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Parameters of spread spectrum signals

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Introduction

Code division multiple-access (CDMA) methods have attracted increasing attention in the past few years because of their advantages over conventional frequency division multiple access and time division multiple access methods. Entirely new applications and opportunities such as personal communications networks, digital mobile cellular phones, indoor wireless communications, very small aperture satellite terminals have created a need for research and development focused on direct sequences (DS) spread spectrum techniques.

Spreading codes play an important role in the implementation of CDMA systems, as the selection of the code directly influences the system performance (signal to noise ratio (SNR), bit error ratio (BER), code synchronization, multipath signal rejection).

The main area of using the spread spectrum signals was communications systems and only a few research works was done with this technology in distance measurement area, especially in ultrasonic measurements equipment, so there is a very wide research area.

The main aim of this paper is to show the difference between criteria for choosing spread spectrum signals for communication systems and distance measurement systems.

Basic DS/SSMA parameters

The signal to noise power ratio at the output from the asynchronous direct sequences/spread spectrum multiple access (DS/SSMA) receiver of the *j*-th channel can be expressed in terms of asynchronous multiple-access interference Q_a and the signal to noise power ratio of the additive white Gaussian noise (AWGN) channel E_b/N_0 [1]:

$$SNR_j = \left\{ \frac{N_0}{2E_b} + Q_a \right\}^{-1} . \tag{1}$$

The SNR at the output of a receiver is one of the most important measures of average performance that can be obtained with reasonable amount of computation [1]. Multiple access interference (MAI) can be expressed in terms of the sum of the average interference parameters (AIP) of *K* simultaneous and asynchronous interfering users. The AIP value ($r_{i,j}$) depends on the aperiodic cross-correlation function ($C_{i,j}$) between the desired (*j*) and undesired (*i*) code of period *p*.

$$Q_a = \frac{1}{6p^3} \sum_{i=1, i\neq j}^{K} r_{i,j} , \tag{2}$$

$$r_{i,j} = 2\mu_{i,j}(0) + \mu_{i,j}(1) , \tag{3}$$

$$\mu_{i,j}(n) = \sum_{l=1-p}^{p-1} C_{i,j}(l) C_{i,j}(l+n) . \tag{4}$$

The parameters $r_{i,j}$ and $\mu_{I,j}(n)$ are considerable important in SSMA communications, as they can be used as a criterion for comparing sequences [4]. The AIP can also be expressed in terms of aperiodic auto-correlation functions that are not independent [2].

$$\mu_{i,j}(n) = \sum_{l=1-p}^{p-1} C_i(l) C_{j(l+n)} . \tag{5}$$

It can be easily proved that the SNR performance of an asynchronous DS/SSMA system depends only on the aperiodic correlation, although the even (periodic) and the odd cross-correlation functions, which exist approximately with equal probability in the dispreading process of a DS/SSMA receiver are taken into consideration in the derivation of Fig. 1-5. The overall average performance parameter depends on the whole spectrum of the aperiodic correlation functions of the codes used, and not just on their peak values, as can be seen from Eq. 4 and 5.

Thus the codes used in SSMA systems should be selected so that all the absolute values of the aperiodic cross-correlation functions are minimized within a discrete delay axis of *2p*. The influence of good aperiodic cross-correlation function can be seen immediately in both even and odd cross-correlation functions. In DS/SSMA communication system, with random binary data modulation, the autocorrelation and cross-correlation magnitudes depends on both the even and odd correlation functions, which both, depend on the aperiodic correlation functions, which are sensitive to the sequence initial phase [3]. The choice of the code family or function set is typically based on the even cross-correlation function, although aperiodic cross-correlation function is also important. The absolute maximum values for the aperiodic and odd cross-correlation function can be minimized by proper choice of the starting phases of the sequences.

