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The modified method for simulation of ultrasonic fields of disk shape transducer

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Introduction

In non - destructive testing it is important to simulate ultrasonic field, excited by direct or angular transducers, in order to predict how waves will propagate in different mediums. The simplest method for simulation of ultrasonic fields is direct application of the Huyghens' principle. Many authors had theoretically studied pressure waveforms radiated into different mediums by an idealised piston source [1-6].

Experimental observations of the pulsed field of a circular ultrasonic transducer where compared with calculated results for an ideal piston radiator [6]. Experimental studies showed that field point waveforms and transmit receive mode responses are in reasonable agreement with theoretical results calculated assuming ideal piston behaviour [5, 6]. These studies had also demonstrated the plane wave and edge wave structure of the radiated field.

Problem arises when implementing this analytical model into a numerical form, because the impulse response function possess singular points. In a far field of the transducers the maximums of the spatial impulse response function are very near each other, because when the distance from the transducer is large, distance from the centre and the edges of the transducers differs very slightly and the delay times from the centre and the edges of the transducer are almost the same. Hence, in order to calculate the impulse function numerically, high sampling frequencies are required, resulting in large computation times [7].

Objective of this work was to modify method for simulation of ultrasonic fields of a disk shape transducer, in order to be able to calculate ultrasonic field of the transducer precisely even in a far field of the transducer. In the next chapter the model for the calculation of transducer field in media without boundaries will be presented, the problems, associated with implementation of the model will be investigated and the propositions for the solving of the problems will be made.

Generalised spatial impulse response function approach

For direct calculation of the time-domain field of a plane piston in an infinite baffle, the Rayleigh's equation is used. This equation expresses the velocity potential at a field point as the sum of the contributions from elementary Huyghens sources, each radiating a hemispherical wave into the fluid [6]:

$$\phi = \iint_S \frac{v(t - r/c)}{2\pi r} dS , \tag{1}$$

where ϕ is the velocity potential, v is the velocity of the piston, r is the distance from the field point to the surface element dS, c is the velocity of sound.

The pressure P in a fluid of the density ρ, is given by

$$P = \rho \frac{\partial \phi}{\partial t} . \tag{2}$$

The pressure due to an arbitrary velocity function $v(t)$ can be derived by convolution. If the piston velocity v is uniform over the piston surface, then [6]

$$v(t - r/c) = v(t) * \delta(t - r/c), \tag{3}$$

where $*$ indicates convolution.

Velocity potential can be expressed [6]:

$$\phi(r,t) = v(t) * \phi_i(r,t), \tag{4}$$

where the impulse response

$$\phi_i = \iint_S \frac{\delta(t - r/c)}{2\pi r} dS . \tag{5}$$

Pressure also can be expressed as a convolution [6]

$$P(r,t) = v(t) * P_i(r,t), \tag{6}$$

where $P_i = \rho \frac{\partial \phi_i}{\partial t}$ is the pressure impulse response.

The pulsed field of the transducer can be calculated using the mathematical model based on the spatial impulse response approach [8]. The spatial pressure impulse response of the disk transducer with radius R is given by the following expressions:

$$h_t(x,z,t) = \begin{cases} -\rho_0 c \delta(t - t_0), & x < R, \quad t_0 \le t \le t_1 \\ -\dfrac{\rho_0 c}{\pi} \dfrac{d\theta}{dt} & x \ne R, \quad t_1 \le t \le t_2 \\ -\rho_0 c \left[\delta(t - t_0) + \dfrac{1}{\pi} \dfrac{d\theta_1}{dt} \right] & x = R, \quad t_0 \le t \le t_2 \end{cases} \tag{7}$$

$$\frac{d\theta}{dt} = \frac{1}{\left(c^2 t^2 - z^2\right)} \times$$
$$\times \frac{-\left[c^2 t \left(c^2 t^2 - z^2 - x^2 + R^2\right)\right]}{\sqrt{\left[2\left(c^2 t^2 - z^2\right)\left(x^2 + R^2\right) - \left(c^2 t^2 - z^2\right)^2 - \left(x^2 - R^2\right)^2\right]}}$$

$$\frac{d\theta_1}{dt} = -\frac{c^2 t}{\sqrt{\left(c^2 t^2 - z^2\right)\left[4R^2 - \left(c^2 t^2 - z^2\right)\right]}},$$

$$t_0 = z / c,$$

$$t_1 = \sqrt{(R - x)^2 + z^2} \Big/ c$$

$$t_2 = \sqrt{(R+x)^2 + z^2} \Big/ c \, .$$

where ρ_0 – density; R – transducer radius; c - ultrasound velocity; t_0 - delay time of the plane wave from the transducer surface to the point with coordinates x,z; t_1 - delay time of the edge wave from the nearest transducer edge to the point with coordinates x,z; t_2 - delay time of the edge wave from the farthest transducer edge to the point with coordinates x,z.

