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Noise robust one-dimensional deconvolution of medical ultrasound images

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Introduction

Ultrasound imaging is an affordable and effective diagnostic tool in a clinical work. However, the diagnostic value of ultrasound medical imaging is reduced by its fairly low spatial resolution produced by the convolution of the imaging pulse and the signal of the interrogated tissue. Even the best state of the art scanners do not allow imaging of tissue elements less than 0.5 mm. This resolution is necessary to see many of the commonly occurring pathological tissue changes. The image resolution or pulse duration is determined by the width of the ultrasound beam and the ultrasound signal bandwidth. Another limiting factor is the maximum pulse frequency that is possible to use for imaging the various parts of the human body. For adults, the upper frequency for external ultrasound examination of the heart and the abdominal content is 5.0-7.5 MHz. The image resolution can be increased by deconvolution of the observed data with an estimate of the pulse derived from the same data.

Several investigations have studied the problem of ultrasound image deconvolution. Analog-to-digital conversion of recorded ultrasound radio frequency (RF) signals permits the use of discrete versions of the various deconvolution methods known from the image restoration field. The non-linear transform that is used to compute the envelope signal makes it invalid to use the envelope signal as the starting point for the deconvolution. Two essential criteria of any good deconvolution method are reasonable computational demands and noise robustness. Locally, the interaction between the pulse and the tissue is assumed spatially invariant. With this assumption the observed RF signal, $p(t)$, can be modeled in terms of the continuous convolution equation,

$$p(t)=\int_{-\infty}^{\infty} g(\tau)\,h(t-\tau)\,\mathrm{d}\tau+\nu(t) \qquad (1)$$

Here, $g(t)$ is the true tissue signal, $h(t)$ is the ultrasound pulse and $n(t)$ is the additive stochastic noise that invariably degrades all real data. The image blurring is explained by the convolution of the pulse, $h(t)$, with the true tissue signal, $g(t)$. As in optics $h(t)$ is often called the point spread function.

The difficult part of the deconvolution task defined by equation (1) is to obtain a reliable estimate of the acoustic pulse, $h(t)$. Several methods have been proposed. Measurements in a water bath of the emitted pulse can not be used. This is because wave front distortions and ultrasound energy absorption cause large changes of the pulse when it propagates through the tissue. Blind parametric and non-parametric methods can be applied to estimate the pulse from the observed ultrasound RF data. Good estimates have been obtained using short segments of RF data and the assumption about spatial invariance of the pulse within the segment. Non-parametric pulse estimation based on homomorphic filtering is proposed in [1,2]. This approach has the potential for a real time implementation since it is a non-iterative method based on the fast Fourier transform. The performance of several homomorphic transforms in ultrasound pulse estimation is studied in [3]. The central difficulty using any of these homomorphic transforms is their sensitivity to the stochastic noise that degrades the RF data. In this paper we present a noise robustness procedure for a homomorphic pulse estimation using phase unwrapping. The performance of this new method is compared to the established homomorphic based pulse estimation methods [4].

Homomorphic Deconvolution

An RF data set for a single ultrasound image consists of M reflected ultrasound beams. Each beam is recorded with a sampling frequency higher than the Nyquist sampling frequency in a fixed time interval $(0,t_0)$, corresponding to a specific tissue depth segment. The continuous signal of the beam is converted into N

discrete samples, $p[n]$, $n=1,...N$. A discrete version of (1) is given by

$$p[n]=h[n]*g[n]+v[n], \qquad n=1,...N \quad (2)$$

where * denotes the discrete 1D convolution operator. A homomorphic filtering is based on a non-linear mapping of the convolution $h[n]*g[n]$ in Eq.(2) [5]. The homomorphic filtering using the complex cepstrum,

$$p[n]\xrightarrow{Z}P(z)\xrightarrow{\log}\hat{P}(z)\xrightarrow{Z^{-1}}\hat{p}[n], \quad (3)$$

transforms the convolution $p[n]=g[n]\,h[n]$ into the sum $\hat{p}(n)=\hat{g}(n)+\hat{h}(n)$. Prior to the homomorphic transform an exponential weighting of the RF data along each beam is necessary for the established homomorphic transforms. The exponential weighting moves the complex zeroes of the sequence $\hat{p}[n]$ away from the complex unit circle and makes the sequence stable. For the new homomorphic transform in this paper the sequence $\hat{p}[n]$ is made stable by the noise robust phase unwrapping procedure. To separate the pulse from the transformed signal of the tissue an ideal linear filtering with n_c cutoff value can be used in cepstrum domain:

$$\hat{h}[n]=l[n]\hat{p}[n], \qquad n=1,...N; \quad (4)$$

$$l=\begin{cases}0, & n_c^+ < n < n_c^- \\ 1, & n \le n_c^+, \quad n \ge n_c^-\end{cases}$$