The SNR of a DS/SSMA receiver can be approximated in the following manner [1]:

$$SNR_j = \left\{ \frac{N_0}{2E_b} + \frac{K-1}{3p} \right\}^{-1} . \tag{6}$$

Eq. 6, which doesn't depend on the type of spreading codes, is based on the assumption that all spreading codes are purely random, independent binary sequences. It is shown in [6] that the second term of the right hand side of Eq. 6 is actually the expectation of asynchronous multiple access interference (Q_a) associated with random sequences. According to [1], the main use of Eq. 6 would be in a preliminary system design. A more detailed investigation of the performance can then be carried out using Eq. 1 for specific code sequences. The expressions Eq. 1-5 can also be used for performance comparison employing spreading code families and code sets of various kinds. The bit error probability of a DS/SSMA systems can be approximately calculated by applying (1) in the following form [1]:

$$\Phi_e = 1 - \Phi(\sqrt{SNR_j}) \ . \qquad (7)$$

The approximation is based on the assumption that the probability distribution function of the MAI is Gaussian, according to the Central Limit Theorem, having in mind that the number of simultaneous signals is sufficiently large and the period of the codes sufficiently long. Some of the latest results regarding the accuracy of the standard Gausian approximation Eq. 7 and improved approximations can be found in [11].

Comparison of the performance of some linear spreading code families

The comparison of some spreading code sets and families are presented in [12], the focus is on optimised AIP values for the various code sets, and little has been written about SNR performance of well known linear code families as well. The accuracy of the approximation Eq. 6 has not been fully studied also.

The AIP values of some sets of Gold, Kasami and M-sequences of short periods are examined in [5…8], where they are minimized by the AO/LSE (auto-optimal/least side lobe energy) and LSE/AO (least side lobe energy/ auto-optimal) criteria. The parameter $\mu_{I,j}(0)$ of Eq. 3 and $r_{i,j}$ [5] can be bounded by the inequalities [2]:

$$p^2 - 2(S_i S_j)^{\frac{1}{2}} \le \mu_{i,j}(0) \le p^2 + 2(S_i S_j)^{\frac{1}{2}} \qquad (8)$$

,

$$r_{i,j} \le 4p^2 + 6(S_i S_j)^{\frac{1}{2}} \ . \qquad (9)$$

If the sidelobe energies [5,6] S_i and S_j of the aperiodic auto correlation function are optimised by the proper choice of sequences of sequence initial phases, the upper bounds of Eq. 8 and 9 will be minimised. The lower bound will be maximized, which is undesirable and the mater of some importance. This means that the benefit derived from the optimisation will be reduced.

In [12] optimised and unoptimised (randomly selected phases of the same codes) codes are compared, and it was shown that the optimisation criteria don't minimize a cross-correlation function directly. They only optimise the shapes of auto-correlation functions. It can be argued that very little can be gained by difficult, time-consuming optimisation because on average the SNR performance of these optimised and unoptimised sets are equal. Consequently no optimisation of the code sets was performed here. The above is true only if minimization of the MAI is the most important goal.

It was also shown [12] that there are no noticeable differences in SNR performance between the families of unoptimzed Gold, Kasami and M-sequences with equal period and size set. This feature is surprising: because it calls into question the accepted belief that some code families are superior for DS/SSMA systems and Gold sequences are better that M-sequences. It is commonly believed that good, even cross-correlation functions for a code family result in a small MAI, as the main choice criterion for a code family. However, an even cross-correlation function will be effective for only about a half of time in a DS/SSMA receiver because of the statistical properties of data bits, in an odd cross-correlation function, which is quite similar for all the well-known code families of equal period, will take up the part of the time. This means that an odd cross-correlation reduces the benefits of a good even cross-correlation, so that M-sequences, for example, are equal in terms of SNR performance in a DS/SSMA system to Gold sequences.

The similarity in SNR performance between these code families can also be predicted from [4]. In this reference it is shown that the expectation of the AIP for randomly selected, unoptimised Gold sequences is

$$E(r_{i,j})_{Gold} = 2p^2 - p - 1 + \frac{2}{p} \ , \qquad (10)$$

and that for randomly selected, unoptimised M-sequences is:

$$E(r_{i,j})_{M-sequences} = 2p^2 - 2 + \frac{2}{p} \ . \qquad (11)$$

It can be easily conclude that the results of Eq. 10 and 11 are about the same even for short periods of sequences.