According to various methods it was shown that acoustic field of plane disk transducers consists of plane and edge waves. Such a presentation has been proved to be valid by many theoretical simulations and practical experiments [2, 3, 8 - 10]. The whole surface of a piston generates a direct plane wave, which propagates in cylindrical region having the piston at its base. From theedge of the transducer diffracted edge waves are radiated, which propagate in all directions.

Typical waveforms of the spatial impulse response, calculated using Eq. 7, are given in Fig. 1. We can see that a big part of energy is concentrated in three pulses. The first pulse corresponds to the arrival time of a plane wave from the surface of a transducer. The second and the third pulses correspond to the arrival times of edges waves from the nearest and farthest edges of disk shape transducer. Outside the direct beam region they correspond to the first and second pulses, because in this case a plane wave is absent.

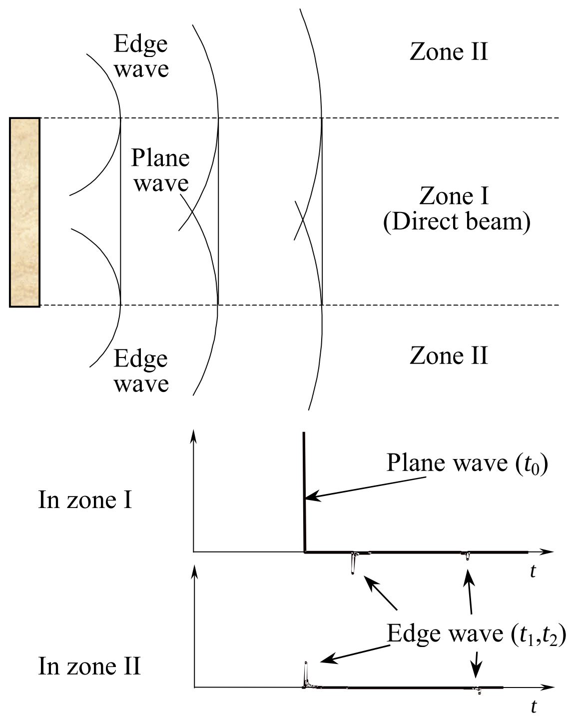


Fig. 1. **Pulse response of disk shape transducer in a homogeneous medium**

The modification of the simulation method

A very important aspect of the model used is that a magnitude of the velocity potential at some instant t is proportional to the length of the arc on the transducer surface (Fig. 2). After a velocity impulse has been applied to a piston at t=0, the field at a points Q_v or Q_i is due to contributions from points on the piston at the distance ct from the points Q_v or Q_i, an arc of a circle on its surface. So the more of the transducer surface located at thedistance ct from the point Q_v or Q_i at the time t, the greater the spatial impulse response function will be at that time [3]. It has been shown that the velocity potential ϕ_i is proportional to the fraction of equidistant arc included on the piston surface [6]:

$$\phi_i(r,t) = \frac{c\theta(ct)}{2\pi} \, , \tag{8}$$

where ϕ is the velocity potential, r is the distance from the field point to the arc on the transducer surface, $\theta(ct)$ is the total angle of the included equidistant arc.

When the distance z from the transducer surface changes, for all points, which have the same coordinate x, amplitude of the velocity potential at the same time instant is due to contributions from the same arc on the transducer surface. The Eq. 8 is for the velocity potential, and pressure is the derivative of the velocity potential in time. So we can assume, that amplitudes of the spatial impulse response for the same coordinate x are the same and only the delay times are different. So amplitudes for each coordinate x can be calculated on the transducer surface, that is, when z=0, and when the z coordinate changes, we have to recalculate only the delay times.

As it was stated before, a problem arises implementing analytical model into a numerical form, because the impulse response function has singular points. In this case a question arise, where in the time array to put maximums of the spatial impulse response, which arrive at the instants t_0, t_1 and t_2, because these times will not coincide with the used time discretization step.

The biggest allowed sampling step in the time domain depends from transducer frequency:

$$\Delta t_{\max} = \frac{1}{n_{\min} f} \, , \tag{9}$$

where $n_{\min}$ is the minimal point number in one period, f is the transducer frequency.

