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Application of analytical and semi-analytic modelling methods for investigation of ultrasonic guided waves propagation in composites

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Abstract

The application of composite materials in different areas of industry leads to the need of inspection of such components. One of the inspection techniques which are already used is based on the ultrasonic guided waves. However, modelling of guided waves propagation in composite components meets multiple problems: geometrical and structural complexity of components, anisotropy of materials, not defined elastic properties. This leads to the fact that the modelling task cannot be solved using only one simulation technique. So, the objective of the investigations carried out was to review analytical and semi-analytic modelling methods related to propagation of ultrasonic guided waves and to define which question can be answered by them.

Keywords: ultrasonic guided waves, numerical modelling, multi-layered composite, analytical methods, semi-analytic finite element method.

1. Introduction

The wide group of modern composite materials, such as fibre metal laminates, honeycombs, glass or carbon fibre reinforced plastics are used as lightweight and high strength materials in component construction of land, air and water transport, also on-shore and off-shore engineering constructions (like wind turbine blades, hydrofoils of tidal power plants and etc.). Such materials during exploitation under dynamic loads, continuous vibrations, fatigue and harsh environmental conditions (for example, marine environment) must be periodically tested during the maintenance in order to avoid dangerous defects, like internal delaminations, disbonds, breakage of fibres, cracks and etc. Over time the constructional components of a structure may degrade and failure of the construction may occur if the strength reduces below the designed value. Therefore, periodical structure health monitoring should be performed.

One of the non-destructive testing techniques which enable to detect defects both during manufacturing or in-service inspection is based on application of ultrasonic guided waves. Such waves propagate in different elongated objects such as plates, rods, pipes, rails and spar profiles. The guided waves propagating in plates are called Lamb waves. The parameters of the Lamb wave modes in composite materials depend on elastic properties of the laminate, plate thickness and thickness of different layers, fibre orientation, lay-up and on the presence of internal discontinuities, such as delaminations, porosity, ply gaps, foreign matter and changes in fibre volume ratio [1, 2]. The basic factors which determine a particular mode of Lamb waves and the operating frequency to be used are the following: dispersion, attenuation, sensitivity, excitability, detectability, selectivity and sensitivity to defects [3].

In the case of interaction of the Lamb waves with defects present on the propagation path, the waves are reflected, scattered and converted into other modes. Also, the attenuation of the guided waves in composites is relatively high comparing to metal plates. Due to these facts, the analysis of the received multi-mode signal affected by the internal non-homogeneities becomes complicated. The identification of the various guided waves modes in the received multi-mode signal is difficult. Usually in order to avoid or at least to simplify these problems the measurements are performed in low frequency ranges where two primary fundamental modes A_0 and S_0 propagate [4]. The group velocities of A_0 and S_0 modes are different, what enables to avoid overlapping of the signals in the time domain. The S_0 mode possesses a higher group velocity and propagates faster then the A_0 mode. The signals used in measurements usually are not narrow band. Due to dispersion, the higher frequency components propagate with different velocity comparing to the lower frequency components. As the result, waveforms of the received waves become elongated [5].

During the long range inspection, the length of the structure which can be inspected from a single location will be determined by the degree of attenuation of the chosen mode. Therefore, it is necessary to choose a mode with the lowest attenuation. The Lamb waves are attenuated due to several phenomena: signal losses due to dispersion and beam divergence, scattering and leakage into a surrounding medium. The reduction of the signal amplitude due to the beam divergence is given by the inverse square root dependency on the distance [3].

The more complex scattering phenomena occur in the case of interaction of Lamb waves with internal structural non-homogeneities and defects of the object being investigated. Such effects complicate interpretation of the received signals and distinguishing of the appropriate mode remains problematic [6].

The shortcomings of effective practical applications of ultrasonic guided waves for investigation of multi-layered composites are presence of many modes, the propagation distance is limited due to presence of leakage losses, many modes may propagate simultaneously at different velocities (effect of dispersions), mode conversions and reflections at internal non-homogeneities of the structure occur. The particular mode of the guided wave and the frequency value should be selected according to the type of the defect to be detected.

2. Review of modelling methods

The objective of this part was to present a review of existing modelling methods (analytical and semi-analytic) being used for analysis of guided waves propagation in anisotropic composites, like the multi-layered structures of wind turbine blades and hydrofoils of tidal power plants.

The dispersion curves, which represent frequency dependence of the guided wave velocities in a particular structure, must be calculated in order to select the appropriate operating frequency for experimental investigation. For calculation of the dispersion curves mainly the analytical methods or semi-analytical finite element methods are being used.

2.1. Analytical methods

Analysis of the propagation of harmonic elastic waves in n-layered anisotropic plates was presented by Nayfeh **[7]**. The solutions for each layer were obtained and expressed in the terms of the wave amplitudes using the transfer matrix method. The wave amplitudes are replaced by stresses and displacements at interlayer interfaces and the global transfer matrix is constructed **[7]**. The expression for the transfer matrix was modified by Hosten and Castaings taking into account the direction of propagation and the attenuation caused by anisotropic structure of the composite [8].

