

## The fast technique for calculation of ultrasonic field of rectangular transducer

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### Abstract

The need for fast calculation technique of ultrasonic field of rectangular transducer is related to design of cylindrical transducer arrays for medical applications. These transducer arrays are 10mm diameter, approximately 5mm length and consist of 16-32 rectangular elements. In order to determine optimal parameter transducer array it is needed to investigate what can be "seen" under different conditions with each type of transducer array. As the total ultrasonic field of cylindrical array is the sum of the field created by the separate elements the key part of the model becomes calculation of the ultrasonic field of rectangular transducer. There are many works devoted to the calculation of the ultrasonic fields of the transducers possessing different contour of active surface.

In the work present there are proposed fast modification of the diffraction based technique for calculation of the ultrasonic field of rectangular transducer. It was show that the velocity potential can be calculated only in the plane of transducer. The velocity potential and pressure field in other point are calculated using interpolation. The application of the adaptive sampling in time domain enables essentially increase calculation time and at the keep necessary accuracy.

**Keywords:** Ultrasonic field, rectangular transducer, diffraction approach, fast modeling.

### Introduction

The conventional phased arrays, used in medicine, usually consist of large number of narrow rectangular elements and are investigated quite widely. The different problems have been met in the development of cylindrical transducer arrays used in ultrasonic endoscope. In this case the number of elements and beam steering possibilities including focusing are limited mainly due to energy recourses. In order to predict images of what quality can be obtained using different number of elements the computer model of a cylindrical transducer array should be developed. The key part of such model is ultrasonic field calculation of a rectangular transducer. There are many techniques and publications devoted to this topic, however main disadvantage of them is the fact that they are relatively slow, mainly due to the need of small sampling steps in the space and time domains at high frequencies.

Objective of this work is to develop method for a fast calculation of ultrasound field of a rectangular shape transducer, which is used in a construction of a cylindrically placed transducers array.

### Basic approach

There are many methods [3,4,6] which enable to calculate the pulse response and ultrasound fields of transducers possessing different geometry. The theory is based on the Huygens's principle which states that the pulse response of the velocity potential  $h(t,x,y,z)$  at any observation point at a given time instant is equal to the sum of ultrasonic waves coming from all elementary segments  $dS$  of the active transducer surface

$$h(t,x,y,z) = \frac{1}{2\pi} \int_S \frac{\delta(t-r/c)}{r} dS, \quad (1)$$

where  $t$  is the time,  $r$  is the distance from the observation point to the transducers elementary segment,  $c$  is the ultrasound velocity in the medium,  $S$  is the active

transducer surface. In many articles [1-6] it was shown that in the case of a piston like vibration of the transducer surface, the pulse response  $h(t,x,y,z)$  at a given time and observation point is proportional to arc of circular equidistant lines intersecting with the transducers face. Then the distribution of the acoustic pressure can be found from the expression

$$p(t,x,y,z) = \rho \frac{\partial h(t,x,y,z)}{\partial t} \otimes u(t), \quad (2)$$

where  $\rho$  is the density of the medium,  $u(t)$  is the wave from of the particle velocity of the transducer surface and the symbol  $\otimes$  denotes the convolution operation.

### Basic method for rectangular transducer

So, in order to calculate the pulse response  $h(t,x,y,z)$  it is necessary to find out the arc angles of equidistance lines intersection with the transducers surface at each time instance. In work [4] and many later it was demonstrated that for a circular transducer these angles can be found analytically. In the case of numerical solutions it can be done for any transducer shape [5, 6] and the rectangular transducer is not exception.

The meaning of these equidistant lines for the case of a rectangular transducer is explained in Fig.1. There can be an unlimited number of the equidistant lines corresponding to the time instances  $t \in [t_1, t_2]$ ,  $t_1 = r_1/c$  corresponds to the arrival time of the waves from the closest elementary segment of the transducer surface and  $t_2 = r_2/c$  to the farthest. The number of these lines used in calculation depends on the sampling interval in the time domain  $\Delta t$ .