Correlation parameters for spreading codes

The basic correlation parameter for spreading codes is the aperiodic correlation function. The general discrete – valued aperiodic cross – correlation function CCF for arbitrary complex – valued sequences *x* and *y* of period *p* is defined as [1]:

$$C_{x,y}(\tau) = \begin{cases} \sum_{i=0}^{p-1-\tau} x(i)[y(i+\tau)] & , \ 0 \le \tau \le p-1, \\ \sum_{i=0}^{p-1+\tau} x(i-\tau)[y(i)] & , \ 1-p \le \tau \le 0, \\ 0 & , \ |\tau| \ge 0. \end{cases} \qquad (12)$$

The even (periodic) CCF and the odd CCF, which exist with about equal probability in the dispreading process of a DS/SSMA receiver, can be expressed with the aid of (12) as:

$$Q_{x,y}(\tau) = C_{x,y}(\tau) + C_{x,y}(\tau - p) \ , \qquad (13)$$

$$\hat{Q}_{x,y}(\tau) = C_{x,y}(\tau) + C_{x,y}(\tau - p) \quad \text{for} \quad 0 \le \tau < p \ . \qquad (14)$$

The maximum value of the odd function depends strongly on the relative phase between sequences. In references [3], [4], [10] and [12] odd correlation functions are analysed. One common method of selecting cede sequences for DS/SSMA systems is to select a family of sequences for which the maximum even cross correlation is relatively small (Gold sequences), in the hope of finding a sub-family for which the maximum odd correlation is also small after sequence phase optimisation.

It has been shown [3] that the average signal to noise power ratio at the output from a BPSK type asynchronous DS/SSMA receiver of the *j*-th signal can be expressed in terms of the sum of the average interference parameters ($r_{i,j}$) of K simultaneous signals, and the SNR of the AWGN channel (E_b/N_0) as follows:

$$SNR_j = \left\{ \frac{N_0}{2E_b} + \frac{1}{6p^3} \sum_{i=1, i\neq j}^{K} r_{i,j} \right\}^{-1} \qquad (15)$$

Where the average interference parameter is defined as $r_{i,j} = 2\mu_{i,j}(0) + \mu_{i,j}(1)$. In case of binary sequences the expression for $\mu_{i,j}(n)$ can be defined as:

$$\mu_{i,j}(n) = \sum_{m=1-p}^{p-1} C_{i,j}(m) C_{i,j}(m+n) \qquad (16)$$

Thus the final average asynchronous SSMA performance, measured with SNR, depends only on aperiodic CCF values between code sequences, although the maximum value of the even CCF is traditionally used as selection criterion for CDMA code family. However the use of the maximum value of the even CCF as a code selection criterion ignores effect of data modulation, i.e. the odd cross-correlation, which is effective for about a half of the time in a DS/SSMA receiver because of the statistical properties of data bits.

The parameters SNR_j and $r_{i,j}$ have been widely used as criteria for comparing PN sequences. The SNR is one of the most important performance measures that can be obtained with reasonable amount of computation [3]. Derivation of the exact average bit error probability is the most interesting performance measure of a SSMA system is complicated even in case of the additive white Gaussian noise (AWGN) channel.

Mean square cross-correlation for characterization of CDMA code families

It can be shown that the following unnormalized equation:

$$\sum_{k=0}^{p-1} \theta_{i,j}^2(k) + \sum_{k=0}^{p-1} \hat{\theta}_{i,j}^2(k) = 2 * \sum_{k=1-p}^{p-1} C_{i,j}^2(k) \ , \qquad (17)$$

holds both for binary and for complex-valued sequences {*i*} and {*j*} [14]. Now with the aid of (17), the average interference parameter can be expressed in the form:

$$r_{i,j} = \sum_{k=0}^{p-1} \theta_{i,j}^2(k) + \sum_{k=0}^{p-1} \hat{\theta}_{i,j}^2(k) + \sum_{k=1-p}^{p-1} C_{i,j}(k) C_{i,j}(k+1) \qquad (18)$$

for a BPSK type DS/SSMA system with binary sequences. From Eq. 18 it can be predicted that the average performance of an asynchronous DS/SSMA system, measured by the SNR, strongly depends on the square sums of both even and odd cross-correlation values between sequences, i.e. the mean square cross-correlation values of both even and odd functions, when the equations are normalized properly by the code period *p*. It is felt that the decomposition technique of the $r_{i,j}$ can be applied for the analysis of other types of SSMA modulation methods with complex-valued spreading sequences.

Comparison of the communication system and the distance measurement system

The model of communication system is shown in Fig.1.

