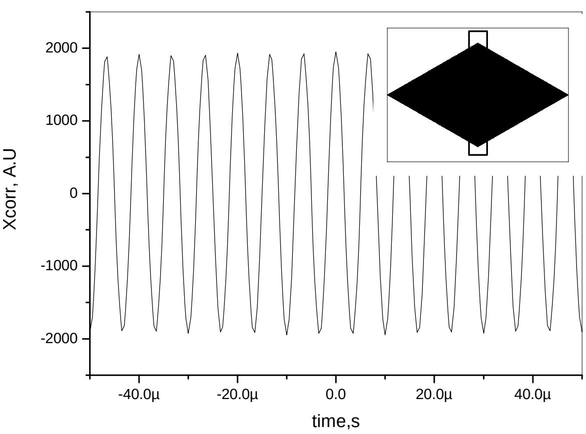

Fig. 1. **The matched filter output of CW burst**

Though the output looks quite steep, the tip zoom in showed on the top right corner is very flat. Even the moderate noise level will cause the peak detection errors which are not accounted in Eq. 1 and random errors will increase significantly. The Fig. 2 is presenting the same temporal window of the same length signal with the only difference that the linear frequency modulation is applied which gives a significant spread of signal spectrum.

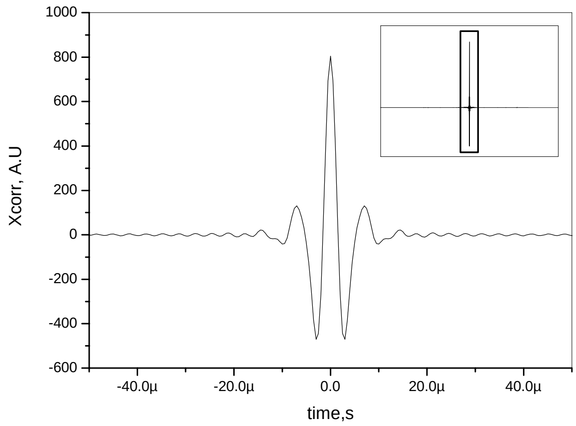

Fig. 2. **The matched filter output of spread-spectrum burst**

It can be seen that peak steepness is increased. Several techniques can be applied for to spread the signal spectrum: phase manipulated sequences, chirp and arbitrary waveform excitation.

The phase manipulated sequences (Fig.3) make the excitation task easer since square waves can be used for the signal generation. Square waves make the excitation hardware less complex because a power amplifier may be replaced by a high speed switch. Application of the orthogonal coded sequences allows easy separate the probing channels [1] so simultaneous surrounding area scanning with several ranging channels can be done.

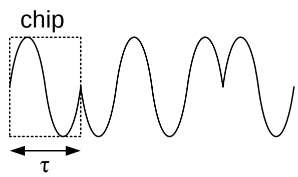

Fig. 3. **Phase manipulated sequence**

In [8] authors were trying to fit the Golay sequences spectrum into the bandwidth of an ultrasonic transducer: the signal bandwidth was either narrower or wider than that of the ultrasonic transducer. The problem is that the resulting signal bandwidth is predicted by the smallest chip (Fig. 3) duration. If the integer number of half-periods must be used for chip this is a disadvantage since then the bandwidth adjustment step is discrete. Another disadvantage is that there is no possibility to shape the signal spectrum in the arbitrary way.

Application of the arbitrary waveform produces any shape of amplitude and phasing spectrum and correlation function. But this technique is used rarely as this technique requires complicated excitation hardware. There are attempts to use some approximation using only limited number or excitation levels. Publication [9] is reporting a quinary excitation method for linear frequency modulation windowing implementation.

Another frequency spreading technique is using a carrier frequency modulation [10, 11] and frequently is addressed as a chirp signal (Fig. 4).

