

Quasi-electrostatic charge distribution analysis of the planar interdigital transducer

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Abstract

This paper presents an analysis of quasi-electrostatic charge density distribution of the surface acoustic wave (SAW) planar interdigital transducer (IDT). Approximation is realized using Chebyshev polynomials and Green's function without transformation in the spatial spectrum domain. The analysis method is suitable for a long IDT's with arbitrary metallization ratios and electrode-polarity sequences. It is assessing the nonuniform charge distribution of electrodes, interaction of local electric field, the end effects. It is easy to calculate the capacity of each electrode. To reduce errors and duration of calculations for a long IDT as far as possible, it is appropriate to refuse from iterative methods and use the analytical expressions of convolutions. The analytic expressions of the convolution the first six terms of the Chebyshev polynomial with the Green's function are obtained. The total error of approximation is found by calculating the difference between the solutions for finite and infinite periodic systems.

Keywords: SAW, interdigital transducer, IDT, quasi-electrostatic charge distribution, Chebyshev polynomial approximation, Green's function, arbitrary metallization ratio.

Introduction

To analyse the quasi-electrostatic charge density distribution of surface acoustic wave transducers (called SAW IDT - Surface Acoustic Wave Interdigital Transducer) the charge density distribution must be calculated counted accurately as possible. Inaccuracy can affect on the SAW transducer frequency response and adequacy of analysis. The calculations complicate the fact that charge density increases sharply on the edge's, as well we cannot examine the individual electrode charge distribution regardless influence of the neighboring electrodes [1, 2]. Real SAW transducer has hundreds of electrodes, whose interactions must be assessed. As a result, it dramatically increases computing duration. Charge distribution analysis simplifies the use of polynomials. Their coefficients are calculated by solving the linear algebraic equation systems. Approximation can be realized using Chebyshev polynomials [3] and the method of Green's function [4, 5]. High accuracy is achieved even when using only six terms in the polynomial. Here, it is assessing the nonuniform charge distribution of electrodes, interaction, the end effects. It is easy to calculate the capacity for each electrode, but such approximation errors are still insufficiently explored. Total approximation error could be evaluated in comparison with the ideal model. To reduce errors and duration of calculations as far much as possible, it is appropriate to refuse from iterative methods, but to use the analytical expressions.

Quasi-electrostatic charge density analysis

For approximation of the infinite absolute integrated function, as the proximity criterion a standard deviation is used. This deviation could minimize, if the function is approximated by series, which consists of orthogonal base functions. It is considered, that the most appropriate basis functions are the Chebyshev polynomial, the weight

$1/\sqrt{1-x^2}$ [3]. Using these, the function is approximated as follows polynomial:

$$\tilde{\sigma}(x, d) = \begin{cases} \left(1 - \left(2\frac{x}{d}\right)^2\right)^{-\frac{1}{2}} \sum_n \alpha_n T_n\left(2\frac{x}{d}\right), & |x| < \frac{d}{2}; \\ 0, & |x| > \frac{d}{2}, \end{cases} \quad (1)$$

where $T_n\left(2\frac{x}{d}\right)$ is the n -th order Chebyshev polynomial; d is the electrode width; α_n - the polynomial coefficients. The formula is valid, when the coordinate of the center of the electrode $x_c = 0$.

For the transducer analysis the Fourier transform of electric charge density distribution is used. The polynomial (1) has the convenient expression which described by the Fourier transform:

$$\begin{aligned} \tilde{\sigma}(\beta, d) &= \frac{2}{d} \int_{-\frac{d}{2}}^{\frac{d}{2}} \tilde{\sigma}(x, d) \exp(-j\beta x) dx = \\ &= \pi \sum_n (-j)^n \alpha_n J_n\left(\beta \frac{d}{2}\right), \end{aligned} \quad (2)$$

where $J_n(\beta)$ is the n -th order Bessel function; β is the wave number; $j = \sqrt{-1}$.

The charge density of the electrodes grating (Fig. 1) is expressed by the Green's function method:

$$\Phi_{ei}(x) = -\frac{1}{\pi(\varepsilon_0 + \varepsilon_p^T)} \sum_{j=1}^N \int_{a_j}^{b_j} \sigma_j(x') \ln|x-x'| dx', \quad (3)$$

where $a_i < x < b_i$, $i = 1, 2 \dots N$; $\sigma_j(x')$ is the j -th electrode's charge density distribution; ε_p^T - the dielectric permittivity at constant mechanical tension; N - the number of electrodes. The points with coordinates x' may be only the j -th electrode's area, and the points with coordinates x includes all electrodes.

