# Love waves in proton-exchanged z-cut lithium niobate 

F.Dobrovolskis, D.Eiplys, R.Rimeika, J.Paðkauskas

Vilnius University, Laboratory of Physical Acoustics,
Physics Faculty, Saulëtekio 9, 2054 Vilnius, Lithuania

## Introduction

Proton-exchanged lithium niobate is very attractive and prospective material for applications in acoustooptics using the interaction between guided optical waves and surface acoustic waves. Relatively simple techniques enables the fabrication of good quality optical guides with a large change of the extraordinary refractive index $\Delta n_{\text {e }}$. The proton exchange results also in creation of a layer at the surface with altered elastic properties of the crystal. It is the reason why the velocity of the Rayleigh surface acoustic waves becomes frequency dependent [1,2]. Moreover, in the system consisting of the substrate covered by a thin layer, the other type of the surface acoustic wave can propagate: it is the Love wave with a purely shear polarisation in the propagation plane. The Rayleigh wave propagation in proton-exchanged lithium niobate has been studied in several works [4-6], but there is a lack of relevant Love wave studies. Meanwhile, these waves may offer new interesting possibilities of the interaction with guided optical modes. It is the purpose of the present work to perform studies of Love wave propagation in $Z$-cut PE:LiNbO crystal and their interaction with guided optical modes. In particular, the dependence of Love wave velocity upon a frequency is calculated using the ray geometry method for the case of an anisotropic layer and an isotropic substrate. The calculated $V(f)$ dependence is compared with the results obtained from the measurements of colinear diffraction angles.

## Calculation of Love wave velocity

Consider the structur shown in Figure 1. As shown in [3] for the Z cut Y -propagation in 3 m symetry group crystal, two independent types y of surface acoustic waves can exist when the velocity of shear bulk waves in the

| $\mathrm{H}_{\mathrm{x}} \mathrm{Li} \mathrm{i}_{1-\mathrm{-x}} \mathrm{NbO}_{3}$ |  |
| :--- | :---: |
| $\mathrm{~V}^{\prime}{ }_{\mathrm{sH}}(\Phi), \mathrm{C}^{\prime}{ }_{44}, \mathrm{C}^{\prime}{ }_{66}, \mathrm{C}^{\prime}{ }_{14}, \mathrm{P}^{\prime}$ |  |
| $\mathrm{LiNbO}_{3}$ | $\mathrm{~V}_{\mathrm{SH}}, \mathrm{C}_{66}, \mathrm{P}$ |

layer is lower than that in the substrate. These are: the piezoelectric Rayleigh wave, possessing two components of a mechanical displacement in sagittal plane, and the Love wave possessing only the displacement component normal to the sagittal plane. In order to find the velocity of the Love wave, we use the ray geometry method similar to that used to describe a guided wave propagation in integrated optics. Consider the horizontally polarized shear plane wave propagating in the anisotropic layer at the angle $\phi$ with respect to the $y$-axis (see Figure 2). The wave is totally reflected at the layer boundaries with air (point 1) and the substrate (point 2). As the condition of the transverse resonance is satisfied, i.e. the phase difference between points $A$ and $B$ is multiple of $2 \pi$, the layer supports a guided acoustic mode which is a shear Love wave. As the layer is anisotropic, the angles $\phi$ and $\phi_{1}$ are not equal to each other. They are related as

$$
\begin{equation*}
\frac{\cos (\phi)}{V_{S H}(\phi)}=\frac{\cos \left(\phi_{1}\right)}{V_{S H}^{\prime}\left(\phi_{1}\right)}, \tag{1}
\end{equation*}
$$

i.e., the longitudinal propagation constant $\beta$ must not change along the propagation direction. Then, the condition of phase matching becomes:

$$
\begin{equation*}
\left(q+q_{1}\right) d-\varphi_{1}-\varphi_{2}=2 m \pi . \tag{2}
\end{equation*}
$$