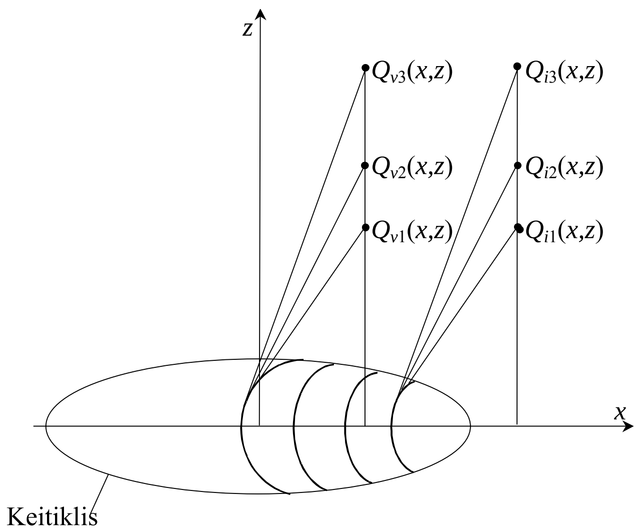


Fig. 2. **Field of the disk transducer at points Q_v or Q_i is due to contributions from points on the piston distant ct from points Q_v or Q_i , an arc of a circle on its surface**

In order to calculate the delay times t_0, t_1 and t_2 precisely, the sampling step in the time domain has to be decreased, and that automatically increases the number of points, which has to be calculated:

$$n = \frac{z_{mx}}{c \cdot \Delta t}, \tag{10}$$

where z_{mx} is the distance from the transducer to the farthest calculating point, Δt is the used sampling step.

Using the method proposed spatial impulse response at the point $P_1(x, z)$ (Fig. 3, a) is calculated in following steps:

1. Two points x_p and x_g, on the surface of the transducer are found, from which the delay times of transmitted rays are equal to the delay times of the edge waves t_1 and t_2 (Fig. 3, b). When the point is outside the transducer boundaries, then:
   $$x_p = R, \quad x_g = -R; \tag{11}$$
   when the calculating point is inside the transducer boundaries, then:
   $$x_p = 2x - R, \quad x_g = -R. \tag{12}$$
2. The step Δx for the calculating of the spatial impulse response is selected:
   $$\Delta x = \frac{(x_g - x_p)}{n_x - 1}. \tag{13}$$
   where n_x is the number of calculated points between t_1 and t_2.
3. The delay times from the each point on the transducer surface to the calculated point are found:
   $$t_i = \frac{\sqrt{x_s^2 + z_1^2}}{c}, \tag{14}$$
   where $x_s = x_1 - (x_p + \Delta x(i-1))$.
4. Amplitudes of the spatial impulse response for the point projection to the transducer plane are found using Eq. 7.
5. The spatial impulse response is multiplied by the correction coefficient k_{kor}, which takes into account the discretization step Δt:
   $$k_{kor} = \frac{t_2 - t_1}{(n_x - 1)\Delta t}. \tag{15}$$
   This correction has to be introduced, because we calculate the same number of points on the spatial impulse response from t_1 to t_2 and that doesn't depend on how far the computed point is.
6. Amplitudes of the spatial impulse response at the time instants t_0, t_1 and t_2 are calculated separately. Equations for the calculation of the amplitudes of the spatial impulse response can be derived from Eq. 7.

However, we are interested in a transducer field but not in the spatial impulse response, so the signal, transmitted by a transducer has to be convolved with the spatial impulse response function. Usually convolution is performed using the Fourier transform, because this method is faster then convolution in the time domain. If we want to convolve signals in frequency domain, the signal has to be sampled with the constant step Δt, which has to be small enough in order to get a large accuracy in a far field of the transducer.

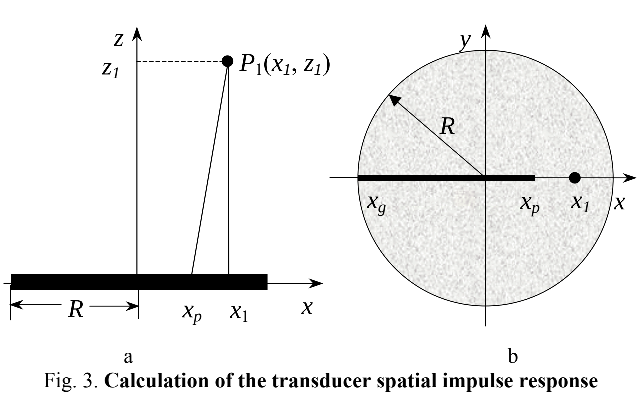
Using the modified method amplitudes and precise times of maximums of the spatial impulse response where computed at fixed number of points. So in this case it was not possible to use convolution in the Fourier domain and it was necessary to perform convolution in the time domain (Fig. 4). It gave a better accuracy and the speed of computations was increased also.