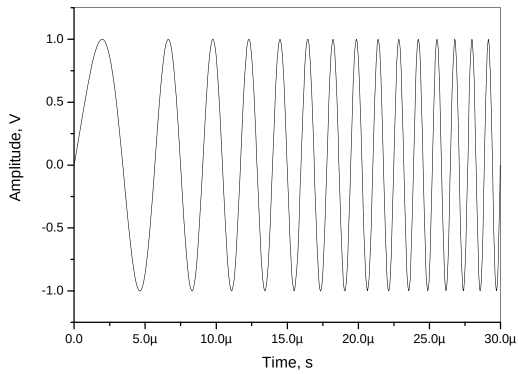

Fig. 4. **Chirp signal**

Chirp signal excitation offers any spectral shape and easy excitation circuit and looks an attractive solution. In [10] an application of chirp signals for air-coupled non-destructive testing is reported. The combination of chirp signals and wideband micromachined capacitance (CMUT) transducers allowed compensation of the signal to noise ratio (SNR) losses present in air-coupled ultrasound applications. Authors did not care nor accounted for transducer and signal bandwidth matching. The signal bandwidth was chosen to be wider than the transducer passband. In publication [11] authors are using a nonlinear frequency modulation of a square wave signal in order to produce the signal spectrum which corresponds to ultrasonic transducer bandwidth shape. But there are no publications which investigate the case when the excitation signal bandwidth is wider than the ultrasonic transducer pass band. As it was indicated earlier, Eq. 1 suggests that using such signal should increase the effective signal bandwidth. Of course, the penalty will be the reduction of a signal energy passed through the filter. Since the ToF variation decreases linearly with the effective signal bandwidth increase and the ToF variation increase due to energy losses is by the square root, one could expect that there still should be a ToF variation reduction gain. Therefore an assumption was made that in a bandlimited channel it is more efficient to increase the effective signal bandwidth despite energy losses.

The numerical experiment setup

In order to check the aforementioned assumption it was decided to make a numerical experiment in order to evaluate how the effective bandwidth (F_e) of the chirp signal is affected by a bandlimiting channel. The main two questions where risen:

i) How effective bandwidth of the chirp signal is affected by bandlimiting?

ii) How random errors of the ToF will behave in a bandlimited channel?

The numerical simulations have been carried out in order to evaluate the influence of a bandlimited channel on a TOF estimation performance. The signal has been simulated as the linear frequency modulation burst.

$$s_T(t) = rect(\tau_{pulse}) \cdot \cos(2\pi(f_1 + f_d t)t), \qquad (6)$$

where τ_{pulse} is the chirp signal duration, f_1 is the start frequency, f_d is the deviation frequency. It should be noted that the upper frequency of the resulting frequency spectrum is twice higher the deviation because of the frequency derivative effects. Therefore a corresponding normalization of the deviation frequency was used. The ultrasonic transducer simulated had the passband of 100 kHz to 200 kHz. The simulation has been carried out in MATLAB. The noise has been simulated using *randn* function, which assumes the additive white Gaussian noise. Random errors of the ToF have been extracted by taking a large number of runs (10,000) and calculating the standard deviation of the ToF value estimated.

Two cases were investigated: a) with sweep frequencies centered around constant center frequency, b) when upper chirp sweep frequency was increasing.

Three types of experiments in MATLAB for the ToF estimation have been carried out:

1) unfiltered measurement signal and reference;

2) unfiltered reference, but filtered measurement signal;

3) filtered measurement signal and reference.

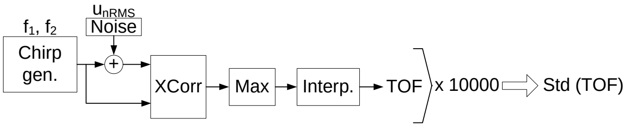

Fig. 5. **Numerical experiment setup 1: unfiltered signal and reference**

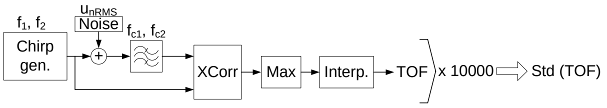

Fig. 6. **Numerical experiment setup 1: filtered signal**

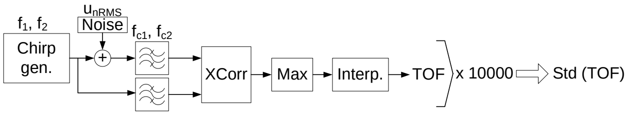

Fig. 7. **Numerical experiment setup 1: filtered signal and reference**

The goal of the numerical simulation was to reveal the influence of the channel bandwidth on the ToF estimation. The spectral response of the chirp signal after passing the bandlimited channel is presented in Fig.8.