The model of distance measurement system is given in Fig. 2. From these two figures we can see the differences between the listed systems. From the first view these systems looks very similar and the main difference is that the distance measurement system don't have modulators and have only one input per channel for a spread spectrum signal. Because evaluating parameters of the communication system wasn't taken into consideration the data signal from the one side we can insist that all parameters of communication system can be used for evaluation of the distance measurement system, but from another side the main measured parameter is a signal delay time. So from this point of view we can make conclusion that only some basic parameters, mainly for evaluating spread spectrum parameters of communication system can be used for evaluating distance measurement systems.

Criteria for evaluating of the spread spectrum signals for the distance measurement system

The common task in comparison between communication systems and distance measurement systems are presented in Table 1. As we can see the main task in communication systems is data transfer between two users and the main task in distance measurement systems is measurement of the signal delay time. Also there are the same tasks in both systems like signal extraction from noise and different tasks like data security or system synchronization.

According to these common tasks it is possible to formulate criteria for choosing spread spectrum signals. These criteria are listed in Table 2. The main difference between these criteria is requirements for the shape of an autocorrelation function of the signal in distance measurement systems. It makes main influence in a distance measurement precision. Of course, these systems have some additional criteria like data security or length of sequence, but these are only additional but not the main criteria.

In fact, using these criteria we can only chose some signals in the beginning of the system design and for evaluating measurement accuracy we need to use additional parameters, which will take in to consideration the signal parameters and the limitation of the signal bandwidth.

In Fig. 3-8 are shown some examples of M-sequences correlation parameters. In the Fig. 3 are shown normalized even cross correlation of sequence No1 with all another M-sequences, the numeration of sequences is according to [6]. This sequence is passing criteria for communication system but it is not passing criteria for a distance measurement system, because level of cross correlation function with other M-sequences is too big. In the Fig. 5 are shown autocorrelation function of the same M-sequence.

In Fig. 4 are shown normalized even cross correlation of M-sequence No76 with all other M-sequences. This sequence is passing criteria for communication system and distance measurement system, because it has a very low level of the cross correlation function with all other sequences and as it shown in Fig. 6, a very low level of side lobes.

The main problem for choosing PN sequences according to the listed criteria is very large amount of computation, because it is necessarily at the beginning to calculate all parameters of the chosen PN sequence or sequence family and only after that to check is it passing all listed criteria.

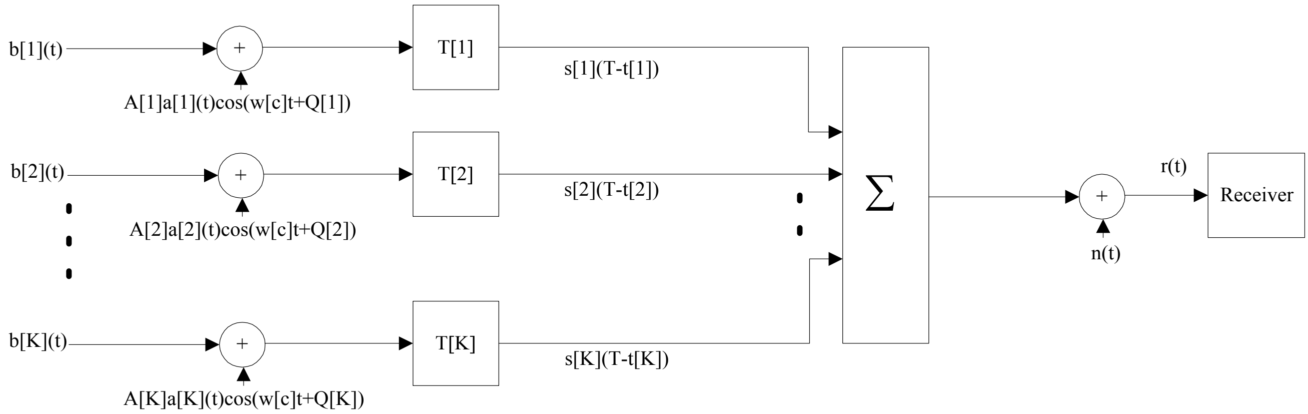

Fig. 1 **Model of an asynchronous biphase coded direct sequence spread spectrum multiple access system**.

