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Influence of the dispersion on measurement of phase and group velocities of Lamb waves

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Abstract

Influence of the dispersion on the measurement of phase and group velocities of Lamb wave in a plate is investigated in the work presented. The velocity of propagating guided waves is important parameter of their application in non-destructive testing. Accurate determination of the Lamb wave velocity is complicated by the fact that it is frequency dependent. In order to determine in what way the dispersion affects the uncertainties of the phase and group velocity measurements the investigation was carried out. The finite element model of the 2 mm thickness and 200 mm length aluminium plate was used in order to obtain the signals for analysis. The A0 mode was excited by adding a shear force to one of the ends of the plate. The excitation signal was 300 kHz burst with the Gaussian envelop. Determination of the velocities was based on measurement of the propagation time. The propagation time was measured using a zero-crossing technique. Investigations demonstrated that the obtained values of phase velocities depend essentially on the number of periods in the burst which was used for measurements by the zero-crossing technique. The obtained regularities enable to compensate part of uncertainties of the phases velocity measurement. Moreover it enables to reconstruct the segment of the phase velocity dispersion curve in narrow frequency ranges.

Keywords: Lamb wave, dispersion, phase and group velocity, measurement, finite element.

Introduction

Guided ultrasonic waves are used in many non-destructive testing (NDT) and material evaluation techniques (NDE). The propagation velocity of these waves is a key parameter defining efficiency and accuracy of these techniques. However, the guided waves possess a dispersion phenomenon. This leads to the presence of two different propagation velocities – phase and group velocities, both dependent on a frequency. The dispersion phenomenon is characterized by dispersion curves which determine the propagation velocity of different guided wave modes at different frequencies. These velocities correspond to propagation of harmonic, single frequency waves. However, in NDT the pulsed ultrasonic waves are mainly used. The waveform of signals usually is some kind of burst with the Gaussian envelope and, of course, in the frequency domain covers some bandwidth. So, in the measurements of the phase and the group velocities the question arises not only about the value of the velocity itself, but also about the frequency to which it corresponds.

In most cases the propagation velocity is determined using measurement of the propagation time. The delay time can be measured using different techniques: signal maximum position in the time domain, zero-crossing technique [1,2], cross-correlation or optimization based [3].

The objective of the work presented is to investigate in detail the Lamb wave velocity measurement technique based on the zero-crossing approach and using the signals obtained by finite element modelling.

The model

The Lamb waves possess infinite number of modes, however in most cases only lowest modes are exploited. The asymmetric A_0 mode of Lamb wave propagating in a 2 mm thickness aluminium plate was selected for analysis. The segment of dispersion curves of this mode in the frequency range 0-500kHz is presented in Fig.1. The dispersion curves were calculated assuming that the propagation velocity of the longitudinal wave is $c_L = 6350$m/s and of the shear waves is $c_T = 3100$ m/s.

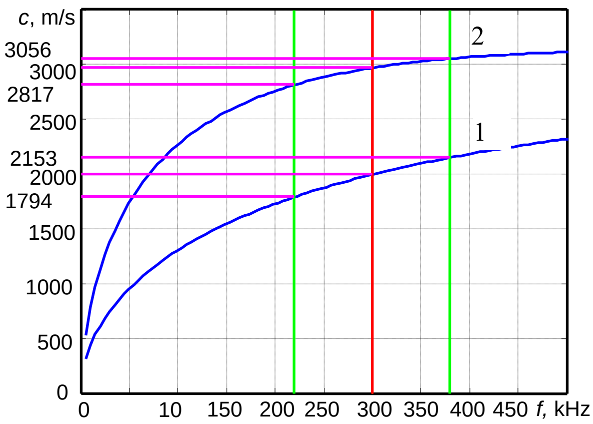

Fig.1. **The dispersion curves of phase (1) and group (2) velocities of A_0 Lamb wave mode propagating in 2mm thickness aluminium plate**