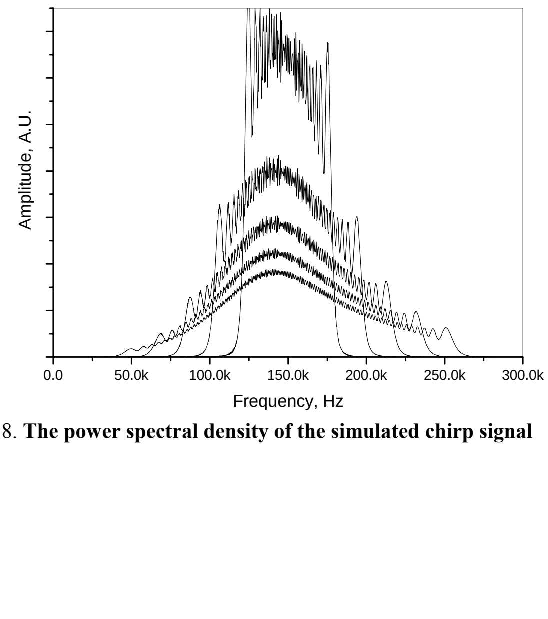

Fig. 8. **The power spectral density of the simulated chirp signal**

It should be noted that the signal energy was kept constant by keeping the same signal amplitude. Such restriction allows to separate the filter effects on the ToF variation.

Sampling influence minimization

Sampling the analog signal introduces convolution in the frequency domain with a period of the sampling frequency f_s. The aliasing will occur for any frequency component (both signal and noise) falling outside the $f_s/2$. This region between zero and the $f_s/2$ is named the Nyquist zone. The antialiasing filter must be used in order to avoid aliasing [6]. Furthermore, aliasing of the noise will decrease the SNR. The sampling frequency was chosen high enough so that the antialiasing filter did not have influence on experimental results. The sampling frequency selection was based on the initial investigation aim of which was to establish the sampling frequency where effect of aliasing is minimized (Fig. 9).

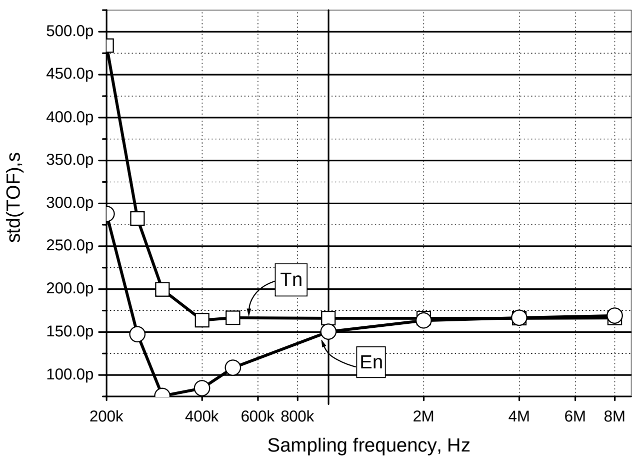

Fig. 9. **ToF deviation versus the sampling frequency**

The sampling frequency was chosen 2 MHz, so that aliasing does not have influence on experimental results and the ToF prediction using Eq. 1 and experimental results are the same.

Results for constant center frequency

The first case experiments were carried out with the chirp spectral response centered at the constant frequency f_0. The transducer has been simulated using the I-st order Butterworth filter with a bandpass filter, the cut off frequencies of which were from 100 kHz to 200 kHz. The center frequency f_0 was 150 kHz and the requested chirp signal bandwidth span was from 30 kHz to 300 kHz (Fig. 10). The chirp signal duration was 2 ms.

In Fig.10 f_{c1} and f_{c2} denote the filter cut off frequencies. The lines f_1 and f_2 show the lowest and the highest chirp signal spectrum frequency.