Here, $\varphi_{1}=0$ is the phase of the reflection coefficient at the boundary with air, and $\varphi_{2}$ is that at the boundary with the substrate:

$$
\begin{aligned}
& \varphi_{2}=\arctan \left(\frac{\rho V_{S H} \sqrt{\left(\frac{V_{S H}}{V_{S H}^{\prime}(\phi)} \cos (\phi)\right)^{2}-1}}{\rho^{\prime} V_{S H}^{\prime}(\phi) \sin (\phi)}\right)+ \\
& \arctan \left(\frac{\rho V_{S H} \sqrt{\left(\frac{V_{S H}}{V_{S H}^{\prime}\left(\phi_{1}\right)} \cos \left(\phi_{1}\right)\right)^{2}-1}}{\rho^{\prime} V_{S H}^{\prime}\left(\phi_{1}\right) \sin \left(\phi_{1}\right)}\right)
\end{aligned}
$$



Fig. 2. Ray geometry model of acoustic mode in

Assigning $\theta=\pi / 2+\phi$ ( Figure 3) one may express the dependence of horizontal shear bulk waves upon the propagation direction in the layer (Figure 4) and in the substrate:

$V_{S H}^{\prime}(\theta)=\sqrt{\frac{c_{44}^{\prime} \cos ^{2}(\theta)+c_{66}^{\prime} \sin ^{2}(\theta)+c_{14}^{\prime} \sin (2 \theta)}{\rho^{\prime}}}$,
$V_{S H}=\sqrt{\frac{c_{66}}{\rho}}$
The transversal components of wave vectors are:
$q=k \sin (\phi)=\frac{\omega}{V_{S H}^{\prime}(\phi)} \sin (\phi)$
$q_{1}=k_{1} \sin \left(\phi_{1}\right)=\frac{\omega}{V_{S H}^{\prime}\left(\phi_{1}\right)} \sin \left(\phi_{1}\right)$
Taking into account the expressions (3) ir (4), one may write the expression (2) in the form


Fig. 4. Section by YZ plane of slowness surface for horizontal shear bulk wave
$2 m \pi=d \omega\left(\frac{\sin (\phi)}{V_{S H}^{\prime}(\phi)}+\frac{\sin \left(\phi_{1}\right)}{V_{S H}^{\prime}\left(\phi_{1}\right)}\right)-$
$\arctan \left(\frac{\rho V_{S H} \sqrt{\left(\frac{V_{S H}}{V_{S H}^{\prime}(\phi)} \cos (\phi)\right)^{2}-1}}{\rho^{\prime} V_{S H}^{\prime}(\phi) \sin (\phi)}\right)-$
$\arctan \left(\frac{\rho V_{S H} \sqrt{\left(\frac{V_{S H}}{V_{S H}^{\prime}\left(\phi_{1}\right)} \cos \left(\phi_{1}\right)\right)^{2}-1}}{\rho^{\prime} V_{S H}^{\prime}\left(\phi_{1}\right) \sin \left(\phi_{1}\right)}\right)$.
This is the characteristic equation describing propagation of the shear guided acoustic waves. Its solution leads to the dependence of the Love wave upon the frequency:
$V_{L}=V_{S H}^{\prime}(\phi) / \cos (\phi)$.
The dependence calculated for PE Zcut lithium niobate is shown in Figure Fig. 5. Dependences of Love wave velocity upon frequency in Z -cut $\mathrm{LiNbO}_{3}$ calculated using elastic constants from [2,9] for isotropic (curve 1) and anisotropic (curve 2) layer; evaluated from experimental measurements with best-fit elastic constants for anisotropic layer model.

In the case the layer is isotropic [9], the dependence $V_{L}(\omega)$ (Figure 5) can be found from the relation
$\tan \left(b^{\prime} k d\right)=\frac{\rho V_{S H}^{2} b}{\rho^{\prime} V_{S H}^{\prime 2} b^{\prime}}$,
where

$$
b=\left[1-\left(\frac{V_{L}}{V_{S H}}\right)^{2}\right]^{1 / 2}, \quad b^{\prime}=\left[\left(\frac{V_{L}}{V_{S H}^{\prime}}\right)^{2}-1\right]^{1 / 2}
$$

$$
(6 \mathrm{a}, \mathrm{~b})
$$

and

$$
\begin{equation*}
V_{S H} \equiv\left(c_{66} / \rho\right)^{1 / 2} \quad, \quad V_{S_{H}^{\prime}} \equiv\left(c_{66}^{\prime} / \rho^{\prime}\right)^{1 / 2} \tag{6c,d}
\end{equation*}
$$

are the velocities of horizontal shear bulk waves, $\mathrm{C}_{66}$ ir $\mathrm{C}^{\prime}{ }_{66}$ are the elastic constants, $\rho$ ir $\rho^{\prime}$ are the densities in the substrate and the layer, respectively.

