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Acoustical studies of the high-frequency hopping conductivity in the quantum Hall regime

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1. Introduction

In the Quantum Hall regime when the Fermi level is situated between two adjacent Landau bands, the electrons are localized. This fact is confirmed by numerous DC measurements of the resistivity of the high-mobility 2-dimensional systems in a magnetic field at low temperatures (see, for example, [1]). In this case the conductivity σ_{xx} is of a hopping kind. However, the nature of the localized states is very difficult to determine in this experiments. The study of high-frequency conductivity σ^{hf} proved to be useful in solving of this problem. If the electrons are "free" the high-frequency conductivity σ^{hf} should be the same as σ^{dc}, measured in DC experiment, and the difference between σ^{hf} and σ^{dc} , from the other hand, points to the carrier localization.

2. The experimental setup

The high-frequency conductivity can be obtained from the propagation measurements of a surface acoustic wave (SAW). When a SAW propagates along the surface of a piezoelectric on which a semiconducting heterostructure with 2-dimensional electrons is superimposed (see Fig.1), the elastic wave is accompanied with an alternating electric field. This field penetrates into the 2-dimensional conductivity canal, thus producing currents, Joule losses, and the SAW attenuation. Sound velocity changes also. All these effects are governed by the high-frequency conductivity of the 2-dimensional system, and consequently if one observes Shubnikov-de Haas oscillations of the 2-dimensional system DC resistance in a magnetic field, similar oscillations should manifest themselves in the SAW attenuation coefficient Γ and velocity change $\Delta V/V$.

In the present work Γ and $\Delta V/V$ have been measured in a magnetic field up to 7*T* in the Si δ-doped GaAs/AlGaAs heterostructures with sheet densities $n=(1.3\text{-}2.7)\cdot 10^{11}\text{cm}^{-2}$ and mobilities $\mu=(1\text{-}2)\cdot 10^{5}\text{cm}^2/\text{Vs}$.

3. Experimental results

The high-frequency conductivity is generally a complex quantity: $\sigma^{hf}=\sigma_1-i\sigma_2$. For Γ and $\Delta V/V$ in this case we have:

$$\Gamma = 8.68\frac{K^2}{2}qA\frac{(4\pi\sigma_1 t(q)/\varepsilon_s V)}{[1+4\pi\sigma_2 t(q)/\varepsilon_s V]^2+[4\pi\sigma_1 t(q)/\varepsilon_s V]^2}$$

$$A=8b(q)(\varepsilon_1+\varepsilon_0)\varepsilon_0^2\varepsilon_s \exp[-2q(a+d)], \qquad (1)$$

$$\frac{\Delta V}{V}=\frac{K^2}{2}A\frac{[1+4\pi\sigma_2 t(q)/\varepsilon_s V]}{[1+4\pi\sigma_2 t(q)/\varepsilon_s V]^2+[4\pi\sigma_1 t(q)/\varepsilon_s V]^2}$$

where K^2 is the electromechanical coupling constant of $LiNbO_3$, *q* and *V* are wavevector and velocity of SAW, respectively, *a* is the gap between the piezodielectric and the heterostructure, *d* is the depth at which the 2-dimensional canal is buried, ε_1, ε_0 and ε_S are the dielectric constants of lithium niobate, vacuum and gallium arsenide respectively, *b* and *t* are some complicated functions of *a*, *k*, *d*, ε_1, ε_0, ε_S.

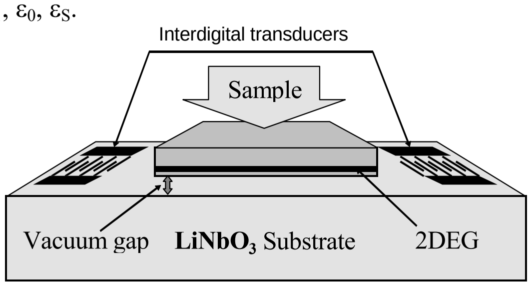

Fig 1. **The experimental setup**

In Fig. 2 the dependences of $\Gamma/(4.34AK^2k)$ and $(\Delta V/V)/(AK^2/2)$ on a magnetic field for a sample with the carrier density $n=2.7\cdot 10^{11}\text{cm}^{-2}$ and mobility $\mu=2\cdot 10^{5}\text{cm}^2/\text{Vs}$ are shown. One can see that these values oscillate with magnetic field, and for large filling factors the attenuation and velocity change peak do coincide, whereas for little filling factors the velocity change maxima coincide with the minima of the attenuation. Such a behaviour of these values could be explained sufficiently well by the Eq. 1.

