Specific features of airborne insulation

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Introduction

Airborn sound insulation is one of the main methods for noise reducation in rooms and in production. The main property of airborne sound insulation that predetermines its effectiveness is the density (mass) of insulation material. However, the most recent theoretical and experimental study and their results have shown that the effect of that property on airborne sound insulation can vary in dependence on the localisation of sound insulation materials in the construction and their performance.

The present paper deals with some of the properties of sound insulation constructions.

General outlines of airborne sound insulation of a Barrier

The mechanism of airborne sound insulation by a barrier consists of the fact that a sound wave that is incident on a barrier drives it into a vibratory motion with a frequency that is equal to the frequency of vibrations of airborne particles in a wave. As a result, the barrier becomes the source of sound and radiates it into the isolated room. For acoustical characteristics of a barrier the concept of the coefficient of sound penetration

$$\tau(\vartheta)=\left|\frac{p_2(\vartheta)}{p_1(\vartheta)}\right|^2, \tag{1}$$

and of an airborne sound insulation are introduced

$$R(\vartheta)=10\lg\frac{1}{\tau(\vartheta)}, \tag{2}$$

where p(ϑ), p(ϑ) is the pressure in a sound wave, which is incident on a barrier and passing through it at an angle ϑ.

At a diffusion incidence of sound waves, the concept of the diffusion coefficient of sound penetration is being introduced, which represents the static value of sound penetration through a barrier at all possible angles of incident sound, i.e.,

$$\tau=\int_o^{\pi/2}\tau(\vartheta)\sin 2\vartheta d\vartheta\,. \tag{3}$$

Accordingly, the airborne sound insulation

$$R=10\lg\frac{1}{\tau}\,. \tag{4}$$

Two approaches exist for calculation of airborne sound insulation of a barrier. The first one, which was proposed by L. Cremer [1], consists of the following.

Having studied the insulation properties of a thin plain sheet (plate) L. Cremer showed that they depend on relative values of phase velocities c_f (motion velocity of a sound wave trace along the plate) and c_u (propagation velocity of sound in the plate). With changes in frequency or the incidence angle of sound waves the velocity c_f can become equal to the velocity c_u, i.e.,

$$c_u=c_f=\frac{c}{\sin\vartheta}, \tag{5}$$

where c is the sound velocity in the air.

In this case a well-known phenomenon of wave coincidence appears and airborne sound insulation becomes low. The lowest frequency where wave coincidence is possible is called the critical (boundary) frequency and it corresponds to the “sliding” incidence angle of sound waves, where $c = c_u$.

Th. Vogel applied another approach of problem solving [3] when studying the plates of finite dimensions. According to his research, the major part of sound energy transmitted through the plate is related with its mode of vibrations, excited at sound wave incidence. If to consider the expression of the energy transmitted in the form of a series, each terms of which is determined by the difference between the frequency of excitation and its eigenfrequency, the major part of energy passed belongs to the resonance terms.

These two approaches are almost similar, if to consider the bending wave as natural vibrations, i.e., as eigenvibrations. The main result of these two approaches is that the equalization of wavelengths or frequencies of a sound wave and frequencies of eigen vibrations leads to the airborne sound insulation of a barrier that becomes low. Since the wall resistance at the resonance comes through a zero value, its value will be less than the resistance of the mass at the absence of resilient forces within the limits of some range of frequencies, containing the resonance frequency. Therefore in these both cases of resonance the value of airborne sound insulation will be less than it is determined by a simple “mass law”.

In addition, the insulation of airborne sound, evidently, depends on the shape of barriers. This is predetermined by the effect of different character of one and the same sound wave on barriers of different shapes as well as on the different types of vibrations of these barriers. However, an approach to solving a problem as well as some basic statements of airborne sound insulation, probably, will be the same. Issues of airborne sound insulation of plates have been studied most extensively.

Airborne sound insulation of double leaf constructions from the resilient wall containing viscous surface

A wide application of absorbing surfaces in engineering constructions causes a certain interest to calculation schemes, containing a description of the condition of absorbing layers in the process of vibrations. Commonly, damping is accounted by the introduction of complex moduli of resilience. In the present report damping material is considered as a layer of resilient material, working on the displacement. The effect of viscosity on airborne sound insulation of a layered construction will be naturally noticeable only at high frequencies, i.e., at random vibrations with a wide spectrum, impacts, etc.