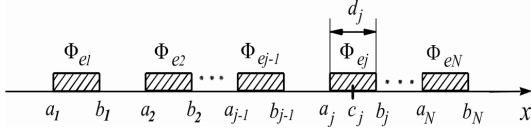


Fig. 1 The electrodes grating

Coordinates must be normalized:

$$\begin{cases} \bar{x} = 2 \frac{x_i - c_j}{d_i}, \\ \bar{x}' = 2 \frac{x_j - c_j}{d_j}, \end{cases} \quad (4)$$

where d_i is the i -th electrode's width; d_j, c_j is the j -th electrode's width and coordinate of the center; x_i, x_j are free to choose the points in the electrodes area.

Then from Eq. 1, 3 and 4 obtained of a charge density expression of the j -th electrode's:

$$\sigma_j(\bar{x}') = 2\pi |\Phi_{ej}| \frac{(\varepsilon_0 + \varepsilon_p^T)}{d_j \sqrt{1 - (\bar{x}')^2}} \sum_{n=0}^5 \alpha_n T_n(\bar{x}'), \quad (5)$$

where $|\Phi_{ej}|$ is the absolute j -th electrode's potential value.

Substituting Eq. 5 to Eq. 3, we get:

$$F_{ei} = - \sum_{j=1}^N |F_{ej}| \int_{-1}^1 \frac{\sum_{n=0}^5 \alpha_n T_n(\bar{x}')}{\sqrt{1 - (\bar{x}')^2}} \ln |\bar{x} - \bar{x}'| d\bar{x}', \quad (6)$$

where $\Phi_{ei} = \Phi_e(\bar{x})$ if the points \bar{x} are on the i -th electrode. For further simplification of the marking, the symbol "-" above the normalized coordinates, we will not show.

The sum charges of the electrodes equal zero and from Eq. 5 and 6 we obtain a system of linear equations:

$$\begin{cases} \frac{1}{2} \sum_{j=1}^N \int_{-1}^1 \Psi(x_{ij}, x_{i+1j}, x') \sum_{l=0}^5 \alpha_l^{(j)} T_l(x') dx' = \Delta F_{ei}, \\ \sum_{j=1}^N \int_{-1}^1 \frac{\sum_{l=0}^5 \alpha_l^{(j)} T_l(x')}{\sqrt{1 - (x')^2}} dx' = 0, \end{cases} \quad (7)$$

where

$$\Psi(x_{ij}, x_{i+1j}, x') = \frac{1}{\sqrt{1 - (x')^2}} \ln \frac{x_{ij} - x'}{x_{i+1j} - x'}; \quad (8)$$

$\Delta \Phi_{ei}$ is the potential difference between the i and $(i+1)$ point; $i = 1, 2 \dots N-1$ is the line number; j is the column number.

x_{ij} is calculated according to Eq. 4. $\Delta \Phi_{ei} = 0$, if the point with coordinates x_i, x_{i+1} is on the same electrode; $\Delta \Phi_{ei} = \pm 1$ if the point x_i, x_{i+1} is on a different electrode.

In order to solve the system of equations (7), it is necessary to choose any six normalized points with coordinates $|x_i| < 1$. Symmetrical points are selected with the same step $|x_i - x_{i+1}| = 0,25$ ($i=1, 2, 3$). It is also necessary to calculate the values of convolutions

$$y_n(x) = \int_{-1}^1 \frac{T_n(x')}{\sqrt{1 - (x')^2}} \ln |x - x'| dx'; \quad n = 0, 1, 2, \dots, 5. \quad (9)$$