The Eq. 1 provides one with σ_1 and σ_2 from the experimentally measured Γ and $\Delta V/V$. In Fig. 3 the dependences of σ_1 and σ_2 on a magnetic field at *T*=1.5K are shown.

As one can see, σ_2 practically vanishes near half-integer filling factors, i.e. when the Fermi level lies within the Landau band. It follows that the electrons are delocalized in this magnetic field region, and the conductivity is determined by its real part $Re(\sigma^{hf})=\sigma_1$, which is very close to the DC conductivity σ^{dc}.

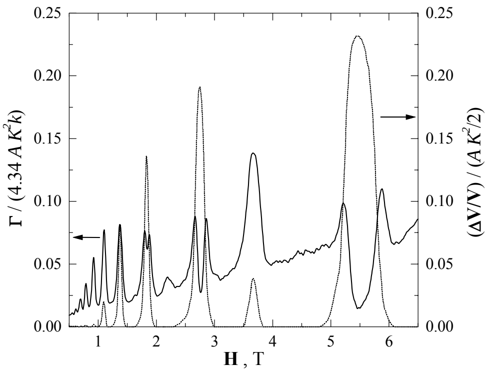

Fig 2. **The experimental dependences of Γ and $\Delta V/V$ on magnetic field H at $T = 1.5$K ($f = 30$MHz).**

With the further increase of the magnetic field the Fermi level leaves the Landau band, a metal-dielectric transition takes place, and the electrons become localized in the random fluctuation potential of the charged impurities.

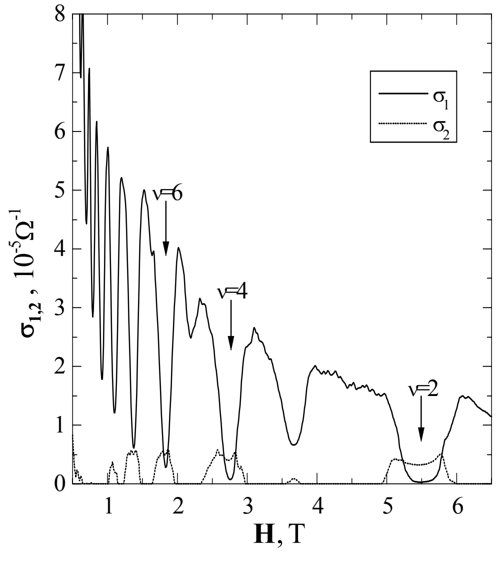

Fig 3**. The dependences of σ_1 and σ_2 on H at $T = 1.5$K (f=30MHz)**

As the Fermi level departs from the Landau band center, σ_1 becomes substantially larger than σ^{dc} . Such a behavior can be qualitatively interpreted [2] as absorption by large clusters ("lakes") disconnected from each other. Inside each cluster the absorption is determined by the value of σ^{dc}. Since the area occupied by the clusters is less than that occupied by the infinite cluster at the mobility edge, the effective $\sigma_1(\omega)$ is less than σ^{dc} at half-integer ν. At the same time $\sigma_1(\omega)$ exceeds σ^{dc} at the given magnetic field because there is no infinite conducting cluster at the Fermi level . The imaginary part, $\sigma_2(\omega)$ increases as the Fermi level departs from the Landau level's center.

In the magnetic fields corresponding to small integer filling factors the Fermi level is in the middle position between the Landau bands. One can see in Fig.4 that in this case σ_2 is far from being equal to zero, but to the contrary, is nearly an order of magnitude higher than σ_1.