Convolution of two signals in the time domaincan be expressed using this recurrental formula:

$$p_{k-1+n_r} = p_{k-1+n_r} + \left(A_{imp}\left(u_{k-1} + \frac{\Delta t - \Delta t_i}{\Delta t}(u_k - u_{k-1})\right)\right), \tag{16}$$

where $\Delta t_i = t_{imp} - (t_p + \Delta t(n_r - 1))$, $n_r = \dfrac{t_{imp} - t_p}{\Delta t} + 1$,

p is the acoustical field pressure, u is the excitation signal, A_{imp} is the amplitude of the impulse, t_{imp} is the time instant, at which impulse arrives, t_p is the time instant, at which the signal is recorded, Δt is the signal sampling step, Δt_i is the delay of impulse from the signal at the some time moment, n_r is the impulse place in array.



a b

Fig. 3. **Calculation of the transducer spatial impulse response**

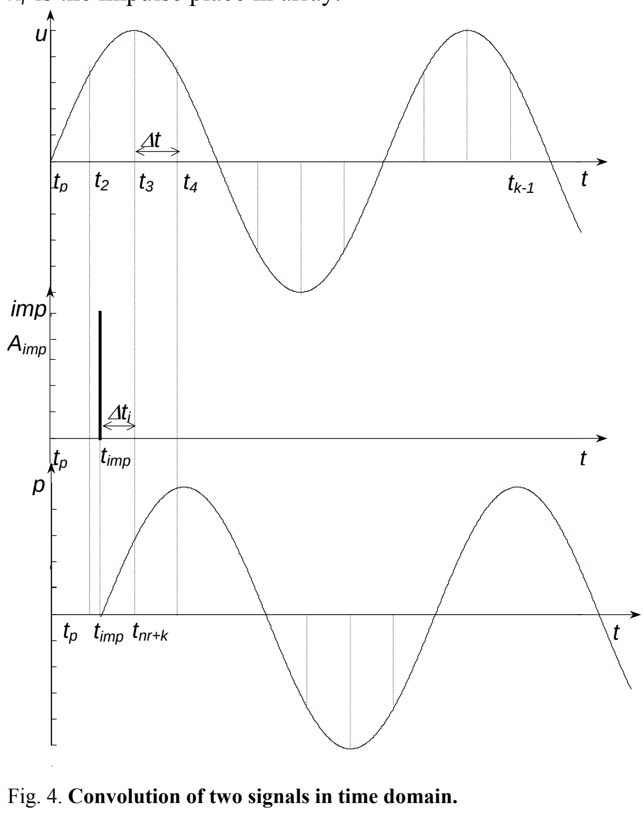
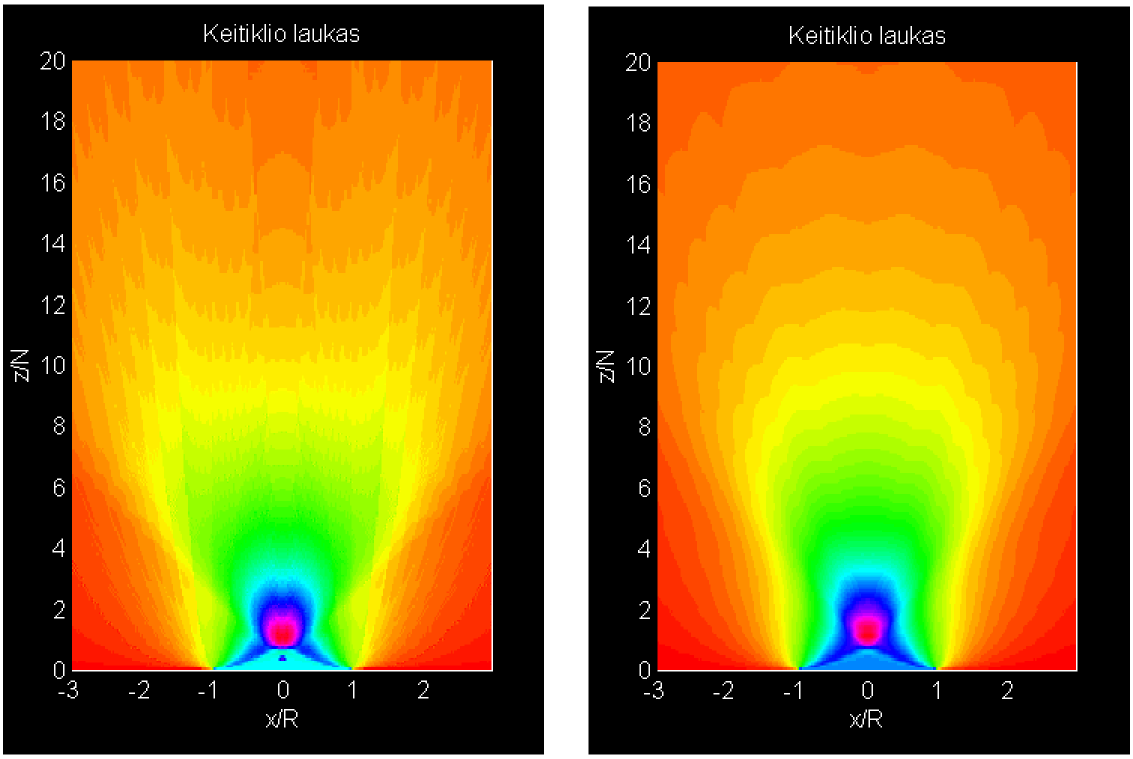