Fig. 11 shows the resulting F_e changes and the constructing components: the envelope bandwidth β and the center frequency f_0 .

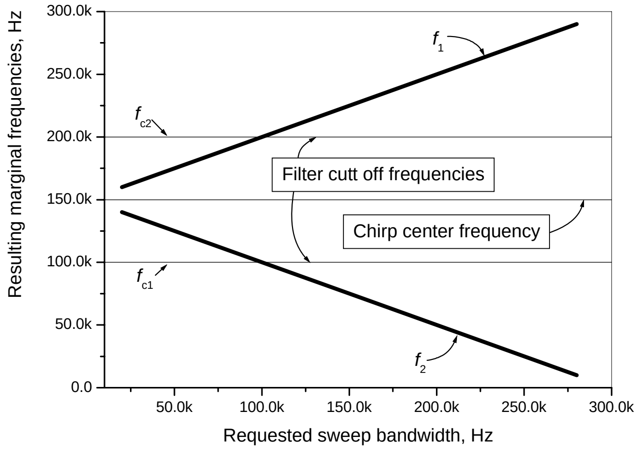

Fig. 10. **Chirp signal: f_1 and f_2 chosenm so f_0 is constant**

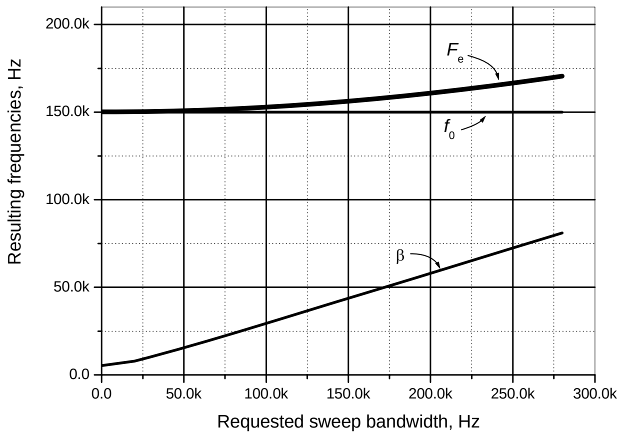

Fig. 11. **Filtered Chirp signal f_0, beta and F_e**

From Fig. 11 it is seen that if f_0 is kept constant, the envelope bandwidth β is lower than the center frequency f_0, the resulting increase of the effective bandwidth F_e is insignificant. It can be expected that the resulting ToF variation decrease should also be negligible (Fig. 12).

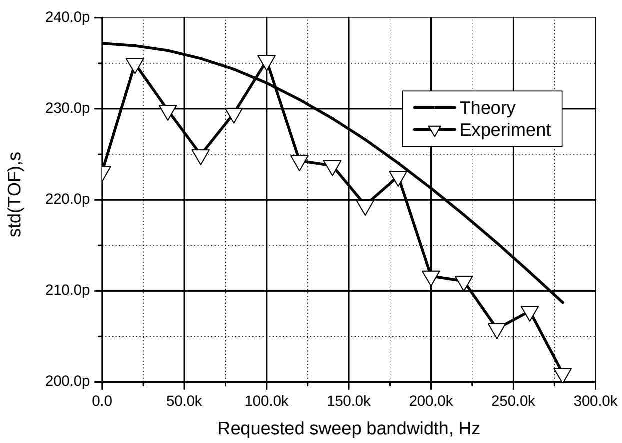

Fig. 12. **Unfiltered chirp signal performance**

The results confirm the assumption above: if the increase of F_e is small, then the total reduction of the standard deviation of ToF is also low.

Another two experiments were aimed to account for the ultrasonic transducer bandlimiting influence. This was accomplished by applying the filter on the measurement signal. Type 2 experiment used the unfiltered reference, but measurement signal was filtered. Type 3 experiment used both the filtered measurement signal and the reference. The obtained results are presented in Fig.13. The experiment results indicate that there is no improve of the ToF variance. This can be explained by the results on Fig. 11 and 12: the increase of the effective bandwidth F_e is insignificant even for unfiltered case.