As one can see from Figure 5, the anisotropy of the layer alters the frequency dependence of Love waves (compare curves 1 and 2)

## Colinear diffraction

When the acoustic and optical waves propagate colinearly, the acousto-optic diffraction takes place leading to the conversion of guided optical mode into the substrate mode, provided the relevant components of photoelastic tensor $p_{i j}$ are not equal to zero, and the phase matching conditions for interacting wave vectors are satisfied. From the photoelastic and electrooptic matrices one can see that the diffraction of the TM modes by the Rigyleiqbsibfaverpesibf cblinearilapffrdaeg not change the polarization of light, while the diffraction by the Love waves rotates the polarization by $90^{\circ}$. In Fig. 6 , possible cases of the phase matching for interacting wave vectors are shown. Three cases can be distinguished.



1. Conversion of the TM guided mode into the extraordinary bulk wave. The diffraction angle is determined by the expression

$$
\begin{equation*}
n_{e}(\Theta) \cos \Theta=n_{m}-f \lambda / V_{R}(f) \tag{7}
\end{equation*}
$$

The diffraction of this type corresponds to the opposite directions of wave vectors and is allowed at the frequencies higher than defined by the condition

$$
f \lambda / V_{R}(f)=n_{m}-n_{e} \cdot(7 a)
$$

2. Conversion of $T M$ guided mode into an ordinary bulk wave.


Fig. 8. Scheme of diffraction angle measurements
a) Acoustic and optical wave vectors are of the same direction. The diffraction angle then satisfies the condition of phase matching

$$
\begin{equation*}
n_{o} \cos \Theta=n_{m}+f \lambda / V_{L}(f) \tag{8}
\end{equation*}
$$

at frequencies, lower than that defined by the relation
$f \lambda / V_{L}(f)=n_{o}-n_{m}$. (8a)
b) Acoustic and optical wave vectors are of the opposite directions. The correspondent phase matching condition now is

$$
\begin{equation*}
n_{o} \cos \Theta=n_{m}-f \lambda / V_{L}(f) . \tag{9}
\end{equation*}
$$

If $n_{m}<n_{0}$, there is no low-frequency cut-off ${ }^{m}$ at all. If $n_{m}>n^{\prime}$, the lowfrequency cut-off is determined by the relation

$$
\begin{equation*}
f \lambda / V_{L}(f)=n_{m}-n_{o} \tag{9a}
\end{equation*}
$$

## Experiment

The $Z$-cut $L^{2} \mathrm{NbO}_{3}$ single crystal plates were used in the experiment. The proton exchange was performed in pure benzoic acid for 8 hours at $230^{\circ} \mathrm{C}$. In order to evaluate the thickness of protonated layer $\mathrm{H}_{\mathrm{x}} \mathrm{Li}_{1-\mathrm{x}} \mathrm{NbO}_{3}$, the spectra of guided mode effective refractive indices were measured by the prism coupling method [7], and the refractive index profiles were reconstructed using the standart IWKB procedure [8]. Well expressed step-like profiles were obtained as it can be seen from Fig. 7.

The measurements of the colinear diffraction were carried out using the arrangement shown in Figure 8. The light from the $\mathrm{He}-\mathrm{Ne}$ laser with $\lambda=0,6328 \mathrm{~mm}$ is coupled to the waveguide by means of GaP prism. The surface acoustic wave is excited by the interdigital transducer placed at the other edge of the sample, In order to prevent the decrease of performance of the transducer , the later was deposited on the non-exchanged part of the surface.

Acousto-optic diffraction of multiple modes, both for polarization conserving as well $90^{\circ}$ polarization rotating cases according to Figure 6, was observed.