In order to obtain signals necessary for analysis of different measurement techniques the 2D finite element model of the aluminium plate was created (Fig.2). The following parameters of the aluminium plate were used in the model: density $\rho = 2780\,\text{kg/m}^3$, Young modulus E = 73.1 GPa, the Poisson's ratio $\nu = 0.3$. The sampling step in the spatial domain was *dx*=0.1mm and *dt*=0.15μs in the time domain. The A_0 mode was excited by attaching a tangential force to one of the plate edges (Fig.2). The waveform of the excitation signal is presented in Fig.3 and the frequency spectrum in Fig.4. As can be seen at - 6 dB level it covers the frequency range from 220 kHz up to 380 kHz. The phase and the group velocities for these frequencies obtained from the dispersion curves are presented in Table 1 and in Fig.1. The propagation of the Lamb wave was modelled for the time interval up to 100 μs.

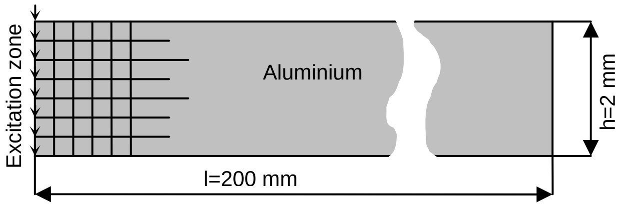

Fig.2. **The finite element model for investigation of propagation of the A_0 mode Lamb wave in aluminium plate**

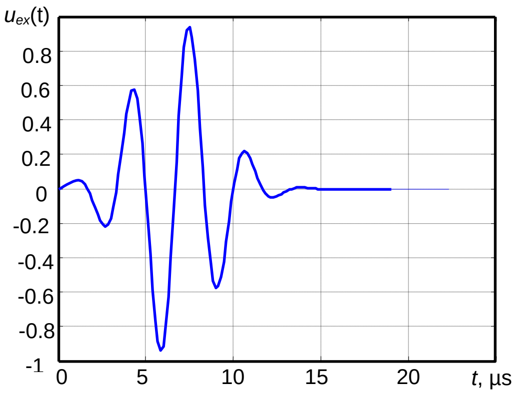

Fig.3. **The waveform of the excitation signal**

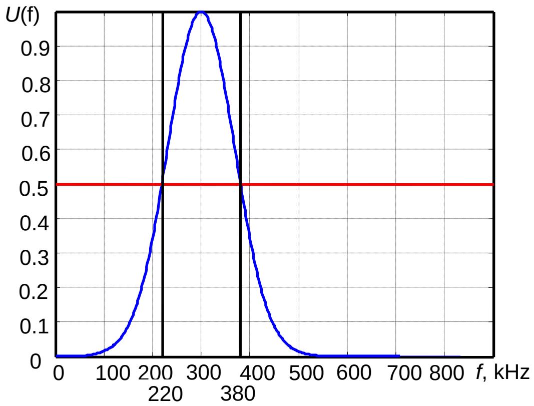

Fig.4. **The frequency spectrum of the excitation signal**

Table 1. **The of phase and group velocities of Lamb waves in aluminium plate at different frequencies**

Frequency, KHz	Phase velocity , m/s	Group velocity, m/s
220	1794	2817
300	1999	2972
380	2153	3056