Furthermore, there is and increase of the ToF variance. This increase is more significant for the reference signal having a wider bandwidth (unfiltered reference). We explain this by a wider bandwidth of the matching wilter which in turn gives a higher level of a penetrating noise.

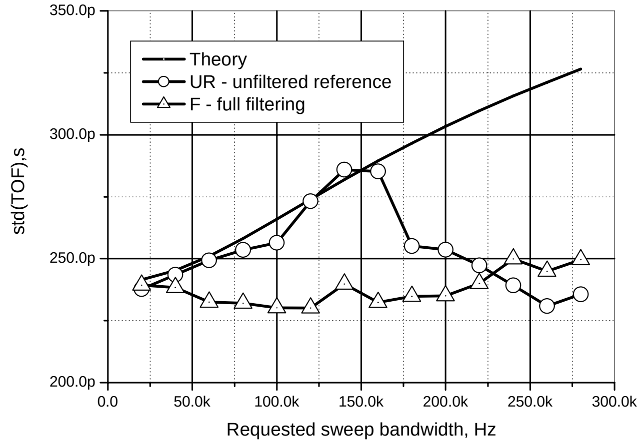

Fig. 13. **Filtered chirp signal performance**

The graph in Fig.14 is used to demonstrate the time shift induced by not even filtering of the reference and the measurement signals. The cross correlation function was normalized by the signals mutual energy geometric average (MATLAB option "coeff").

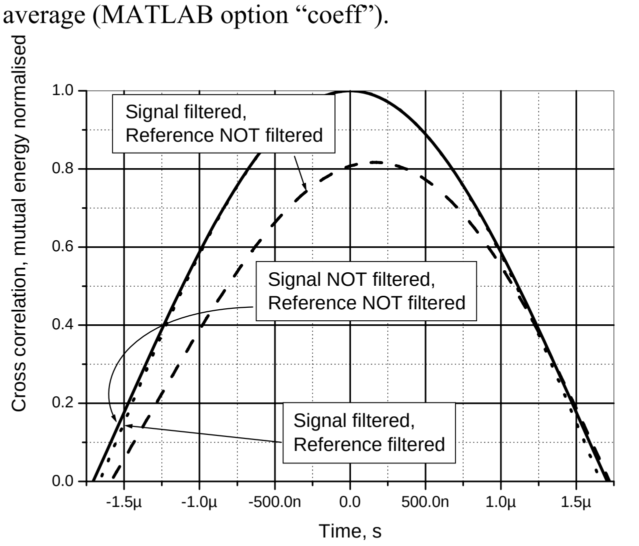

Fig. 14. **The correlation function peak area for all three experiments**

The conclusion can be drawn that the reference signal should be filtered in the bandlimited correlation processing in order to get the optimal results.

Results for increasing center frequency

The previous experiments indicated that the envelope bandwidth β should be comparable to the center frequency

f_0 in order to have a significant influence on the resulting increase of the effective bandwidth F_e, so the resulting in ToF variation decrease.

A new set of experiments was aimed to investigate the case when the upper chirp sweep frequency was increasing so the center frequency was no longer the same and it was increasing. With the starting sweep frequency f_1 held at 100 kHz, the upper limit f_2 of the sweep was increasing. The increase of the center frequency suppose to cause the increase of the effective bandwidth F_e resulting in the ToF variation decrease. Fig.15 is used to demonstrate the effect of such modulation on the envelope bandwidth β and the center frequency f_0 and the effective bandwidth F_e change.

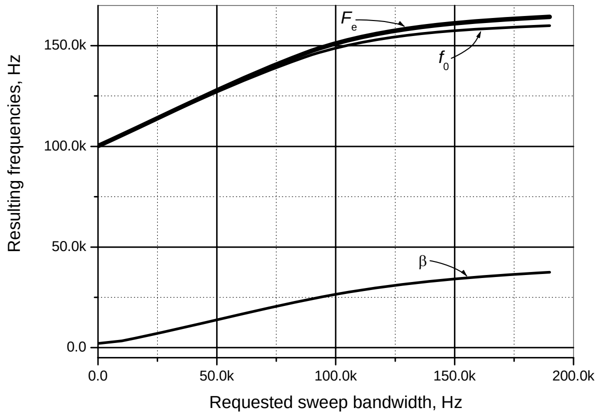

Fig. 15. **Filtered Chirp with f_1=100 kHz, f_2 increasing**

Again, despite the increase of the envelope bandwidth β its influence on the effective bandwidth F_e is negligible since the center frequency f_0 is higher. But here there is an increase of the center frequency f_0 and the resulting effective bandwidth F_e change is significant. The resulting the ToF variation can be examined in Fig. 16.