The angle of diffraction $\Theta$ and the angle of refraction at the end face of the sample $\varphi$ are related by the expression

$$
\begin{equation*}
n_{B} \sin (\Theta-\beta)=\sin \varphi . \tag{10}
\end{equation*}
$$



The angle $\varphi$ was measured by the precise goniometer. Measured dependences of diffraction angles upon the frequency for various optical modes in the case of polarization $90^{\circ}$ rotation are plotted in Figure 9. In the same figure, the dependences $\Theta(f)$ calculated from the relations (8) ir (9) using the Love wave velocity as the parameter which satisfies the equation (5) are shown. The frequency dependence of the best-fit Love wave velocity found is plotted in Figure 5. The bulk wave values thus obtained are: $V_{\mathrm{sH}}=3940 \mathrm{~m} / \mathrm{s}$, $V^{\prime}{ }_{\text {sh }}=3600 \mathrm{~m} / \mathrm{s}$. It follows from (6c,d) that the elastic constant $\mathrm{C}_{66}=\left(\mathrm{C}_{11}\right.$ $\mathrm{C}_{12}$ )/2 is decreased by $19 \% \quad\left(\mathrm{C}_{66}=7.2 \times 10^{10}\right.$ Pa, $C^{\prime}{ }_{66}=5.85 \times 10^{10} \mathrm{~Pa}$ ) due to the proton exchange. These data are in agreement with the values of elastic constants obtained by the other methods for pure and exchanged lithium niobate.

## Conclusions

Proton exchange in Z -cut $\mathrm{LiNbO}_{3}$ creates a structure where the propagation of horizontal shear surface acoustic waves (Love waves) becomes possible. This wave causes the colinear diffraction of $T M$ guided optical modes with a polarization rotation. The calculations of the Love wave velocity showed the importance of accounting for the crystal anisotropy. In the present work, only the anisotropy of the layer was taken into account, and the calculations of $V_{\mathrm{L}}$ were performedusing a simple ray geometry method. The measurements demonstrated that the effective acousto-optic diffraction with a polarization rotation takes place which can be attributed to the interaction with Love waves. From the velocity dependence on a frequency and its zero-frequency value the elastic constant $\mathrm{C}_{66}$ both in the layer and


Fig. 9. Dependences of diffraction angle on SAW frequency for various optical modes. Protonexchanged $Z$ - cut lithium niobate; colinear diffraction with polarization rotation. Points experiment, lines - theory; m is the mode order. a) Optical and acoustic wave vectors are of the same direction, b) contrary directions.
substrate, is evaluated. The proton exchange of the $Z$-cut $\mathrm{LiNbO}_{3}$ for 8 h at $230^{\circ} \mathrm{C}$ causes the $19 \%$ decrease of $\mathrm{C}_{66}$

Table 1. Elastic moduli of pur and protonated lithium niobate.

|  | Substrate ( LiNbO $\left.{ }_{3}\right)$ |  | Layer ( $\left.\mathrm{H}_{x} \mathrm{Li} \mathrm{i}_{1-\mathrm{x}} \mathrm{NbO}_{3}\right)$ |  |
| :---: | :---: | :---: | :---: | :---: |
| Elastic <br> constant, $10^{10} \mathrm{~Pa}$ | Reference [9] | Our value | Reference [2] | Our value |
| $\mathrm{C}_{66}$ | 7.5 | 7.2 | 5.5 | 5.85 |

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Ô．Ä 1 áð̂̂âîëüñêèñ，Ä．×èieèèñ，Đ．Đèiåéêà，P．Ïàøêáóñêàñ

## Âiëíú Ëäâà â ỉð̂òôííî－îáiàííìi íèááàòå èèòèÿ

Đåçbìa

 êîiñòàíò â ïð̀̀iinaaåðõiinñòîîì ñëîå．Ýòî äåëàào âîçî̀æíuì̀ ðàniiðiñòðàiåíèa îîâåðõíinnoóíûõ àéóñòè〒åñêèõ âîëí Ëÿâà â ñòðóéòóðå $\mathrm{H}_{\mathrm{x}} \quad \mathrm{Li}_{1-x} \mathrm{NbO}_{3}$－



 äèôðàêöèè äëÿ ðàçíúõ îîä â äèàìaç̧íå àêóñòè〒åñêèõ 〒àñôîò 30－300 Ĩãö
 îái̊áîì èçiàíåíàå óïðóã̂é e êîñòàíôû $\mathrm{C}_{66}$ ．