The Lamb waves usually are measured by scanning the contact type transducer over top surface of the plate. Such transducer is sensitive to the normal component of the particle velocity of propagating waves. In order to obtain the signals as much as possible closer to experimental ones the particle velocity at each node of a finite element grid corresponding to the upper surface of the plate were recorded. The obtained B-scan image is presented in Fig.5. The direct Lamb wave and reflected by the end of the plate can be observed clearly. Also it can be seen that phase and group velocities are different. The propagation time of the phase velocity was estimated using the zero-crossing technique. In short it is explained by Fig.6. According to this technique some threshold level U_{thr} is set in advance. Using this level the approximate position in the time domain of the signal corresponding to the propagating wave is determined. In the second step accurately the time instants at which the signal crosses the zero amplitude line are determined. As it is shown in Fig.6 the four zero crossing instants $t_1(x_n), t_2(x_n), t_3(x_n), t_4(x_n)$ are determined for each signal $u_{x_n}(t)$, where x_n is the measurement positions along the plate, $n = 1 \div N$, N is the total number of the measurement positions. The dependency of the propagation time measured using the first zero-crossing point $t_1(x)$ versus the distance is presented in Fig.7. As can be seen in some distance intervals it increases almost linearly, but then jumps sharply down. It is explained by the fact that the phase and group velocities are different and the half periods of signal with the distance in some sense "moves" inside the signal envelop. At the "jump" position the threshold technique fits to another period of the signal. The distance between two "jumps" is very important because it determines the maximal distance at which the phase velocity can be measured. The propagation time measured using other zero-crossing points $t_2(x)$, $t_2(x)$ $t_3(x)$ possesses similar regularities

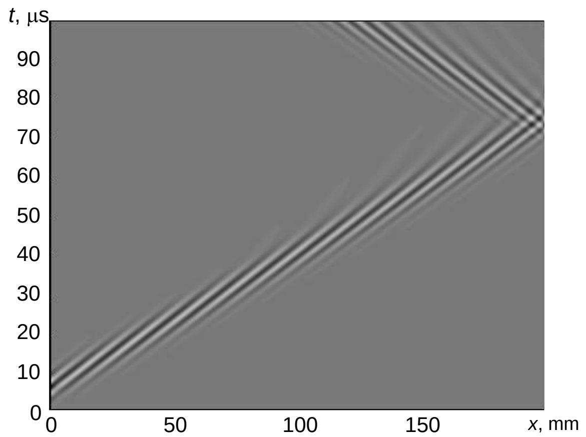

Fig.5. **The B-scan image of the normal component of the particle velocity on the surface of the plate**

In general the zero-crossing instants correspond to the zero phase of the signal. So, the phase velocity can be determined from

$$c_{ph} = \frac{x_{n_2} - x_{n_1}}{t_m\left(x_{n_2}\right) - t_m\left(x_{n_1}\right)}, \tag{1}$$

where m = 1, 2, 3 or 4. However simple application of this equation by using measurements at two positions along x axis leads to big uncertainties. On the other hand in the distance intervals between two "jumps" $x \in \left]x_k \div x_{k-1}\right[$, the propagation time can be approximated by equation

$$t_m\left(x_n\right) = \frac{x_n}{c_{A_0}} + t_0, \tag{2}$$

where $k = 1 \div K$, K is the total number of jumps. So, the c_{A_0} was determined by the least-squares method using Eq.2. Depending on the number of "jumps" several values of c_{A0} can be obtained, which correspond to different propagation distance $\bar{x}_k$, where $\bar{x}_k = (x_k + x_{k-1})/2$.

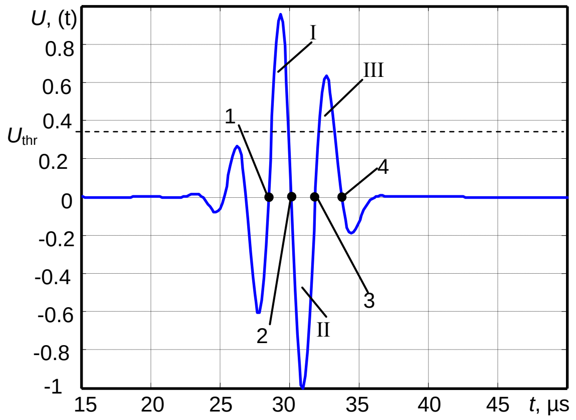

Fig.6. **Explanation of the zero-crossing technique. 1 ,2, 3, 4 are the zero crossing points used in phase velocity estimation; I,II,III are half periods of the signal used in the analysis; U_{thr} is the threshold level**