The frequencies of a plate, loaded on both sides by a pressure difference, are expressed by the following equation:

$$B\frac{\partial^4 u}{\partial x^4} - \rho h\omega^2 u - \sigma_{zz} = p_1 - p_2, \tag{6}$$

where B, h, ρ are the cylindrical rigidity, thickness and density correspondingly, u is the displacement; x is the coordinate, directed along the plate in the plane of incidence of outer wave pressure; σ_{zz} is the normal to the plate of tensor component of viscous tension; ω is the angular frequency of vibrations.

For accounting of viscous resistance of the environment it is sufficient to take into account only a displacement potential, since the compressibility of an absorbing surface is of no considerable importance.

Let axis z be directed along the normal to the layers. Then the displacement potential is determined by an equation:

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial z^2} + \frac{i\rho_0\omega}{\eta} f = 0. \tag{7}$$

Moreover, velocity components $\dot{u}_z$ and $\dot{u}_x$, the value σ_{zz} are determined through f in the following way:

$$\dot{u}_z = -\frac{\partial f}{\partial x}; \qquad \dot{u}_x = -\frac{\partial f}{\partial z}; \qquad \sigma_{zz} = -2\eta\frac{\partial^2 f}{\partial x \partial z}. \tag{8}$$

Here η and ρ_0 is the displacement viscosity and the density of the environment.

For the equation used the boundary-value problem is being set:

$$\frac{\partial f}{\partial x} = \dot{u}_z \quad \text{at} \qquad z=0;$$

$$\frac{\partial^2 f}{\partial x \partial z} = 0 \quad \text{at} \qquad z=H. \tag{9}$$

The solution of that problem is written in the form:

$$f = e^{iqx - i\omega t}\left(A\sin\aleph z + B\cos\aleph z);\right)$$

$$q^2 + \aleph^2 = \frac{i\rho_0\omega}{\eta}. \tag{10}$$

Constants A and B are determined from boundary conditions and thus we find:

$$f = -\frac{\dot{u}}{i\rho}\left(tg\aleph h\sin\aleph z + \cos\aleph z\right)e^{i\rho x - i\omega t}. \tag{11}$$

Now we can determine a tensor component σ_{zz} at $z=0$:

$$\sigma_{zz} = 2\eta\dot{u}\aleph tg\aleph H. \tag{12}$$

We note that for a thick layer of surface the following equation takes place $|\aleph H| >> 1$: $\sigma_{zz} = 2\sqrt{i\rho_0\omega\eta}\dot{u}$, since $\aleph$ has a complex value.

From here it is evident that use of too thick layers with a free surface does not lead to the efficient results. In the case of a thin layer $tg\aleph H \cong \aleph H$ and, consequently,

$$\sigma_{zz} \cong \left(2i\rho_0\omega H\right)\dot{u}. \tag{13}$$

Thus too thin layers also do not lead to the effect of damping.

Now, excluding a tensor component σ_{zz} with the help of the formula (12), we come to the following equation of plate motion:

$$B\frac{\partial^4 u}{\partial x^4} - \rho h\omega^2 u + fi\omega u = p_1 - p_2; \; f = 2\eta\aleph tg\aleph H,$$

$$f = 2\eta\aleph tg\aleph H,$$

i.e., we obtain a usual problem of airborne sound insulation, taking the incident, reflected and transmitted waves as factors acting on the plate:

$$\rho_i = e^{-i\omega t + i\frac{\omega y}{c}\cos\Theta + i\frac{\omega x}{c}\sin\Theta};$$

$$\rho_r = \beta e^{-i\omega t - i\frac{\omega y}{c}\cos\Theta + i\frac{\omega x}{c}\sin\Theta};$$

$$\rho_t = \alpha e^{-i\omega t + i\frac{\omega y}{c}\cos\Theta + i\frac{\omega x}{c}\sin\Theta},$$

where θ is the plane wave incidence angle.