In order to calculate the angle of the arc intersecting with the surface of the transducer it is necessary to determine the angles in polar coordinates of the points at which the equidistant line crosses the contour line of the transducer (Fig.2).

Then the velocity potential can be expressed as

$$h(t, x, y, z) = \frac{c}{2\pi} \sum_{i=1}^{N(t)} [\Phi_2^{(i)}(t) - \Phi_1^{(i)}(t)] \quad (3)$$

where  $N(t)$  is the total number of arc segments for the observation point  $P(x, y, z)$  intersecting with transducers boundaries for a given time instance,  $\Phi_1^{(i)}(t)$  and  $\Phi_2^{(i)}(t)$  is in and out angles for the equidistant line corresponding to the time instance  $t$ . It is necessary to state that the equidistant line for an arbitrary selected point can cross the contour of the transducer many times. In the case of the rectangular transducer the maximal possible number of cross-section points is 8. This procedure must be repeated for all time instances  $t \in [t_1, t_2]$ .

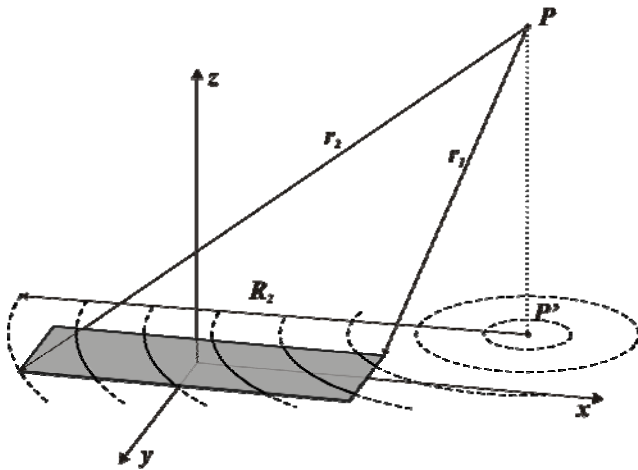


Fig. 1. Explanation of the equidistant lines for the case of a rectangular transducer

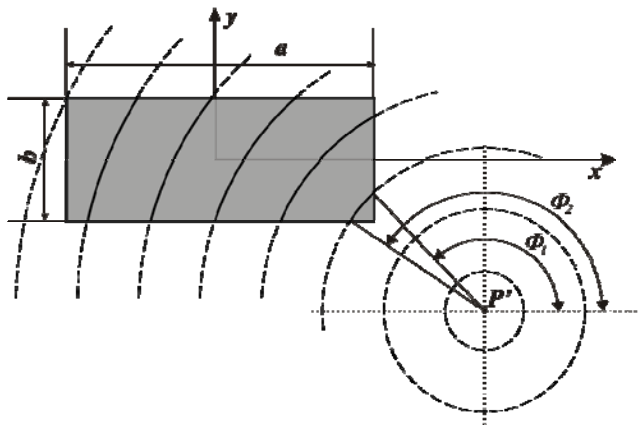


Fig. 2. Explanation of in and out arc angles  $\Phi_1$  and  $\Phi_2$  used in calculation of a velocity potential.

However, this a rather simple approach possesses one very essential disadvantage. The relatively high frequency (up to 10MHz) leads to small sampling steps in the space and time domains and as a consequence to the huge number of points at which the calculation should be performed. So, the calculation of a total field requires a long time of processing. Additionally, the time instances  $t_1$ ,  $t_2$  of the arrival wave from the closest and farthest points of the transducer surface becomes closer to each other for a spatial point situated at bigger distances, that is the time interval  $\Delta t_{12} = t_2 - t_1$  reduces. So, in order to keep the

same accuracy in a pulse response calculation, the sampling step in the time domain should be smaller for the point situated at bigger distances from transducers. Usually in a conventional pulse response point the same sampling step in the time domain is used for all special points under the analysis. This leads to the over-sampling at close distances and as a consequence to the long calculation time.