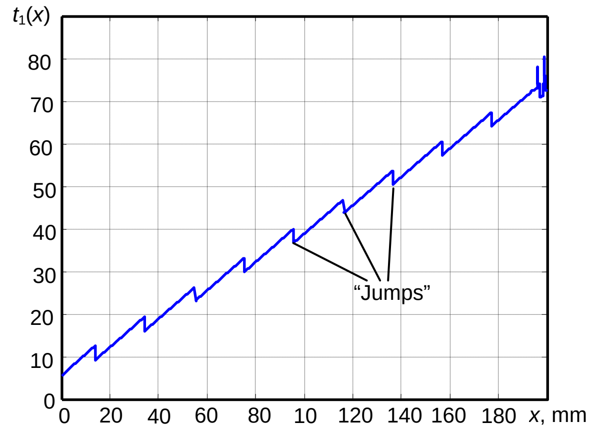

Fig.7. **The propagation time versus distance determined using the first zero-crossing point.**

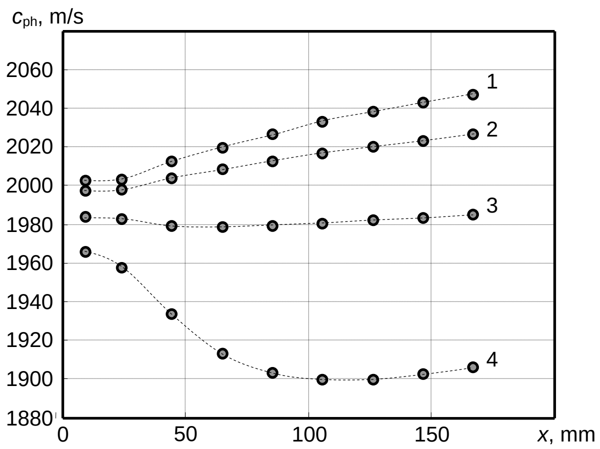

Fig.8. **The propagation velocity of A_0 mode Lamb wave measured at different distances using different zero-crossing points. 1, 2, 3, 4 correspond to the number of zero-crossing point in the burst of the signal which was used for phase velocity determination**

Moreover, the set of these values $\{c_{A_0,k,m}\}$ is obtained using different zero crossing points (Fig.6). The obtained results are presented in Fig.8.

As can be seen the value of phase velocity obtained using the first and the second zero-crossing points increases with a distance. The phase velocity obtained using the fourth zero crossing point demonstrates opposite regularity. Most stable results are obtained using the third zero-crossing point. These dependencies have a systematic character and probably are related to the dispersion phenomena and can be explained by following considerations:

1. The signal is not harmonic, but possess some bandwidth;
2. At the excitation point the components corresponding to different frequencies are more or less uniformly distributed in the signal (in the time domain);
3. During propagation of the Lamb waves the faster components start to concentrates in the front part of the signal and slower ones at the end of the signal;
4. Depending on which part of the signal in the time domain is used for phase velocity estimation it will correspond not to the central frequency, but to the frequency which is concentrating in this part of the signal.

So, the question arise how the redistribution of the different frequency components can be detected or measured. Analysis in the frequency domain using moving narrow window will be not efficient because a narrow window in the time domain lead to big uncertainties in the frequency domain. One of the ways is to measure accurately the duration of each half period in the signal and to monitor their variations depending on a distance. As the propagation times are calculated according to different zero-crossing points, the duration of half periods can be determined according to

$$T_{0.5,\mathrm{I}}(x) = t_2(x) - t_1(x),$$
$$T_{0.5,\mathrm{II}}(x) = t_3(x) - t_2(x), \qquad (3)$$
$$T_{0.5,\mathrm{III}}(x) = t_4(x) - t_3(x),$$