We obtain the following analytical expressions of convolutions Eq. 9:

$$y_0(x) = \begin{cases} -\pi \ln 2, & |x| < 1; \\ \pi \ln \frac{|x| + \sqrt{x^2 - 1}}{2}, & |x| > 1; \end{cases} \quad (10)$$

$$y_1(x) = \begin{cases} -\pi x, & |x| < 1; \\ -\pi \left(x - \text{sign}(x) \sqrt{x^2 - 1} \right), & |x| > 1; \end{cases} \quad (11)$$

$$y_2(x) = \begin{cases} \frac{\pi}{2} - \pi x^2, & |x| < 1; \\ \frac{\pi}{2} - \pi \left(x^2 - |x| \sqrt{x^2 - 1} \right), & |x| > 1; \end{cases} \quad (12)$$

$$y_3(x) = \begin{cases} \pi \left(-\frac{4}{3} x^3 + x \right), & |x| < 1; \\ \pi \left(-\frac{4}{3} x^3 + x + \right. \\ \left. + \text{sign}(x) \left(\frac{4}{3} \left(\sqrt{x^2 - 1} \right)^3 + \sqrt{x^2 - 1} \right) \right), & |x| > 1; \end{cases} \quad (13)$$

$$y_4(x) = \begin{cases} \pi \left(-2x^4 + 2x^2 - \frac{1}{4} \right), & |x| < 1; \\ \pi \left(-2x^4 + 2x^2 - \frac{1}{4} + |x| \left(\sqrt{x^2 - 1} \right)^3 + \right. \\ \left. + |x|^3 \sqrt{x^2 - 1} \right), & |x| > 1; \end{cases} \quad (14)$$

$$y_5(x) = \begin{cases} \pi \left(-\frac{16}{5} x^5 + 4x^3 - x \right), & |x| < 1; \\ \pi \left(-\frac{16}{5} x^5 + 4x^3 - x - \text{sign}(x) \left(\frac{12}{5} \left(\sqrt{x^2 - 1} \right)^3 - \right. \right. \\ \left. \left. - \left(\frac{16}{5} x^4 - \frac{11}{5} \right) \sqrt{x^2 - 1} \right) \right), & |x| > 1. \end{cases} \quad (15)$$

The Chebyshev polynomials are orthogonal, so we get:

$$\int_{-1}^1 \frac{T_n(x)}{\sqrt{1 - x^2}} dx = \begin{cases} \pi, & n = 0; \\ 0, & n = 1, 2, 3, \dots \end{cases} \quad (16)$$

and from Eq. 7 we find:

$$\alpha_0^{(1)} + \alpha_0^{(2)} + \alpha_0^{(3)} + \dots + \alpha_0^{(N)} = 0, \quad (17)$$

where $\alpha_0^{(j)}$ are the coefficients that define the electrode charge density. These coefficients allow to calculate easy the SAW transducer electrode's capacities.

If the electrode's potential's difference is $\pm 1V$, then j -electrode capacity against of the rest electrode system is described by the expression:

$$C_0^{(j)} = \frac{1}{2} \pi^2 (\epsilon_0 + \epsilon_p^T) W |\alpha_0^{(j)}|. \quad (18)$$

where W is the IDT's aperture.

Using analytical expressions of convolutions Eq. 10–15 by solving (Eq. 7) speeds up the calculations for transducers, which contain a large number of electrodes. Fig. 2 shows results using Mathcad internal integration algorithm and analytic expressions Eq. 10 – 15.

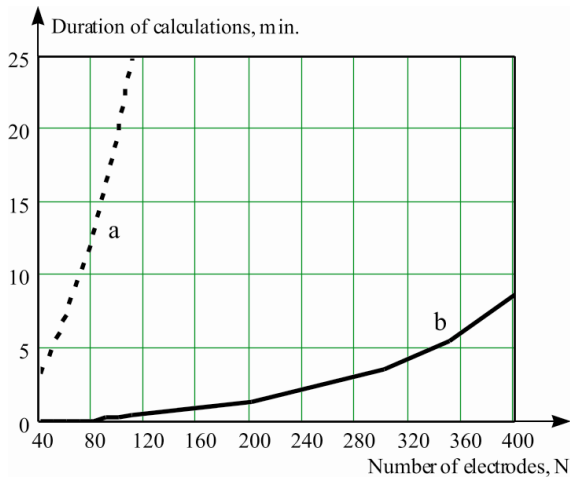


Fig. 2. The duration of calculations using Mathcad internal integration algorithm (a) and analytic expressions of convolutions (b)

Calculations were performed with a Mathcad installed on PC AMD Athlon 64 X2 Dual-Core 4200+, 1 GB of RAM, OS Windows XP.

Approximation error analysis

In general, approximations possess two types of errors: error in the method and calculation errors. Total approximation error can be evaluated in comparison to the exact solution an ideal model. It can be found only for the simplest topologies, for example, consisting of a few strips, or for an infinite periodic system of electrodes [2].