The transfer matrix method assumes that for the N-layer composite laminate, four waves exist at the boundary of each layer, assuming that all of them have the same frequency and spatial properties at each interface. At each frequency the infinite number of the wavenumbers can be obtained by an iterative root-finding method. In order to apply the matrix techniques for simulation of anisotropic layers it is necessary to extend the matrix to six equations, adding six bulk waves in each layer [9]. The limitation of such method is that it becomes unstable in the case of large fd (frequency multiplied by thickness of the plate). The application of the global matrix method for high frequencies or thick plates was proposed by Knopoff [10]. However, the matrix becomes more complex due to large number of elements and the method becomes slower [6,9,10]. The global matrix model was implemented into commercially available software "DISPERSE" for calculation of the dispersion curves [11].

Nayfeh and Chimenti have presented a continuum-mixture theory for uniaxial, fibrous composites where the fibres posses transverse isotropy and derived the equations of motion for a fluid-coupled transversely isotropic plate. The total transmission curves were calculated for a fluid-coupled composite plate [12].

Towfighi et. al. solved the problem to obtain dispersion curves for the curved anisotropic plates by coupled differential equations in the case of wave propagation in the circumferential direction. It was provided a systematic and unifying solution method based on the Fourier series expansion of the unknown quantities [13].

Karpfinger et. al. presented the algorithm which is based on the spectral method which discretizes the underlying wave equations with the help of spectral differentiation matrices and solves the corresponding equations as a generalized eigenvalue problem. For a given frequency the eigenvalues correspond to the wave numbers of different modes. The advantages of such method is that it only solves the generalized eigenvalue problem without involving special functions, therefore it is easy to implement for cases where traditional root-finding methods are strongly limited or complicated in implementation due to attenuative, anisotropic and poroelastic media [14].

The limitations of analytic methods is that they allow to calculate only the dispersion curves, however there is no possibility to visualize the mechanical deformations, wave propagation dynamic and mechanism of waves interaction with internal defects.

As the illustrative example, the plate made of unidirectional (0°) CFRP laminate (having the thickness of 40 mm) has been taken for analysis. The calculated dispersion curves are presented in Fig.1. It is possible to observe, that up to 40 kHz only fundamental modes of guided waves (A_0 and S_0) exists.

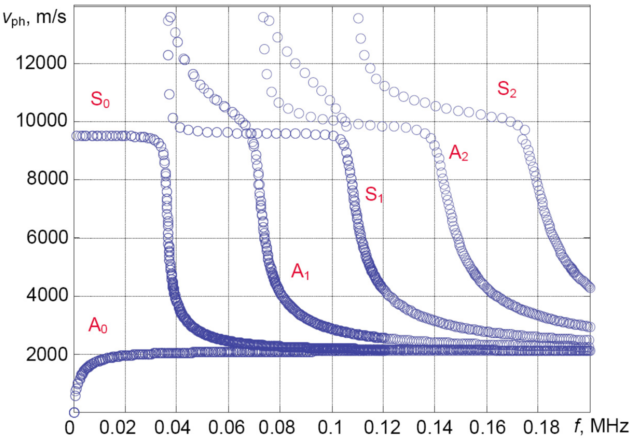

Fig.1. **The dispersion curves in the unidirectional CFRP laminate (thickness of 40 mm)**

2.2. Semi-analytic finite element (SAFE) method

The disadvantage of the analytic methods is that they are applicable only for waveguides possessing relatively simple geometry: plates, cylindrical bars or pipes. The semi-analytic finite element (SAFE) method in general enables to determine dispersion curves of the waveguides possessing arbitrary shaped cross-section. The only cross-section of the arbitrary shape is divided in the finite elements (finite element grid). Therefore, it is always a 2D task [5, 15, 16]. SAFE method is applicable to a wide range of structures, starting from layered plates up to the bars with arbitrary cross-sections [5].