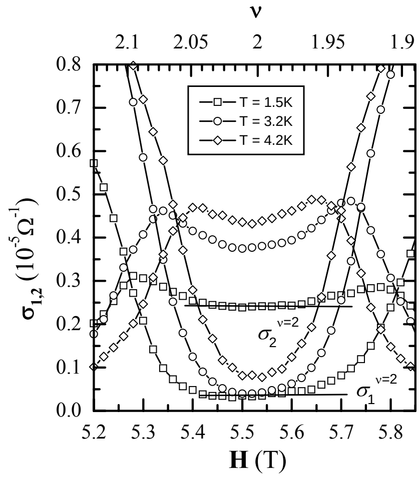

Fig 4. **Magnetic field dependences of σ_1 and σ_2 near the filling factor ν=2 at T=1.5-4.2K, $f=\omega/2\pi$=30MHz for the sample with $n=2.7\cdot10^{11}$cm^{-2}**

4. Discussion

Fig. 5 depicts the $\sigma_1(T)$ dependences (f = 30MHz) at the magnetic corresponding to the mid-points of Hall plateau fields. One can see a crossover from a smooth temperature dependence in a strong magnetic field (5.5T) to a rather steep increase with temperature in weaker fields. Such behavior is compatible with the idea that there are two contributions to the conductivity. The first is due to the extended states near the adjacent upper Landau level, while the second one is caused by the localized states at the Fermi level.

As the temperature grows, more and more 2D electrons appear at upper Landau level, due to their activation from the bound states at Fermi level. This leads to the growth of the weight of σ_T in the total conductivity. Obviously, this effect is the more pronounced at small magnetic fields.

We now turn to the region of low temperatures and the filling factors close to 2, where hopping between the localized states gives the main contribution to dielectric response. To analyze the experimental results we adopt the co-called two-site approximation, according to which an electron hops between states with close energies localized at two different impurity centers. These states form pair complexes which do not overlap. Therefore they do not contribute to the static conductivity but are important for the AC response. In the following we will use the 2D-version of the theory [3]. Some details of the discussion depend on the assumptions regarding both the density of localized states and the ralaxation mechanisms of their population.

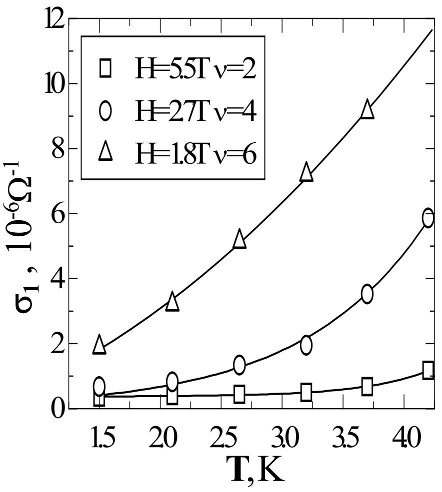

Fig. 5. **The experimental dependence of σ_1 on temperature at magnetic fields corresponding to integer filling factors (*f*=30MHz)**

As is well known, there are two specific contributions to the high-frequency absorption. The first contribution, the so-called resonant, is due to direct absorption of microwave quanta accompanied by inter-level transitions. The second one, relaxational, or phonon-assisted, is due to phonon-assisted transitions which lead to a lag of the levels population with respect to the microwave-induced variation in the inter-level spacing. The relative importance of the two mechanisms depends on the frequency ω, the temperature T, as well on the sample parameters. The most important of them is the relaxation rate $\gamma_0\ (T)$ of symmetric pairs with inter-level spacing $E=kT$. At $\omega \leq (kT\gamma_0/\hbar)^{1/2}$ the relaxation contribution to $\sigma_1(\omega)$ dominates, and only this one will be taken into account.