So the total error of approximation and its dependence on the number of neighboring electrodes can be found by calculating the difference between the solutions for finite and infinite periodic systems.

Suppose that the active electrode with the potential $\Phi_{en} = +1$ is located at the center of the infinite array of regular electrodes with high values of metallization ratio η (Fig. 3). On both sides of the center electrode the array of passive electrodes with potential $\Phi_{en\pm j} = 0$ is arranged. Analysis of the charge distribution (Fig. 3) shows that the normalized charge density $\bar{\sigma}_{en}(x)$ under a given electrode depends only on the dimensions and potential of that electrode and its nearest and next-nearest neighbors. In the

present case increasing the distance from the center electrode on both sides, the charge density of neighboring electrodes decreases rapidly: local electric fields are not influenced by electrodes more distant than the next-nearest neighbors. Therefore, to calculate the total error of approximation at various metallization ratios η , we must take into account the interaction of about 2 – 3 electrodes on each side ($N = 5...7$).

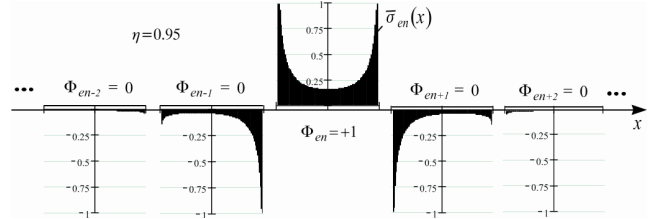


Fig. 3. The array of regular electrodes and electric charge density distribution.

The basic transducer (Fig. 4) frequency response is given by:

$$H_A(j\omega) = (\omega W G_s) \frac{1}{2} \sum_{n=1}^N a_n \sigma_{en}(\omega, \eta) e^{-j \frac{\omega}{v} x_n}, \quad (19)$$

where Γ_s is the a constant depending on the material; ω is the frequency; $\sigma_{en}(\omega, \eta)$ the Fourier's transform of charge density distribution; v the SAW velocity; $\eta = d/(d + a)$ the metallization ratio.

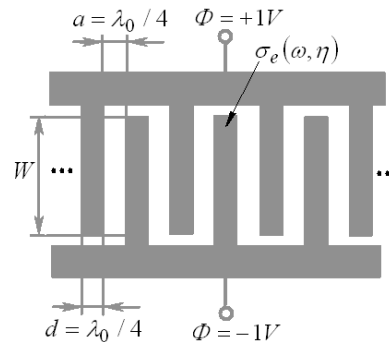


Fig. 4. Basic interdigital transducer

For longer transducers we can ignore end effects and assume that the charge distributions for all electrodes are equal $\sigma_e(\omega, \eta)$. Considering this (Eq. 19) can be written:

$$H_A(j\omega) = (\omega_0 W G_s) \frac{1}{2} \left(\frac{\omega}{\omega_0} \right)^{\frac{1}{2}} |\sigma_e(\omega, \eta)| \times \sum_{n=1}^N (-1)^n a_n e^{-j \frac{\omega}{v} L_g(n-1)}. \quad (20)$$

This expression consists of three parts: A_0 is the constant, $G(\omega, \eta)$ is the frequency response of the electrodes, $H_T(j\omega)$ is the frequency response of transversal filters. These are respectively equal to:

$$A_0 = (\omega_0 W G_s) \frac{1}{2}, \quad (21)$$

$$G(\omega, \eta) = \left(\frac{\omega}{\omega_0} \right)^{\frac{1}{2}} |\sigma_e(\omega, \eta)|, \quad (22)$$

$$H_T(j\omega) = \sum_{n=1}^N (-1)^n a_n e^{-j \frac{\omega}{v} L_g(n-1)}. \quad (23)$$

To estimate error, the frequency responses of the center electrode for finite (approximated solution $\tilde{G}(\omega, \eta)$) and infinite (exact solution $\bar{G}(\omega, \eta)$) basic transducers can be compared. Then the absolute error of the frequency responses $\varepsilon(\omega_M, \eta)$ can be expressed:

$$\begin{aligned} \varepsilon(\omega_M, \eta) &= \left| \bar{G}(\omega, \eta) - \tilde{G}(\omega, \eta) \right| = \\ &= \sqrt{\frac{\omega_M}{\omega_0}} \left[\left| \frac{\pi P_m(\cos \pi \eta)}{K(\sqrt{(1 + \cos \pi \eta)/2})} - \right. \right. \\ &\quad \left. \left. - \pi^2 \sum_{n=0}^2 (-1)^n \alpha_{2n} J_{2n} \left(\frac{1}{2} M \pi \eta \right) \right| \right], \quad (24) \end{aligned}$$

where $m = \frac{M-1}{2}$, $M = 1, 3, 5, \dots$ is the harmonic number;

$\sqrt{\frac{\omega_M}{\omega_0}} = \sqrt{M}$; $P_m(\cos \pi \eta)$ is the Legendre polynomial; $K(\sqrt{(1 + \cos \pi \eta)/2})$ is the complete elliptic integral of the first kind.