Fig. 4. **Convolution of two signals in time domain.**



a **b**

Fig. 5. **Transducer field, calculated using ordinary (a) and modified (b) method**

In Fig. 5 the field of the 2,5 MHz frequency transducer is represented calculated using conventional (a) and modified (b) method. As it can be seen in the pictures presented, when the field is calculated using conventional method, in the far field of the transducer errors in the form of the needles reveals (Fig. 5, a), which, as it was mentioned before, arise because the maximums of the spatial impulse response are close each to other. Meanwhile, after modification of the method, numerical errors where avoided (Fig. 5, b).

Using modified method computations takes less time, because amplitudes of the impulse response function are calculated one time, and delay times are calculated at the fixed number of points and that doesn't depend on the distance from the transducer. Also, the errors of the delay times at singular points and amplitude fluctuations in the calculated field where avoided.

Simulation results

As it was mentioned before, often in non-destructive testing ultrasonic wave propagate through the different media. So we wanted to investigate, how the field of the ultrasonic wave changes, after the wave is transmitted through the different mediums.

To calculate the field of the ultrasonic transducer, the programs in TURBO PASCAL language have been developed. Calculation of the field, which has 128x128 points, takes less then 3 minutes using PC with 300 MHz *Pentium* processor. The transducer field in Fig.6 is represented as absolute maximal values of the acoustical pressure impulse response.

Simulations were performed for the transducer, which is most widespread in non-destructive testing. It was assumed that the transducer radiates pulse with Gaussian envelope, duration of which was two periods.

Using the described model the ultrasonic field, radiated by the disc transducer, in various mediums (water, plexiglas, steel) was calculated. The calculations were performed for the disk type transducer of the radius R=5 mm and with the frequency f=3 MHz. In Fig.6 the simulation results in various mediums are presented: a – water, c=1,48 mm/μs; b - plexiglas, c=2,76 mm/μs and c – steel, c=5,85 mm/μs. From the presented pictures it can be seen that the structure of the ultrasonic field in various materials is similar, only the near field of the transducer ends at different distances from the transducer.

Conclusions

In this paper the method and fast algorithm for simulation of ultrasonic fields excited by the disk shape transducers is presented. The model for the calculation of transducer field in media without boundaries is presented, the problems, associated with implementation of the model are investigated and the propositions for the solving of the problems are made.

After implementation of the modified method computations takes less time, because amplitudes of the impulse response function are calculated only once, and the delay times are calculated at a fixed number of points, which doesn't depend on the distance from the transducer. Also the errors of the delay times at singular points and amplitude fluctuations in the calculated field where avoided.