The estimate of the pulse in the ordinary frequency domain is obtained by the inverse transforms

$$\hat{h}[n]\xrightarrow{Z}\hat{H}(z)\xrightarrow{\exp}\hat{H}(z). \quad (5)$$

To reduce the effects of a noise on the pulse estimate, the mean value of the pulse estimates of all beams is calculated in the complex cepstrum domain. After estimating the pulse with homomorphic filtering, we use the ordinary Wiener filter to perform the actual deconvolution in the frequency domain:

$$G(z)=P(z)\frac{H^*(z)}{|H(z)|^2+q}. \quad (6)$$

The final transformation to get the deconvolved RF image in the time domain is $G(z)\xrightarrow{Z^{-1}}g[n]$.

Noise Robust Phase Unwrapping

The computation of the complex cepstrum signal assumes that the frequency signal $\hat{P}(z)=\log\|P(z)\|+j\arg[P(z)]$, where $z=e^{j\omega}$, is analytic and consequently continuous. When processing the observed RF data, the continuity assumption is fulfilled for the amplitude, but not for the phase. Only the discontinuous, principal phase values are available for $-\pi\le\omega\le\pi$. These principal values, $-\pi\le\mathrm{ARG}[P(z)]\le\pi$, and the continuous phase, $\arg[P(z)]$, are related through the expression $\arg[P(z)]=\mathrm{ARG}[P(z)]+2\pi r(\omega)$; $r(\omega)$ takes on the appropriate integer values for the corresponding frequency ω to unwrap the principal value of the phase to retrieve the continuous phase function. After phase unwrapping, $\arg[P(z)]$ will in general still be discontinuous at $\omega=\pm\pi$ due to the presence of a linear phase component. This linear component must also be removed to make $\arg[P(z)]$ continuous in the interval $-\pi\le\omega\le\pi$.

Simple algorithms for phase unwrapping with phase tracking and correction of phase discontinuities give unreliable results. This is due to that adjacent phase wraps in the RF data may be greater than π radians in magnitude because of noise and aliasing.

The problem of recovering a reliable, unwrapped phase can be solved by minimizing the difference between the wrapped phase values and the unwrapped phase values in a least-squares-error sense [6]. Denoting the continuous unwrapped phase $\varphi(\omega)$, its principal value $\psi(\omega)$ and the stochastic noise contribution to the measured phase value $v(\omega)$, these phases are related by

$$\psi(\omega)=\varphi(\omega)+v(\omega)+2\pi r(\omega), \quad (7)$$

where $r(\omega)$ is an unknown integer value such that $-\pi\le\psi(\omega)\le\pi$. Eq.(7) can be written as $\psi=W[\varphi+v]$, using the wrapping operator W. With no noise the least squared unwrapped phase solution is given by the differential equation

$$\frac{\partial^2\phi}{\partial\omega^2}=\rho(\omega) \quad (8)$$

with appropriate boundary conditions. In Eq(8) $\rho(\omega)=\Delta(\omega+1)-\Delta(\omega)$, where $\Delta(\omega)=W(\psi(\omega+1)-\psi(\omega))$ is the observed, wrapped first order phase difference. This equation models in general the minimization of a nonquadratic cost functional which excludes the use of fast minimization procedures. Using special boundary conditions, Ghiglia at al. [6] have published a fast phase unwrapping algorithm based on Eq.(8) and a specific form of a the discrete cosine transform. It assumes noise free observations or observations with a high signal-to-noise ratio, which can not be guaranteed in practice.