### Optimization of the method

In optimization was exploited one of the essential features of the method described above. In general, the arc angles are the same for all spatial points which possess the same projection on the transducer surface. Of course, equidistant arcs correspond to different propagation time.

So, the idea is that in the case when the planar rectangular transducer is placed in the  $xOy$  plane (Fig.1), only calculation of the velocity potential at the point  $P'(x, y, 0)$  is needed in order to obtain the pulse response at any point  $P(x, y, z)$  possessing the same projection  $P'(x, y, 0)$ . It means that once the velocity potential is found at the point  $P'(x, y, 0)$  the pulse responses of the velocity potential at the points  $P(x, y, z)$  can be obtained simply using interpolation.

Such an approach was implemented in the MatLab medium. The method was designed in such a way that all main calculations are done with data arrays instead of application of *for* loops. Such technique also enables to reduce the calculation time and simplifies task description in the programs. The general algorithm (without application of interpolation) can be described by following steps:

1. The vectors  $\mathbf{X}, \mathbf{Y}, \mathbf{Z}$  containing the spatial coordinates of a point under analysis are created. The size of the vector is a number of points  $N_P$ ;
2. The distances  $\mathbf{R}_2$  between observation points projection and the furthest point of the transducer surface are determined

$$\mathbf{R}_2 = \sqrt{\left(\mathbf{X} + \frac{a}{2}\right)^2 + \left(\mathbf{Y} + \frac{b}{2}\right)^2}, \quad (4)$$

where  $a$  and  $b$  are the length and the width of the transducer (Fig.2);

3. The distances  $\mathbf{r}_2$  between observation points and the furthest point of the transducer surface are calculated

$$\mathbf{r}_2 = \sqrt{\mathbf{Z}^2 + \mathbf{R}_2^2}; \quad (5)$$

4. The time instances corresponding to arrival of the waves from the farthest point and from the transducer plane are determined

$$\mathbf{T}_{max} = \frac{\mathbf{r}_2}{c}, \quad (6)$$

$$\mathbf{T}_{min} = \frac{\mathbf{Z}}{c}; \quad (7)$$

5. The maximal number of samples between  $\mathbf{T}_{min}$  and  $\mathbf{T}_{max}$  is determined

$$N_{st} = \max\left(\frac{\mathbf{T}_{max} - \mathbf{T}_{min}}{\Delta t} + 1\right), \quad (8)$$

where  $\Delta t$  is the interval of the sampling in the time domain. This value is needed for determination of the sizes of arrays used in following steps;

6. The matrix containing all time instances necessary for calculation of the pulse response of velocity potential for all point under analysis is created

$$\mathbf{T}_{int} = \mathbf{T}_{min}^T \cdot \mathbf{1} + \mathbf{1} \cdot [(0 \dots N_{st} - 1) \cdot \Delta t], \quad (9)$$

where  $\mathbf{1}$  is the vector of ones with the same size as the vector  $\mathbf{T}_{min}$ , the upper index  $T$  denotes the transpose operation. The size of the new matrix is  $N_p$  by  $N_{st}$ ;

7. The radii of all equidistant arcs necessary in modeling are determined

$$\mathbf{R} = \sqrt{c^2 \cdot \mathbf{T}_{int}^2 - \mathbf{Z}^2}. \quad (10)$$

8. The coordinates of intersection points between the equidistant arcs and extended lines of the boundaries of the rectangular transducer are found

$$\mathbf{H}_{p1} = \pm \sqrt{\mathbf{R}^2 - \left(\frac{a}{2} - \mathbf{X}\right)^2}, \quad (11)$$

$$\mathbf{H}_{p2} = \pm \sqrt{\mathbf{R}^2 - \left(-\frac{a}{2} - \mathbf{X}\right)^2}, \quad (12)$$

$$\mathbf{V}_{p1} = \pm \sqrt{\mathbf{R}^2 - \left(\frac{b}{2} - \mathbf{Y}\right)^2}, \quad (13)$$

$$\mathbf{V}_{p2} = \pm \sqrt{\mathbf{R}^2 - \left(-\frac{b}{2} - \mathbf{Y}\right)^2}. \quad (14)$$