Investigations of these errors are shown in Fig 5 and 6.

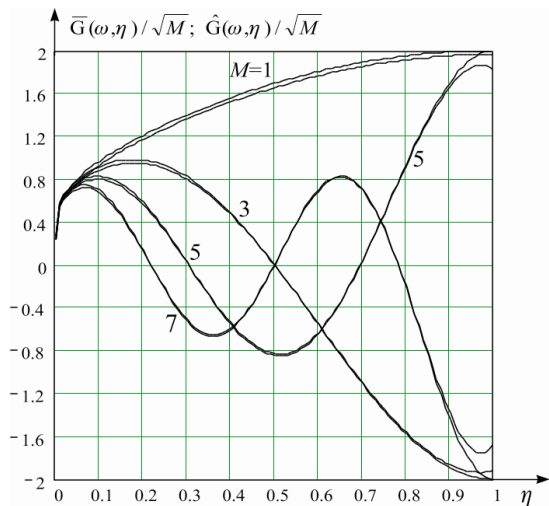


Fig. 5. The frequency spectrum of charge density of the electrode

Conclusions

This paper presents analysis of quasi electrostatic charge density distribution of the interdigital transducer. The Green's function method and polynomial approximation are applied for calculating the charge density on the IDT electrode surfaces. The method is suitable for a long IDT with arbitrary metallization ratios

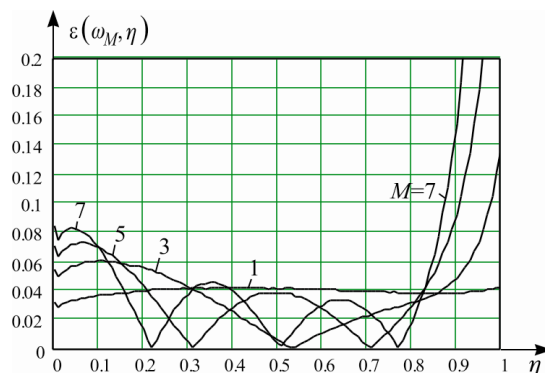


Fig. 6. The absolute error of the frequency responses

and electrode polarity sequences. It is assessing the nonuniform charge distribution of electrodes, interaction of local electric field and the end effects. It is easy to calculate the capacity of each electrode. To reduce errors and duration of calculations for a large IDT as much as possible, it is appropriate to use the analytical expressions of convolutions. The total error of approximation was found by calculating the difference between the solutions for finite and infinite periodic systems.

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Planariojo sunertųjų elektrodų keitklio kvazielektrstatinio krūvio pasiskirstymo analizė

Reziumė

Pasiūlytas kvazielektrstatinio krūvio tankio pasiskirstymo sunertųjų elektrodų keitklyje analizės metodas, kuriam nereikalinga transformacija į erdvinio spektro sritį. Keitklio elektrodų paviršiaus krūvio tankiui apskaičiuoti pasinaudota Grino funkcija ir aproksimacija Čebyševo polinomais. Analizės metodas tinka sunertiesiems keitkliams su daugeliu elektrodų, esant įvairioms metalizacijos koeficiento reikšmėms ir elektrodų poliarumui. Jis įvertina krūvio pasiskirstymo netolygumą, sąveiką su lokaliuoju elektriniu lauku ir supaprastina kiekvieno elektrodo talpos skaičiavimą. Skaičiavimų paklaidai sumažinti ir skaičiavimų laikui sutrumpinti pasiūlyta atsakyti iteracinių metodų ir naudoti analitines konvoliucijų išraiškas. Išvestos šešių pirmųjų Čebyševo polinomo narių ir Grino funkcijos konvoliucijų analitinės išraiškos. Pateikti krūvio tankio aproksimacijos paklaidos įverčiai.

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