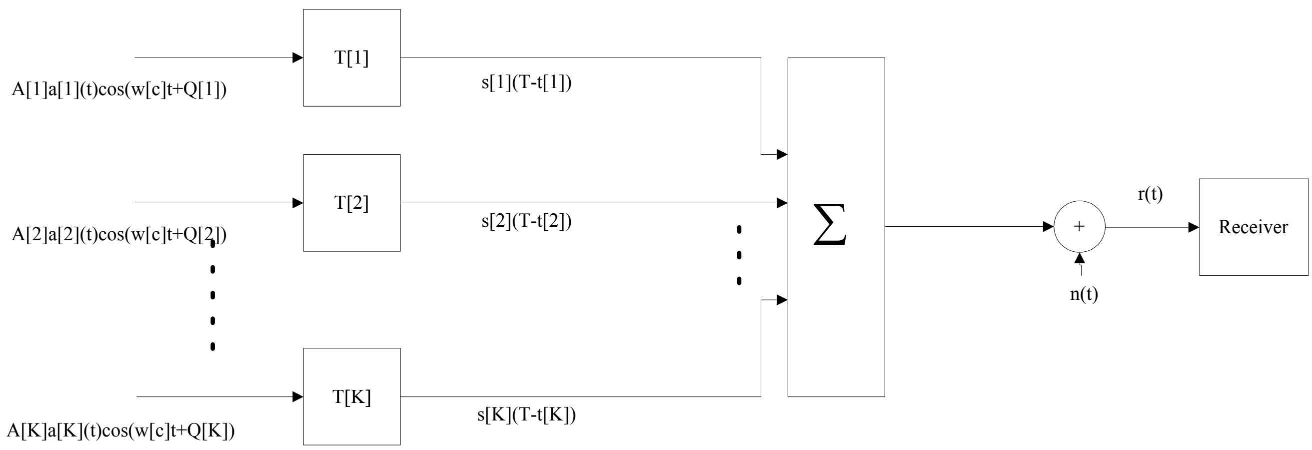

Fig. 2 **Model of a distance measurement system**.

Table 1. **Common tasks**

Communication system	Distance measurement system
1. Data transmission 2. Signal extraction from noise 3. Signal extraction from similar PN signals 4. Data security (if needed) 5. Synchronization between systems (if needed)	1. Measurement of the system delay time. 2. Extraction of the delayed signal from noise 3. Extraction of delayed signal from similar PN signals

Table 2. **Signal selection criteria**

Communication system	Distance measurement system
1. Crosscorrelation between signal and noise must be close to zero. 2. Crosscorrelation between two codes must be close to zero. 3. Special criteria (if needed) for data security and system synchronization.	1. Autocorrelation function must be very narrow. 2. The side lobes of autocorrelation function are very small. 3. Crosscorrelation between signal and noise must be close to zero. 4. Crosscorrelation between two codes must be close to zero. 5. Duration of the sequence must be finite.

Conclusions

1. The distance measurement system has bigger requirements for PN sequence quality.
2. PN sequences that are good for communication systems are not always good for the distance measurement

systems, however PN sequences which are good for the distance measurement system is almost always good for a communication system.

3. There is no noticeable difference in SNR performance between the families of unoptimized Gold, Kasami and m-sequences with an equal period and set of size.

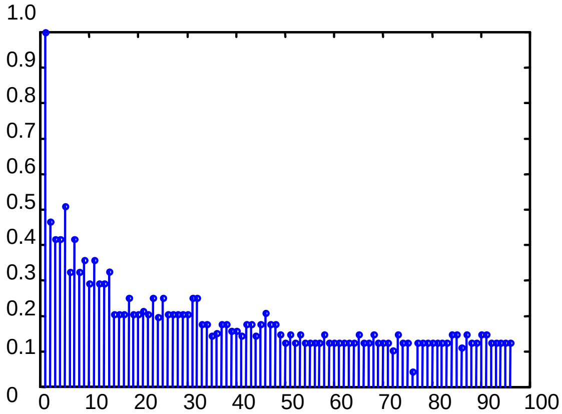

Fig. 3 **Normalized even cross correlation between M-sequence No1 and all another M-sequences**

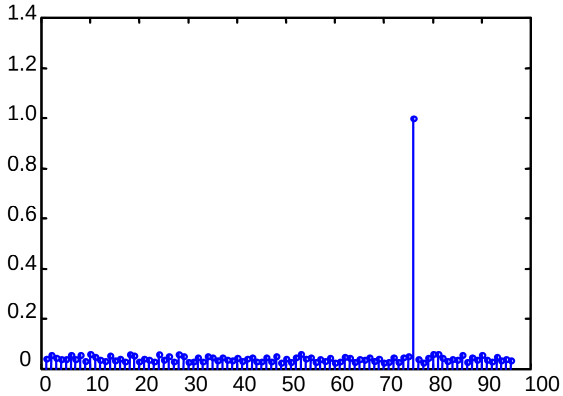

Fig. 4 **Normalized even cross correlation between M-sequence No76 and all another M-sequences**.