9. The intersections angles are calculated using four-quadrant inverse tangent

$$\Phi_{H1} = \arctan\left(\frac{\mathbf{H}_{p1}}{a/2 - \mathbf{X}}\right), \quad (15)$$

$$\Phi_{H2} = \arctan\left(\frac{\mathbf{H}_{p2}}{-a/2 - \mathbf{X}}\right), \quad (16)$$

$$\Phi_{V1} = \arctan\left(\frac{b/2 - \mathbf{Y}}{\mathbf{V}_{p1}}\right), \quad (17)$$

$$\Phi_{V2} = \arctan\left(\frac{-b/2 - \mathbf{Y}}{\mathbf{V}_{p2}}\right). \quad (18)$$

10. Correction of the angles depending on the observation point position with respect to the transducer is performed. If the point is inside boundary, the angles of all arcs not reaching any edge of the transducer are set to  $2\pi$ . The angles of arcs for the points which are on the edge and arc does not crosses other boundaries are set to  $\pi$ . If the point is outside the transducer boundaries, the angles of arcs not reaching any edge of transducer are set to zero.
11. The angles are sorted in ascending order and the angles with odd numbers are subtracted from the angles with even number. This gives arc segments

intersecting with the transducer active surface. These segments are used for calculation of the pulse response of velocity potential according to Eq.3.

12. The acoustic pressure field is obtained according to Eq.2.

It is necessary to mention that due to the symmetry of a rectangular transducer with respect to  $x$  and  $y$  axis (Fig.3) it is reasonable to perform calculations only for a quarter, what also enables to save a modeling time.

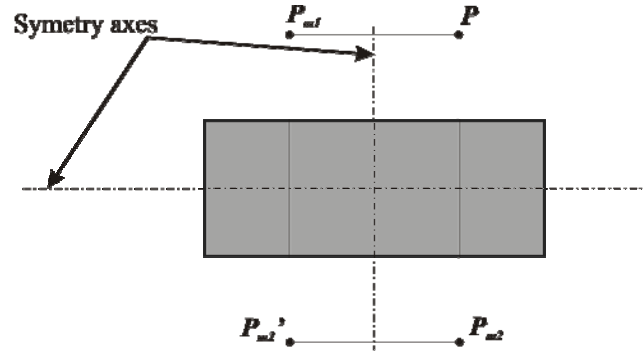


Fig. 3. Transducer geometry viewed from a top with symmetry axes and the observation point  $P$  related to mirror points  $P_{m1}$ ,  $P_{m2}$  and  $P_{m2}$ .

As have been mentioned before the selection of the sampling frequency or interval is very important issue for the technique under analysis. If the sampling interval will be set uniformly for the whole calculation problem, then or the pulse response at further distance will be not accurate enough or the pulse response of closest points will be over-sampled. The purposed solution for this problem is to use a dynamic sampling frequency which depends on the distance of the observation point with respect to the transducer surface. In this case not the sampling interval is constant for all observation points, but the number of points in the pulse response. In Fig. 4 are shown three observation points  $P_1$ ,  $P_2$ ,  $P_3$  with the same  $x$  and  $y$  coordinates, but with different distances from the transducers surface plane.

At first the velocity potential is calculated for the point  $P_1$  according to the method described above. In the next steps the velocity potentials of points  $P_2$  and  $P_3$  are calculated using linear interpolations from the  $P_1$  point velocity potential. The number of points in the pulse responses of points  $P_1$ ,  $P_2$ ,  $P_3$  are the same and defined by Eq.8. So, interpolation is done with the same number of time instances for all points with the same  $x$  and  $y$  coordinates. Of course, the sampling step at each point is different and determined by

$$\Delta t = \frac{\mathbf{T}_{max} - \mathbf{T}_{min}}{N_{st}}. \quad (19)$$

In the following step the velocity potential  $h(t,x,y,z)$  is interpolated at time instances corresponding to the calculated radii  $\mathbf{R}(t,x,y,z)$ . According to the approach proposed instead of the calculation of the velocity potential function for points above  $xOy$  plane only the time axis is recalculated and interpolated from the pulse response of the observation point projection. The velocity potential pulse responses for point  $P_1$ ,  $P_2$ ,  $P_3$  are shown in Fig. 5.