In the analysis of the plates, according to the SAFE method a cross-section of the plate is divided in the thickness direction into layered element and waves in the propagating direction x are described by the orthogonal function $\exp(i\cdot\xi\cdot x)$, where ξ is the wavenumber of the Lamb wave. Such function is used to express the distribution of the displacement field in the longitudinal direction. The m^{th} eigenvalue ξ_m of the eigensystem denotes the wavenumber of the m^{th} resonance mode of guided wave [5, 17]. The wave distribution in the longitudinal direction, can be expressed using orthogonal functions [5]. The iterative calculations of eigenvalues at each frequency lead to the dispersion curves of the phase velocity. Then, displacement fields can be calculated for each mode or for group of the modes using the obtained eigenvalues, eigenvectors and orthogonal function $\exp(i \cdot \xi \cdot x)$. The wave propagation can be simulated by collecting these displacement data for all frequencies in the corresponding frequency bandwidth [17]. So, using the semi-analytic finite element model it is possible to get the approximate solutions for phase velocity, group velocity and displacement of guided waves. The SAFE method also enables to investigate different excitation conditions also. Verification of the dispersion curves obtained by this method can be performed comparing them with the experimental ones [18]. The experimental dispersion curves can be obtained from conventional B-scan data using the 2D Fourier transform. It enables to create the dual-space image: wave number versus frequency or phase velocity versus frequency. SAFE method has been applied by Lanza Di Scalea et. al. for investigation of joints between wing skin and spar having two different types of bond defects, like poorly cured adhesive and disbonded interfaces [19].

The advantage of the SAFE method is that it is suitable for acquiring of dispersion curves for bars of arbitrary complicated cross-sectional geometries such as rail, spar or longeron [17]. Hayashi proposed to use the combination of the SAFE and finite element (FE) method for modeling of guided wave propagation in the elongated structures with the defects. In this case the FE method is used only in the region with the crack, while SAFE is used for non-defected semi-infinite pipe sections [5]. The Lamb waves scattering by crack and inclusion in the composite laminates using a combined finite element–strip element approach was analysed by Liu [20].

The limitation of the SAFE method is that it can handle only bar-like structures with constant material properties in the longitudinal direction [5]. Also, the dispersion curves obtained from SAFE calculations are difficult to separate from each other. Therefore, a mode sorting method should be established in order to distinguish modes by their orthogonality [21]. The example of application of the SAFE method for analysis of the mode shapes and dispersion curves for CFRP rods is presented by Raisutis et.al. [22].

As the illustrative example, the plate made of GFRP laminate has been taken for analysis. Specification of the sample is presented in Table 1.

Table 1. **Specification of the sample**

Type of sample	Thickness	Skin
GFRP (Glass fibre) laminate	4 mm	+45º/-45º - Woven and biaxial (6 plies)

According to specification of GFRP laminate sample (Table 1) the dispersion curves were calculated using the SAFE method [15-17]. The calculated dispersion curves of the guided wave modes propagating along the laminate structure are presented in Fig.2. It is possible to observe, that up to 150 kHz only fundamental modes of guided waves exists.

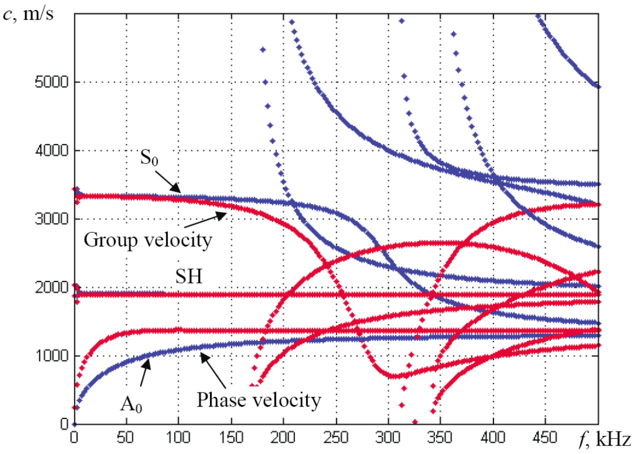

Fig.2. **The dispersion curves of phase and group velocities in the GFRP laminate (thickness of 4 mm)**

Conclusions

The review of analytical and semi-analytical methods concerning simulation of ultrasonic guided waves propagation in the composite materials, structures of which are close to the multi-layered composite structures of hydrofoils of tidal power plants, has been performed. The advantages and limitations of each technique were clarified. The analytical techniques can be used just for very rough estimation in 1D approach and do not take into account the peculiarities of geometry. It was shown that the most suitable for the determination of the dispersion curves of guided waves propagating in the object under investigation is the semi-analytical finite element method. However, the main problem usually is caused by non-accurate definition of the elastic properties which should be adjusted using the experimental measurements.

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Analitinio ir pusiau analitinio modeliavimo metodų taikymas nukreiptųjų bangų sklidimui kompozituose tirti

Reziumė

Kompozitų ir iš jų pagamintų konstrukcijų tyrimų poreikį lemia tai, kad jie plačiai taikomi įvairiuose pramonės sektoriuose. Vienas iš tyrimo metodų pagrįstas nukreiptosiomis ultragarso bangomis. Šių bangų sklidimą modeliuoti yra sudėtinga dėl tiriamų objektų geometrijos ir vidinės struktūros sudėtingumo, anizotropijos, netiksliai žinomų elastinių konstančių. Todėl modeliuoti turi būti naudojami skirtingi metodai. Tyrimų tikslas buvo atlikti analitinių ir pusiau analitinių metodų, taikomų nukreiptosioms bangos modeliuoti, analizę.

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