To increase the robustness of the least square error approach to phase unwrapping Marroquin at al. [7] have proposed a regularization technique.The main idea of regularization is that, if an object of interest **x** is related to the observed data **y** corrupted with additive noise **v** by the observation model $\mathbf{y}=\mathbf{Ax}+\nu$, one can obtain an exact solution $\mathbf{x}(\lambda,\mathbf{y})$ from the imperfect data $\mathbf{y}$ by defining a class of admissible solutions $\{\mathbf{x}:\|\mathbf{y}-\mathbf{Ax}\|\leq\|\nu\|\}$. Within this class an acceptable solution is selected ,which is consistent with the prior information [8]. The term λ is the regularization coefficient and controls the tradeoff between the data source and the prior source of information. The solution, $\mathbf{x}(\lambda,\mathbf{y})$, can be found as a minimizer of the criterion $\mathbf{Ix}=\mathbf{G}(\mathbf{y}-\mathbf{Ax})+\lambda\mathbf{F}(\mathbf{x})$, $0<\lambda<\infty$. In the standard regularization approach $\mathbf{F}$ and $\mathbf{G}$ are both quadratic: $\mathbf{G}(\mathbf{y}-\mathbf{Ax})=\|\mathbf{y}-\mathbf{Ax}\|^2$, $\mathbf{F}(\mathbf{x})=\|\mathbf{D}_k\mathbf{x}\|^2$., which lead to a second order cost functional. Here $\mathbf{D}_k$ is the regularization operator and is formed by a linear combination of derivatives.

As it is shown in [7], the regularized solution $\mathbf{x}(\lambda,\mathbf{y})$ can be interpreted as the maximum posterior estimator of the true phase φ. In the Markov random field context, for 1D case φ and ψ are both random fields on an $N\times1$ lattice and the noise ν is a spatially independent Gaussian random variable with common variance σ^2. The prior information about the true phase φ is expressed in the form of the prior probability distribution $P_\varphi(\varphi)$ Bayes' rule allows the combination of this prior distribution and the distribution of the observed data $P_\psi(\varphi)$, to obtain the posterior probability distribution $P_{\varphi|\psi}(\varphi)$. This latter distribution is related to the standard regularization cost functional through a discrete approximation to a regularization operator $\mathbf{D}_k$ and through the regularization term λ, which is equal to twice the common noise variance σ^2. This interpretation gives insight into the meaning of the term λ in the representation of the second order cost functional for least squared phase unwrapping:

$$U(\varphi)=\|\mathrm{L}(\varphi)-h\|^2+\lambda\|D_2\varphi\|^2 \qquad (9)$$

Here L is a noninjective linear operator and in this case the first order difference, h is the observation model and is represented with the observed wrapped phase difference $\Delta(\omega)$, D_2 is the second order regularizer, which is a combination of second order phase differences.

Minimization of the cost functional is done by setting the gradient of U with respect to φ equal to zero. The 1D normal equation for phase unwrapping becomes

$$\frac{\partial^2\varphi}{\partial\omega^2}+\lambda\frac{\partial^4\varphi}{\partial\varphi^4}=\rho(\omega), \qquad 10)$$

The solution of this fourth order differential equation with the discrete cosine transform to obtain an expression for non-iterative computation of the unwrapped phase is described in [4].

Experimental Results

Two different homomorphic filtering methods were implemented to study the potential of Bayesian phase unwrapping in the homomorphic deconvolution of ultrasound images. The first was the ordinary complex cepstrum method using phase unwrapping and the second was the generalized cepstrum method using phase unwrapping [9]. These methods were compared to the corresponding methods with an ordinary phase unwrapping.

The RF data were recorded with annular mechanical probes of 3.5 or 5.0 MHz using the Vingmed CFM sector scanner 750. The number of beams per segment was from 56 up to 128. The original data were obtained with an 8-bit A/D converter at 16 MHz sampling frequency with 512 samples per a beam. To remove a low and high frequency noise the RF images were prefiltered both in the radial and the lateral direction. After deconvolution the data were demodulated and scanconverted into Cartesian coordinates to reproduce a correct tissue geometry for a visual inspection.