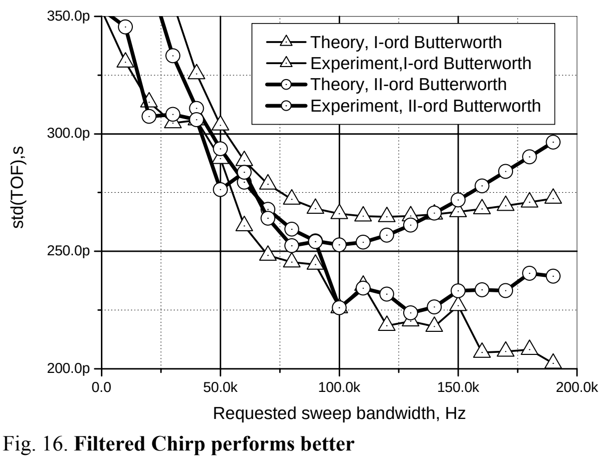

Fig. 16. **Filtered Chirp performs better**

A clear optimum can be located here. In addition, there is a nice match between the theory prediction by Eq. 1 and the numerical experiment results. The optimum is located at the filter passband (refer to Fig. 10). The higher order filter (steeper transition band) has optimum in a higher span. For it there is an optimum which is beyond the filter bandwidth.

Conclusions

It has been assumed that for spread spectrum signals there should be some bonus in ToF random errors reduction thanks to the effective bandwidth increase even if the chirp spectrum exceeds the filter bandwidth. The investigation revealed that if chirp sweep frequencies are centered on a passband center frequency, then the effective bandwidth gain (so the TOF deviation) is insignificant.

Numerical experiments indicate that if the chirp spectrum is shifted towards higher frequencies, then there is some gain in the effective bandwidth, therefore the random ToF estimation errors are reduced. There is an optimum in the case of a higher order filter which is beyond the filter bandwidth.

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Skleisto spektro signalų efektyvumo ribotos juostos kanaluose įvertinimas

Reziumė

Buvo tiriamas skleisto spektro signalų panaudojimas ribotos juostos kanaluose, aptarti galimi spektro išplėtimo metodai. Skleisto spektro signalo efektyvumui įvertinti naudotas signalo sklidimo laiko įvertinimo tikslumas, apskaičiuotas tiesioginės koreliacijos metodu. Straipsnyje pateikiami skaitmeninio modeliavimo rezultatai. Modeliuojant ultragarsinis keitiklis buvo interpretuojamas kaip ribotos pralaidos juostos kanalas. Nagrinėta, kaip sklidimo laiko nustatymo tikslumą veikia skleisto spektro signalai, pereinantys per ribotos pralaidos juostos kanalą. Analizuoti du atvejai: i) kai signalo juosta plečiama į abi puses apie centrinį dažnį taip, kad spektro svorio centras nekistų; ii) kai plečiamo signalo spektro svorio centras slenka į aukštesnių dažnių pusę. Nustatyta, kad plečiant skleisto spektro signalo juostą simetriškai apie centrinį dažnį, signalo sklidimo laiko nustatymo atsitiktinės paklaidos kinta mažai. Jei plečiamo signalo spektro svorio centras slenka į aukštesnių dažnių pusę, tada atsitiktinės sklidimo laiko nustatymo paklaidos gali būti sumažintos. Tyrimo rezultatai parodė, jog tam tikram ribotos juostos kanalui parinkto optimalaus skleisto spektro signalo juosta gali būti ir platesnė už kanalo juostą.

Pateikta spaudai 2008 12 03