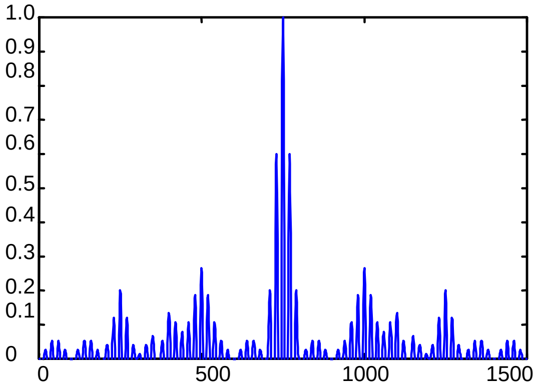

Fig. 5 **Normalized autocorrelation function of M-sequence No1**.

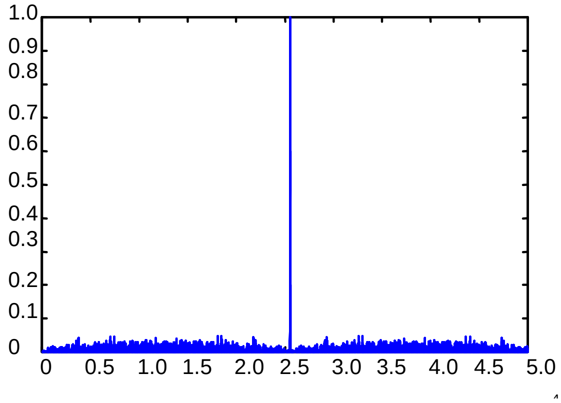

Fig. 6 **Normalized autocorrelation function of M-sequence No76**

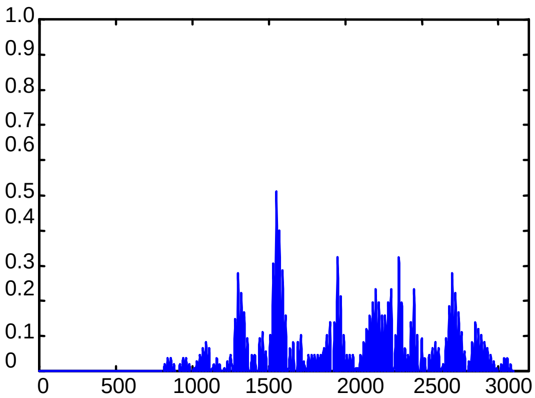

Fig. 7 **Normalized even cross correlation between M-sequences No5 and No1**

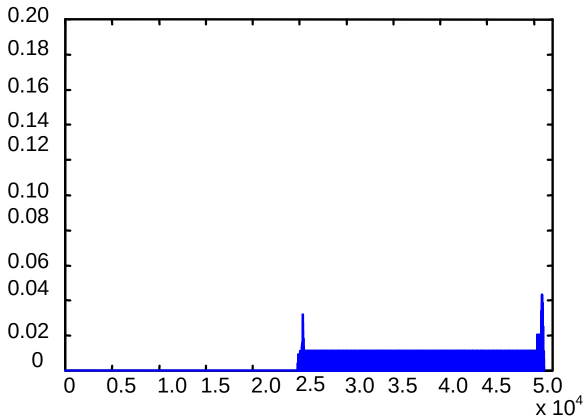

Fig. 8 **Normalized even cross correlation between M-sequences No76 and No1**

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Paskirstytojo spektro signalų parametrai

Reziumė

Išanalizavus, kaip plėstinio spektro signalai panaudojami komunikacinėse ir atstumo matavimo sistemose, prieita prie išvados, kad atstumo matavimo sistemose naudojamiems plėstinio spektro signalams keliami aukštesni kokybės reikalavimai. Šie kokybės reikalavimai daugiausia liečia autokoreliacinės funkcijos šoninių lapelių lygį ir signalų tarpusavio koreliacijos lygį, kadangi tai turi tiesioginę įtaką atstumo matavimo paklaidai.

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