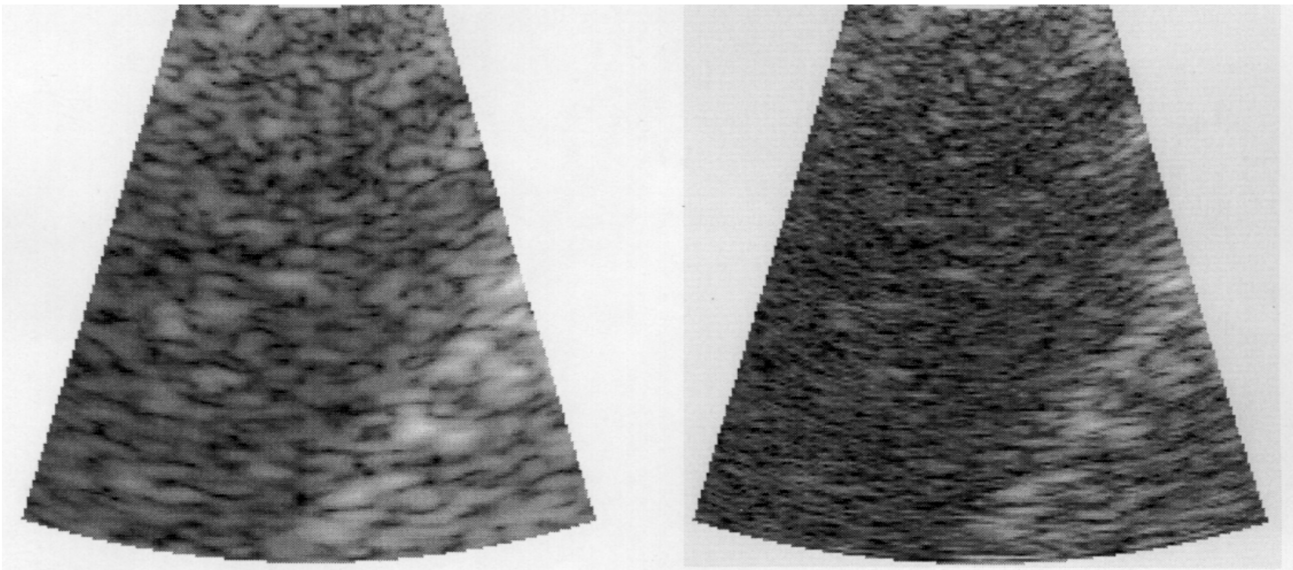

Fig.1. **Radial deconvolution: a- scan converted human tissue image (3,25 MHz probe) with no deconvolution; b- the same image after deconvolution using Bayesian phase unwrapping.**

In the methods with the ordinary phase unwrapping exponential weighting was done with α=0.975 for the radial deconvolution and α=0.96 for the lateral deconvolution. Deconvolution using the minimum phase or the mixed phase pulse assumption was investigated. The cutoff values of the radial ideal low-pass filter were n_c^+=7 for both minimum and mixed phase versions and n_c^- =1 for the mixed phase version. The cutoff values of the lateral ideal low-pass filter were m_c^+=5 and m_c^- =2 respectively. The Bayesian phase unwrapping gave the best results with the noise control parameter λ= 3.0.

Six short ultrasound RF image sequences were selected after preliminary adjustment of parameters to be processed by all four homomorphic filtering methods. To characterize results both visual and statistical evaluations were done. The single frame resolution, the single frame noise level and their variance through the frame sequence were taken into account in the visual evaluation. Reduced blurring and increased visibility of anatomical structures were the main criteria for the visual ranking. The statistical resolution gain was evaluated with the autocorrelation function using its mean value through the frame sequence.

The evaluation showed that both methods using the Markov random field model gave considerably better deconvolution results than the methods with ordinary phase unwrapping. An exzample of the radial deconvolution results is represented in Fig.1.

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Triukðmui atspari vienmatë dekonvoliucija ultragarsiniams medicininiams vaizdams filtruoti

Reziumë

Homomorfiniam filtravimui atlikti gali bûti panaudota signalø, esanèiø skirtingose kepstro srities zonose, tiesioginë konvoliucija. Taèiau ið literatûros þinoma, kad ðis metodas jautrus triukðmui. Ðis jautrumas kyla ið neteisingai suformuotos fazës iðskleidimo procedûros arba, kas ekvivalentu, netinkamo iðvestinës skaièiavimo daþniø srityje. Norëdami teisingai sudaryti fazës iðskleidimo procedûrà naudojome Markovo modeliu pagrástà, triukðmui nejautrø Bayesiano fazës iðskleidimo metodà. Atsitiktiniam triukðmui modeliuoti buvo panaudotas papildomas koreguojantis narys. Gautasis homomorfinis metodas buvo pritaikytas ultragarsiniams medicininiams vaizdams filtruoti. Vizualinis ultragarsiniø vaizdø ávertinimas parodë, kad Markovo modeliu pagrástas vienmatis Bayesiano fazës iðskleidimas daþniø srityje yra ðiuo metu geriausias homomorfinës dekonvoliucijos metodas.