Following [4] we obtain

$$\sigma_1 = \frac{\pi^2 g^2 \xi^3 \omega e^4}{\varepsilon_s}\left(L_T + L_\omega/2\right)^2 \qquad (2)$$

$$r_\omega = \left(L_T + L_\omega/2\right)\xi$$

$$\frac{\sigma_2}{\sigma_1} = \frac{2L_\omega\left(L_T{}^2 + \frac{1}{2}L_T L_\omega + \frac{1}{12}L_\omega{}^2\right) + 4L_T{}^2 L_C}{\pi\left(L_T{}^2 + L_T L_\omega + \frac{1}{4}L_\omega{}^2\right)},$$

$$L_\omega = 1n\left(\frac{1}{\omega\tau_0}\right),\quad L_T = 1n\left(\frac{J_0}{k_B T}\right),\quad L_C = 1n\left(\hbar\frac{\omega_C}{k_B T}\right),$$

$$\frac{1}{\tau_0} = \frac{4\pi_e{}^2 K^2 k_B T}{\varepsilon s_\hbar{}^2 V},$$

where ξ is the localization length, r_ω is the distance between localized states within one pair, e is the electron charge and g is the density of states on the Fermi level, k_B is the Boltzman constant and $J_0 \approx$ Bohr energy.

For σ_1 given, one could with the aid of Eq.2 obtain the absolute value of the localization length of the 2D electrons. To do this one needs the one-electron density of states g in the magnetic field, corresponding to the case of the Fermi level in the middle position between two adjacent Landau levels. In the works of von Klitzing [5] and Kukushkin [6] (from temperature dependence measurements of conductivity in the activation regime) and in Pudalov's work [7] (from the capacity studies) it has been shown that for small even filling factors the density of states in the magnetic field region corresponding to the Hall's plateau is finite and does not depend on a magnetic field.

Using the density of states versus mobility curve from [1] obtained for a sample similar to ours, we estimate the density of states as $g=2.5\cdot10^{24}$ $cm^{-2}\cdot erg^{-1}$. On the other hand, according to [3], the density of states as a function of magnetic field H can be expressed by the interpolation formula

$$g(H) = \frac{g_0}{1+\sqrt{\mu H}} \qquad (3)$$

where μ is the mobility of the 2D-electrons while $g_0=2m/(\pi\hbar^2)$ is the 2D density of states at $H=0$. From (3) we obtain for H=5.5T the density of states $g=1.7\ 10^{24}$cm-2·erg^{-1}.

Using the first estimate for the density of states one obtains $\xi=6.5\cdot10^{-6}$ cm, that is about 1.6 times greater than the spacer thickness, $l_{sp}=4\cdot10^{-6}$ cm. On the other hand, it is the spacer width, which characterizes the random potential correlation length in the 2DEG layer. Hence, this fact contradicts to our interpretation of experimental results as in terms of pure nearest-neighbor pair hopping.

To solve the controversy, we assume that the high-frequency hopping conductivity of the 2DEG channel is shunted by the hopping along the doping Si δ-layer.

This assumption can be substantiated as follows. Let us suppose that in the middle of the Hall plateau $\sigma_1{}^{\nu=2}=4\cdot10^{-7}$ Ohm^{-1} and $\sigma_2{}^{\nu=2}=2.4\cdot10^{-6}$ Ohm^{-1} is entirely determined by the hopping conductivity along the Si δ-layer. Such a contribution is only weakly dependent on magnetic field because the latter is too weak to deform substantially the wave functions of Si-dopants. Then the contributions to σ_i associated with the 2DEG channel is just a difference between the experimentally measured σ_i in a given magnetic field and their values at $\nu=2$.

Let us analyze dependences of the differences $F_1 \equiv \sigma_1 - \sigma_1{}^{\nu=2}$ and $F_2 \equiv \sigma_2 - \sigma_2{}^{\nu=2}$ on the filling factor ν. The plots lg F_i versus ν are shown in Fig.6. Both curves tend to straight lines, and in this way they can be extrapolated to ν=2.Using the extrapolation we have obtained $F_1{}^{\{\nu=2\}}=10^{-8}$ Ohm^{-1} and $F_2{}^{(\nu=2)}=5\cdot10^{-8}$ Ohm^{-1}. It should be noticed here that the extrapolated values of $F_i{}^{\nu=2}$ are two order of magnitude smaller than the quantities of $\sigma_i{}^{\nu=2}$, associated with the hopping along Si-δ-layer.