For a transmission coefficient we find the following solution:

$$\alpha = 2/\left\{1 + \frac{i\cos\Theta}{\rho_0\omega c}\left[B\frac{\omega^4}{c^4}\sin^4\Theta - \rho h\omega^2 + i\omega f\right]\right\}.$$

We find airborne sound insulation:

$$R = -10\lg|\alpha|^2. \tag{14}$$

Airborne sound insulation of symmetrical four-layered system with airborne space and damping layers

Let two plates, supplied with absorbing surfaces, be separated by an air space with a thickness l. The problem of airborne sound insulation is related to the determination of the transmission coefficient α of the wave, transmitted though a layered system:

$$p_1 = e^{-i\omega t}\left(e^{i\frac{\omega}{c}z\cos\Theta + i\frac{\omega}{c}x\sin\Theta} + \mu e^{-i\frac{\omega}{c}z\cos\Theta + i\frac{\omega}{c}x\sin\Theta}\right);$$

$$p_2 = e^{-i\omega t}\left(\beta e^{i\frac{\omega}{c}z\cos\Theta + i\frac{\omega}{c}x\sin\Theta} + \gamma e^{-i\frac{\omega}{c}z\cos\Theta + i\frac{\omega}{c}x\sin\Theta}\right);$$

$$p_3 = \alpha e^{-i\omega t + i\frac{\omega}{c}(z-i)\cos\Theta + i\frac{\omega}{c}x\sin\Theta}.$$

Barrier constructions of such a type possessing good thermal insulation properties can be used successfully also for solution of problems of optimal sound insulation.

The given system of solutions of the wave equation contains four constants, which we shall define from the algebraic system of equations at substituting these solutions into boundary equations:

$$B_1 \frac{\partial^4 u_1}{\partial x^4} - \rho_1 h_1 \omega^2 u_1 + i\omega f_1 u_1 = p_1 - p_2;$$

$$\frac{\partial p_1}{\partial z} = \frac{\partial p_2}{\partial z} = \rho_0 \omega^2 u_1, \qquad (z=0);$$

$$B_2 \frac{\partial^4 u_1}{\partial x^4} - \rho_2 h_2 \omega^2 u_2 + i\omega f_2 u_2 = p_2 - p_3;$$

$$\frac{\partial p_2}{\partial z} = \frac{\partial p_3}{\partial z} = \rho_0 \omega^2 u_2, \qquad (z=l).$$

Therefore,

$$u_1\left[B_1\left(\frac{\omega}{c}\sin\Theta\right)^4 - \rho_1 h_1 \omega^2 + i\omega f_1\right] = 1+\mu-\beta-\gamma;$$

$$(1-\mu)\frac{i\cos\Theta}{\omega\rho_0 c} = (\beta-\gamma)\frac{i\cos\Theta}{\omega\rho_0 c} = u_1;$$

$$u_2\left[B_2\left(\frac{\omega}{c}\sin\Theta\right)^4 - \rho_2 h_2 \omega^2 + i\omega f_2\right] =$$

$$= \beta e^{i\frac{\omega l}{c}\cos\Theta} + \gamma e^{-i\frac{\omega l}{c}\cos\Theta} - \alpha.$$

$$\left(\beta e^{+i\frac{\omega l}{c}\cos\Theta} - \gamma e^{-i\frac{\omega l}{c}\cos\Theta}\right)\frac{i\cos\Theta}{\omega\rho_0 c} = \alpha\frac{i\cos\Theta}{\omega\rho_0 c} = u_2;$$

The system obtained leads into the following system of 4th order

$$\beta\{Q_1\} + \gamma\{Q_2\} - \mu = 1;$$

$$\beta - \gamma + \mu = 1;$$

$$\alpha\{Q_3\}2\sigma e^{-i\frac{\omega l}{c}\cos\Theta} = 0;$$

$$\alpha - \beta e^{i\frac{\omega l}{c}\cos\Theta} - \gamma e^{-i\frac{\omega l}{c}\cos\Theta} = 0.$$