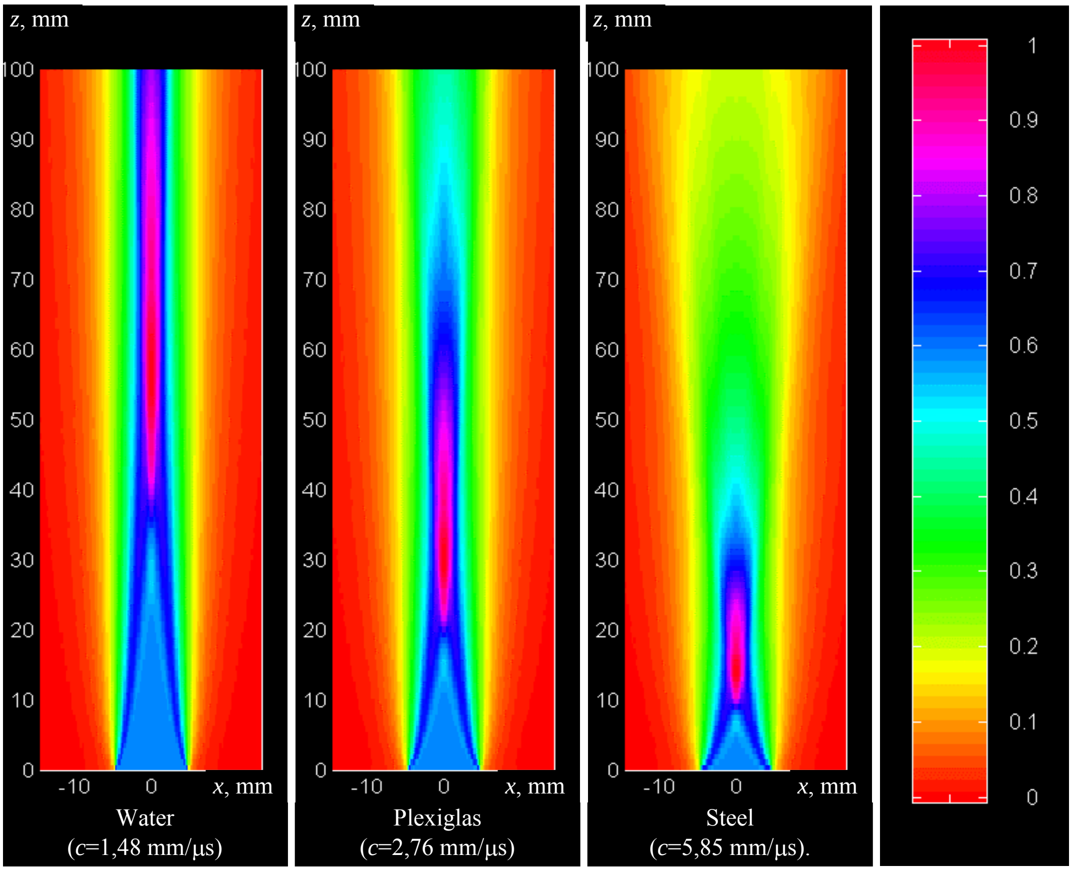


Fig. 6. **Ultrasound field in various mediums (*R*=5 mm; *f*=3 MHz)**

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Patobulintas diskinio keitiklio ultragarso laukų modeliavimo metodas

Reziume

Straipsnyje aprašėme modelį, naudojamą keitiklio laukui vienalytėje aplinkoje apskaičiuoti, išnagrinėjome su jo diegimu susijusias problemas ir pateikėme pasiūlymų, kaip jų išvengti. Siūlomas metodas remiasi diskinio keitiklio akustinio lauko modeliavimo metodo modifikacija. Impulsinis keitiklio laukas homogeninėje aplinkoje gali būti apskaičiuotas naudojantis matematiniu erdvinių impulsinių charakteristikų skaičiavimo modeliu.

Pasiūlytas ultragarsinio keitiklio lauko skaičiavimo metodas yra pakankamai greitas, nereikia didelių kompiuterinių resursų, todėl galima modeliuoti asmeniniu kompiuteriu. Skaičiavimą paspartina pasiūlytas algoritmas, kuriuo naudojantis impulsinio lauko amplitudės skaičiuojamos tik vieną kartą, o laikai nepriklausomai nuo atstumo iki keitiklio skaičiuojami fiksuotame taškų skaičiuje. Pasiūlytame algoritme pašalintos skaitmeninės paklaidos, atsirandančios dėl singuliarinių keitiklio impulsinės charakteristikos taškų, todėl buvo išvengta laikų skaičiavimo paklaidų ir amplitudinių fliuktuacijų apskaičiuotame lauke. Straipsnyje pateiktos kompiuterinio modeliavimo metodikos ir rezultatai.

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