Using the extrapolated values of F_1 and F_2 to extract the 2DEG contributions to σ_1 and σ_2, one can calculate the electron localization length at $\nu=2$ from Eq.2.

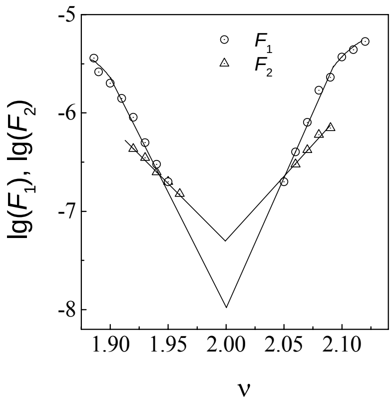

Fig. 6. **The dependences of $\lg F_1=\lg(\sigma_1-\sigma_1^{\nu=2})$ and $\lg F_1=lg(\sigma_1-\sigma_1^{\nu=2})$ versus the filling factor ν near ν=2. T=1.5K, f=30MHz**

This procedure is corroborated by the fact that the experimental ratio F_2/F_1= 5 is close to the theoretical value 4.2 coming from Eq.2. The localization length at ν=2 obtained in this way is $\xi=2\cdot10^{-6}$ cm, which is half of the spacer width. This estimate makes realistic the "two-site model" which we have extensively used. It should be emphasized, however, that from the above value of ξ the hopping length r_ω is estimated to be $1.4\cdot10^{-5}$ cm. Consequently, there is an interplay between hops to the nearest and more remote neighbors. A more rigorous theory for this situation should be worked out. Such a theory should also explain why the magnetic field dependences of σ_1 and σ_2 at the vicinity of ν=2 appear to be different - the σ_1(H)-dependence is more pronounced than the σ_2(H)-one. According to the two-site model, both are determined by the respective dependence of the localization length on the magnetic field and should be similar. Indeed, their ratio, from Eq.2, is almost field-independent. It follows from the experimental data that there exists an additional mechanism leading to the pronounced decrease of σ_2 as the Fermi level falls into the extended states region. A probable mechanism is thermal activation of electrons from the Fermi level to the upper Landau band, leading, firstly, to a decrease of the number of pairs responsible for the hopping conductivity, and, secondly to a screening of the electric field amplitude produced by the SAW. We hope to work out a proper quantitative theory in future.

Conclusions

By means of acoustic methods it has been shown that in the Si δ-doped GaAs/AlGaAs heterostructures with the carrier densities $n=(1.3\text{-}2.8)\cdot10^{11}$cm^{-2} and mobilities $\mu=(1\text{-}2)\cdot10^5$cm^2/Vs at T=1.5K in the IQHE-regime the 2D hopping conductivity takes place. Real and imaginary parts of this conductivity and their ratio as well as temperature and magnetic field dependencies have been obtained.

It has been shown that in magnetic fields at ν=2, 4 hf hopping conductivity in Si δ-layer shunts hopping conductivity in a 2D channel. The method of derivation of hf hopping conductivity in a 2D channel is suggested in this case. The localization length value for electrons in 2D channel is calculated.

The dependences ξ(H) in the vicinity of ν=2 and 4 are similar to those obtained in the Furlan's work [1] using conventional DC method. One should notice that this acoustical method allows one to obtain the localization length value in magnetic fields corresponding to the quantum Hall's plateau centers which is impossible in DC technique.

Acknowledgments

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Aukšto dažnio kintančio laidumo kvantiniu Hallo režimu akustiniai tyrimai

Straipsnyje akustiniais metodais parodyta, kad siliciu legiruotose GaAs/AlGaAs heterostruktūrose atsiranda 2D kintantis laidumas. Nustatyta, kad aukšto dažnio laidumą galima rasti atliekant paviršinių akustinių bangų greičio pokyčio matavimus.

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