Here

$$Q_1 = \left\{1 + \frac{i\cos\Theta}{\omega\rho_0 c}\left[B_1\left(\frac{\omega}{c}\sin\Theta\right)^4 - \rho_1 h\omega^2 + i\omega f_1\right]\right\};$$

$$Q_2 = \left\{1 - \frac{i\cos\Theta}{\omega\rho_0 c}\left[B_1\left(\frac{\omega}{c}\sin\Theta\right)^4 - \rho_1 h_1\omega^2 + i\omega f_1\right]\right\};$$

$$Q_3 = \left\{\frac{i\cos\Theta}{\omega\rho_0 c}\left[B_2\left(\frac{\omega}{c}\sin\Theta\right)^4 - \rho_2 h_2\omega^2 + i\omega f_2\right]\right\};$$

Solution of the given system can be written in the form α=4/Δ:

$$\alpha = \frac{4}{\{Q_1\}\{Q_2\}e^{-i\frac{\omega l}{c}\cos\Theta} - \left(\frac{i\cos\Theta}{\omega\rho_0 c}\right)^2 [Q_3'][Q_4']e^{+i\frac{\omega l}{c}\cos\Theta}}.$$

Here,

$$Q_1' = \left\{2 + \frac{i\cos\Theta}{\rho_0\omega c}\left[B_1\left(\frac{\omega}{c}\sin\Theta\right)^4 - \rho_1 h_1\omega^2 + i\omega f_1\right]\right\};$$

$$Q_2' = \left\{2 - \frac{i\cos\Theta}{\rho_0\omega c}\left[B_2\left(\frac{\omega}{c}\sin\Theta\right)^4 - \rho_2 h_2\omega^2 + i\omega f_2\right]\right\};$$

$$Q_3' = \left[B_1\left(\frac{\omega}{c}\sin\Theta\right)^4 - \rho_1 h_1\omega^2 + i\omega f_1\right];$$

$$Q_4' = \left[B_2\left(\frac{\omega}{c}\sin\Theta\right)^4 - \rho_2 h_2\omega^2 + i\omega f_2\right].$$

Mechanical models of three-layered sound insulation constructions with a light filler

At present three-layered symmetrical constructions filled with glasswool, porious materials or with honeycomb fillers are widely popular.

Calculation of similar products with the help of three-dimensional equations of theoretical resilience is complex and from the engineering point of view is too detailed , since the behaviour of a filler, strictly speaking, cant be not subject of the rheology of the solid resilient body. Two types of fillers are being considered in the literature: filler operating on the contraction and filler operating on the displacement. In the first case we obtain the following equation for a three-layered plate:

$$B_1 = \frac{\partial^4 u_1}{\partial x^4} - \rho_1 h_1 \omega^2 u_1 + E/l(u_1 - u_2) = p_1;$$

$$B_2 \frac{\partial^4 u_2}{\partial x^4} - \rho_2 h_2 \omega^2 u_2 + E/l(u_2 - u_1) = p_2,$$

where E is the Young's modulus of the filler; l is the layer thickness. In the second case

$$(B_1 + B_2)\frac{\partial^4 u}{\partial x^4} - (\rho_1 h_1 + \rho_2 h_2)\omega^2 u + Gl\frac{\partial^2 u}{\partial x^2} = p_1 - p_2,$$

where G is the displacement modulus of the filler; l is its thickness.

The corresponding problem of airborne sound insulation leads to the following transmission coefficient:

$$\alpha = 2/\left\{1 + \frac{i\cos\Theta}{\rho_0\omega c}\left[B\frac{\omega^4}{c^4}\sin^4\Theta + Gl\frac{\omega^2}{c^2}\sin^2\Theta - \rho h\omega^2\right]\right\},$$

where B, ρ, h are the parameters of the plate.

Conclusions

Airborne sound insulation of the construction depends on the mass of the barrier, its dimensions, joints with other constructions, the physical and geometrical indicators of the latter, the frequency of vibrations, and the angle of sound wave propagation.

The specific features of insulation materials are characterised by the ranges of their resonance frequencies.

Research of airborne sound insulation double sheet and multi-layered constructions shows that airborne sound insulation of the said constructions can be improved without increasing the mass of the construction.

References

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Orinio garso izoliacijos ypatumai

Reziumė

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