where $T_{0.5,\mathrm{I}}, T_{0.5,\mathrm{II}}, T_{0.5,\mathrm{III}}$ are duration of the first, the second and the third half periods of the signal. The obtained dependencies are presented in Fig.9. As can be seen the duration of the half period varies essentially. These variations can be explained by the fact that due to difference of the phase and the group velocities each of the half periods "moves" inside the signal during wave propagation. For a better understanding the obtained durations $T_{0.5,\mathrm{I}}, T_{0.5,\mathrm{II}}, T_{0.5,\mathrm{III}}$ can be easily converted into the equivalent frequencies by

$$f_{0.5,\mathrm{I}}(x) = 0.5/T_{0.5,\mathrm{I}},$$
$$f_{0.5,\mathrm{II}}(x) = 0.5/T_{0.5,\mathrm{II}}, \qquad (4)$$
$$f_{0.5,\mathrm{III}}(x) = 0.5/T_{0.5,\mathrm{III}}.$$

In order to obtain more reliable results the mean values $\bar{f}_{0.5,\mathrm{I}}(x), \bar{f}_{0.5,\mathrm{II}}(x), \bar{f}_{0.5,\mathrm{III}}(x)$ of these equivalent frequencies were calculated for each interval between "jumps". The obtained results are shown in Fig.9. As can be seen the equivalent frequency of the first half period increases at bigger propagation distances. The equivalent frequency of the third half period demonstrates opposite regularity - it decreases with distance. It can be explained by the fact that in the frequency ranges under analysis phase and group velocities increase with a distance. So, components corresponding to higher frequencies are faster and arrive earlier and as a consequence they are more concentrated in the first part of the signal.

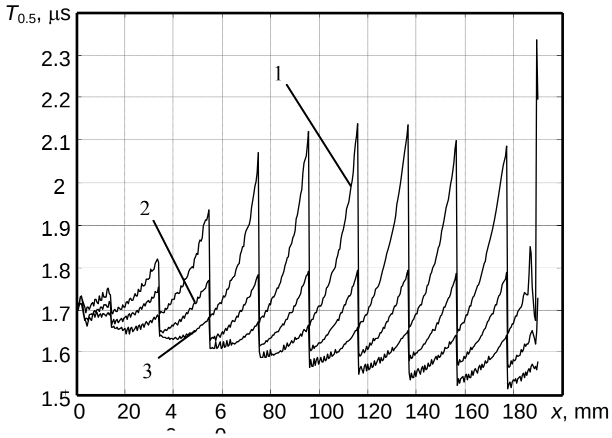

Fig.9. **Durations of different half periods of the signal versus propagation distance. 1, 2, 3 are the numbers of the half period**

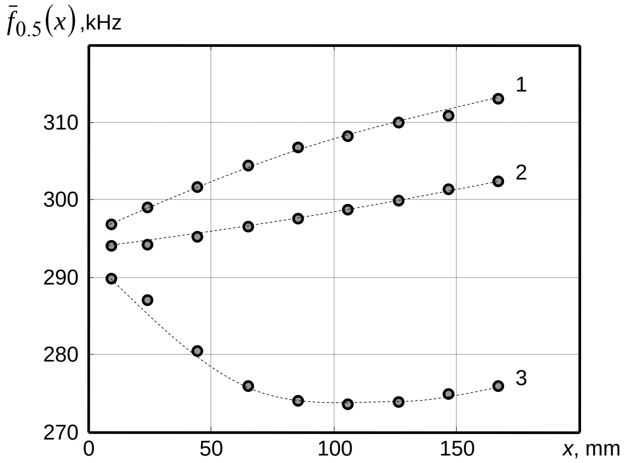

Fig.10. **The equivalent frequencies of different half periods of the signal; 1, 2, 3 are the numbers of the half period**

The obtained regularities can be exploited for more accurate estimation of the phase velocity. As it was shown in Fig.6, the estimated phase velocity value depends essentially on the number of a zero-crossing point which was used. It can be assumed that these values are not just scattering the results caused by some uncertainties but correspond to different frequencies. In order to test this hypothesis the following correction was introduced:

1. The second zero-crossing point is between the first and the second half period and the third zero-crossing point is between the second and the third half period, so the mean values of the equivalent frequency were calculated

$$\bar{f}_{0.5,1}(x) = \left(f_{0.5,\mathrm{I}}(x) + f_{0.5,\mathrm{II}}(x)\right)/2,$$
$$\bar{f}_{0.5,2}(x) = \left(f_{0.5,\mathrm{II}}(x) + f_{0.5,\mathrm{III}}(x)\right)/2. \quad (5)$$

2. These equivalent frequency values were related to the corresponding phase velocities and both values overlaid on the dispersion curve graph (Fig.11).

The obtained results demonstrate a very good coincidence of the phase velocity values calculated using modelled signals and theoretical dispersion curve.

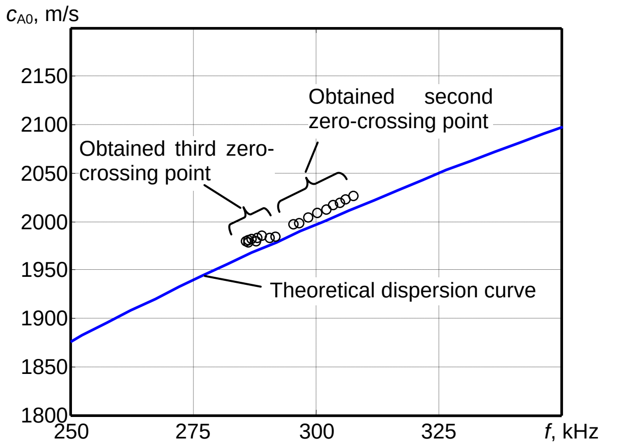

Fig.11. **The phase velocities of A_0 mode Lamb wave in 2mm thickness aluminium plate obtained using simulated signals and theoretical dispersion curve**

Conclusions

It was demonstrated that measurements of the phase velocity of guided waves are much more complicated comparing to conventional ultrasonic measurements. It was proposed to use precise measurement of the duration of different half periods of the signal and to use this information for compensation of uncertainties of the phase velocity measurement. The proposed technique enables not only to reduce uncertainties, but also to reconstruct segment of the dispersion curve.

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Dispersijos įtakos Lembo bangų fazinio ir grupinio greičio matavimams tyrimas

Reziumė

Atliktas dispersijos įtakos nukreiptųjų bangų fazinio ir grupinio greičio matavimams tyrimas. Naudojant Lembo bangas ultragarsiniuose neardomuosiuose bandymuose, svarbu turėti tikslius duomenis apie šių bangų greitį tiriamajame objekte. Lembo bangų greičio matavimo rezultatų tikslumą apsunkina šių bangų fazinio ir grupinio greičio priklausomybė nuo dažnio. Šiai įtakai nustatyti buvo ištirtas fazinio ir grupinio greičio matavimo metodas, pagrįstas sklidimo laiko matavimu naudojant perėjimo per nulį būdą. Tyrimo metu buvo naudojami signalai, gauti baigtinių elementų metodu modeliuojant Lembo bangos A_0 modos sklidimą 2 mm storio ir 200 mm ilgio aliuminio plokštelėje. Bangoms žadinti buvo naudojamas įprastas ultragarsinių neardomųjų bandymų 300 kHz Gauso amplitudės signalas. Tyrimais nustatyta, kad skirtingų signalų periodų trukmė kinta bangai sklindant plokštele ir tai sąlygoja rezultatų priklausomybę nuo matavimams naudoto signalo periodo bendrajame vietos signale. Nustatytieji dėsningumai ne tik įgalino sumažinti vidutinės išmatuotos fazinio greičio vertės neapibrėžtį, bet ir atkurti dispersinės kreivės segmentą šiame dažnių diapazone.

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