The velocity potential interpolation is faster than its direct calculation. It is complicated to estimate the level of compilation time reduction because it is related to the accuracy of the modeling and should be estimated in more details. However it can be stated that the calculation time was reduced at least a few times.

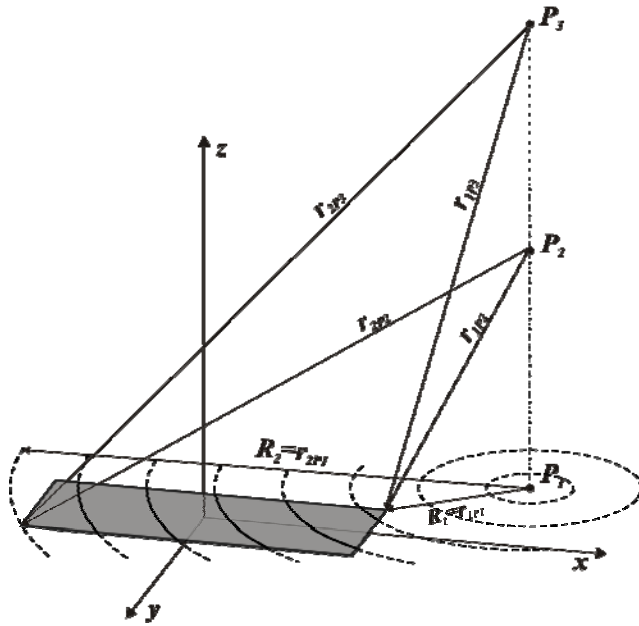


Fig. 4. Three points  $P_1$ ,  $P_2$  and  $P_3$  with the same  $x$  and  $y$  coordinates but with different  $z$  coordinates. Velocity potential patterns for these points are the same.

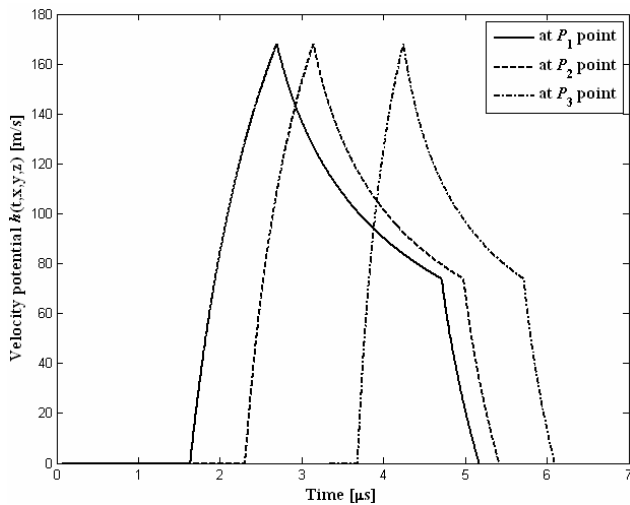


Fig. 5. Velocity potential patterns for three different points with the same  $x=4.3$  mm and  $y=2.5$  mm coordinates, but different  $z$  coordinate for the 2x5 mm transducer.

### Calculation results

In order to test the model developed the ultrasonic field of the rectangular transducer with the length 5mm and the width 2mm was calculated. The 5 MHz transducer generates 3 period burst with the Gaussian envelope. The waveform of the signal used in the modeling is presented in Fig.6. The obtained ultrasonic field in the plane across the transducer is presented in Fig.7. In order to see better the details of the field the results are presented as a surface in a 3D space and lightened with a light source.

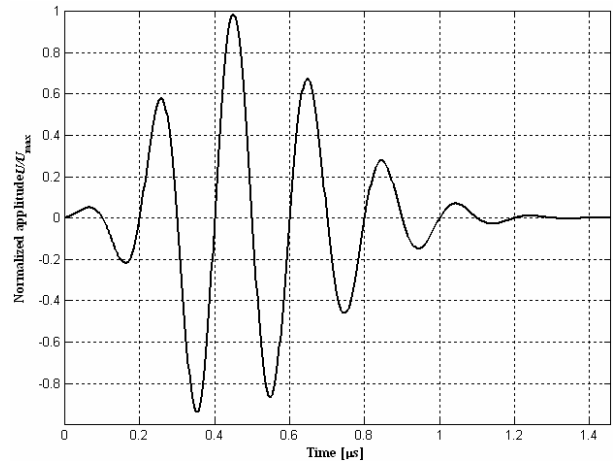


Fig. 6. Signal used in calculations. The central frequency 5 MHz, the number of periods 3, the asymmetry coefficient 0.5

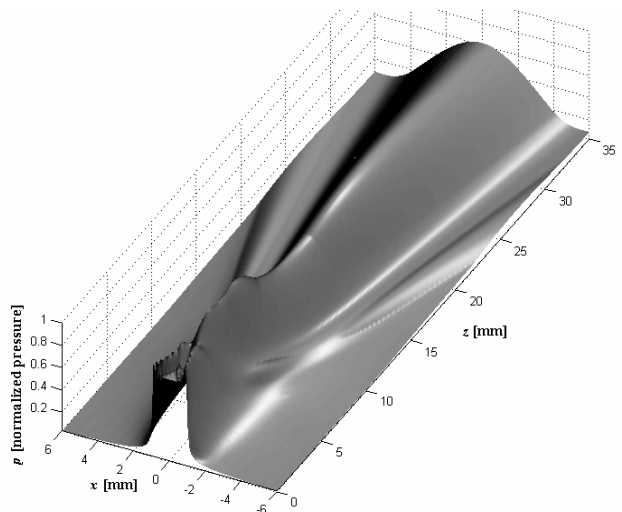


Fig. 7. Calculated spatial peak to peak pressure distribution of transducer 2x5 mm in  $xOz$  plane perpendicularly to the transducer surface at 5 MHz frequency.

### Conclusions and comments

A new fast technique for calculation of an ultrasonic field of the rectangular transducer was developed, which enables essentially, at least a few times to reduce the modeling time.

In the next stages of the investigation it is planned to perform a detailed evaluation of the accuracy of the method comparing the results with ultrasonic fields obtained by other models and measured experimentally.

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**Spartusis stačiakampio keitiklio lauko apskaičiavimo metodas**

**Reziumė**

Tokio metodo poreikis atsirado projektuojant mažas ultragarsines cilindrinės gardeles taikyti medicinoje. Jos yra apie 10 mm skersmens ir 6 mm ilgio ir susideda iš 16-32 stačiakampių pjezoelementų. Šių elementų ilgis yra apie 5 mm, plotis - tarp 0,5-2 mm (priklauso nuo jų bendro skaičiaus). Norint nustatyti optimalius šių gardelių parametrus, faktiškai reikia ištirti, ką ir kokių tikslumu galima matuoti ir „matyti“ su kiekvieno tipo gardele. Kadangi bendras gardelės sukuriamas laukas yra suma laukų, sukuriamų kiekvieno elemento atskirai, stačiakampio keitiklio lauko apskaičiavimo metodas tampa esminis. Yra gana daug keitiklių laukų skaičiavimo metodų, tačiau jie dažniausiai per lėti. Šiame darbe pateikiama žinomo difrakcinio metodo atmaina, pritaikyta išimtinai stačiakampio keitiklio laukui apskaičiuoti. Parodyta, kad skaičiavimo spartą galima padidinti apskaičiuojant greičio potencialą keitiklio plokštumoje, o jo laikinė priklausomybė bet kuriame erdvės taške gaunama atliekant interpoliaciją. Straipsnyje pateiktas metodo aprašymas, duota gaunamų laukų pavyzdžių.

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