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Mathematical modelling of steering mechanism link vibration process in a low power tractor

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Vibration of a steering mechanism link in a low power agricultural wheel-tractor and its possible mathematical modelling are analysed in the article. The developed mathematical model of the mechanical system vibration can be used to calculate longitudinal and transversal vibrations, frequency functions of vibration amplitude and to analyse the influence of construction elements on this process.

Introduction

Engine, transmission, chassis and implements of the aggregated agricultural machinery are the main sources of vibration in mobile agricultural machines [6, 7]. High vibration levels influence the person, who works on agricultural tractor, through driving control and steering mechanism [6]. When a person driving a tractor accumulates vibrations, feels discomfort, his working-capacity decreases. Information about vibroacoustic properties of tractor steering mechanism links is always helpful to decrease the vibration generated by driving control and steering mechanism elements. Some of the vibroacoustic properties can be used in practical solving of vibration related problems. Mathematical model of vibration phenomenon in the steering mechanism link is developed with the purpose to determine the vibroacoustic properties. The results of this mathematical model enables efficiently reduce vibrations in mobile agricultural machines.

The objective of this investigation was development the mathematical model of steering mechanism and its links, i. e., description of vibrations of longitudinal and transversal direction by differential equations.

Theoretical analysis

For calculation of the displacement amplitude $\vec{u}(x,y,z,t)=u\vec{i}+v\vec{j}+w\vec{k}$ of one of the steering mechanism links – steering column the dynamic Lame's equations are used [1, 2, 3]:

$$\begin{cases}(\lambda+G)\dfrac{\partial e}{\partial x}+G\nabla^2 u+X=\rho\dfrac{\partial^2 u}{\partial t^2},\\(\lambda+G)\dfrac{\partial e}{\partial y}+G\nabla^2 v+Y=\rho\dfrac{\partial^2 v}{\partial t^2},\\(\lambda+G)\dfrac{\partial e}{\partial z}+G\nabla^2 w+Z=\rho\dfrac{\partial^2 w}{\partial t^2},\end{cases}$$

where $\nabla^2=\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial y^2}+\dfrac{\partial^2}{\partial z^2}$ is the Laplas operator,

$e=\varepsilon_x+\varepsilon_y+\varepsilon_z=\dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}+\dfrac{\partial w}{\partial z}$ is the volume deformation;

$\lambda=\dfrac{2G\mu}{1-2\mu}$ is the Lame's coefficient; ρ is the body density; $G=\dfrac{E}{2(1+\mu)}$.

The system of the Lame's partial differential equations is solved by approximate methods, using corresponding initial and boundary conditions. This is a complicated way, which is difficult to apply for practical processes[7].

As a mechanical system steering column is a thin - walled cylinder (pipe) of R radius and thickness h which is fixed like a holder beam [6]. Then symmetric dynamic problem of the elastic theory can be solved with respect to the steering device axis 0z. Then movements $u(r,z)\approx u(R,z)$ (where R – radius of the cylinder transversal section wall inside the circle). The main stresses in the deformation process are σ_z and σ_θ, while σ_r and τ_{rz} are small, i. e. $\sigma_r\approx 0$ and $\tau_{rz}\approx 0$. The Hooke's law in a cylindric coordinate system can be expressed in the following way:

$$\begin{cases}\varepsilon_z=\dfrac{1}{E}(\sigma_z-\mu\sigma_\Theta),\\\varepsilon_\Theta=\dfrac{1}{E}(\sigma_\Theta-\mu\sigma_z),\quad \varepsilon_r\approx 0;\gamma_{rz}\approx 0,\end{cases}$$

or

$$\begin{cases}\sigma_z=\dfrac{E}{1-\mu^2}(\varepsilon_z+\mu\varepsilon_\Theta),\\\sigma_\Theta=\dfrac{E}{1-\mu^2}(\varepsilon_\Theta+\mu\varepsilon_z).\end{cases}\qquad(1)$$

General longitudinal movement $w=w_0+w_1$, $w_1=A_0A\sin\alpha$, $\tan\alpha=\dfrac{\partial u}{\partial z}$. As α is small, so $\tan\alpha\approx\alpha=\dfrac{\partial u}{\partial z}$ and $\sin\alpha\approx\dfrac{\partial u}{\partial z}$, $w_1=\xi\dfrac{\partial u}{\partial z}$.

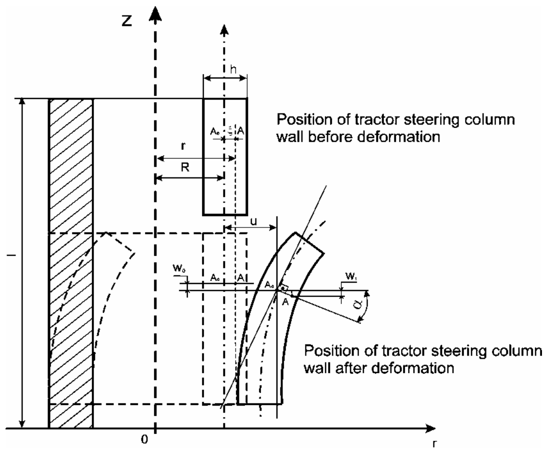


Fig. 1. **Scheme of cylinder walls deformation**

When deformations are small, $w_0 \approx 0$ and $w = \xi \frac{\partial u}{\partial z} = (r - R)\frac{\partial u}{\partial z}$, while

$$\begin{cases} \varepsilon_z = \dfrac{\partial w}{\partial z} = (r - R)\dfrac{\partial^2 u}{\partial z^2}, \\ \varepsilon_\Theta = \dfrac{u}{r} \approx \dfrac{u}{R}. \end{cases} \quad (2)$$

Stresses σ_z and σ_Θ obtained subtistating Eq.2 into Eq.1 obtained:

$$\begin{cases} \sigma_z = \dfrac{E}{1-\mu^2}\left((r-R)\dfrac{\partial^2 u}{\partial z^2} + \dfrac{\mu u}{R}\right), \\ \sigma_\Theta = \dfrac{E}{1-\mu^2}\left(\dfrac{u}{R} + \mu(r-R)\dfrac{\partial^2 u}{\partial z^2}\right). \end{cases} \quad (3)$$

The dynamic equilibrium equations are written in the cylindrical system of axes (Fig.2).

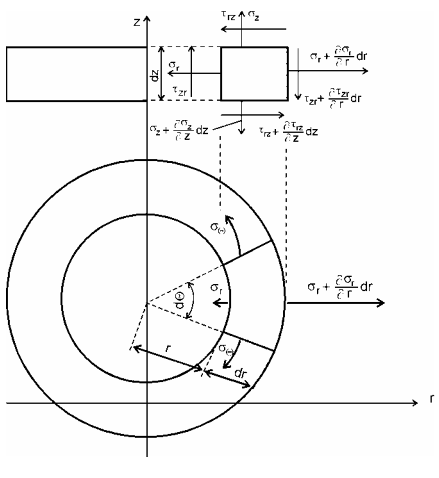


Fig. 2. **Deformation of steering column element dV**

All forces, influencing the element *dV=rdΘdrdz*, are projected to the axes 0*z* and 0*r*. The following equations are obtained:

$$\begin{cases} -\dfrac{\partial \sigma_z}{\partial z} dzrdrd\Theta + \tau_{zr} rd\Theta dz - \left(\tau_{zr} + \dfrac{\partial \tau_{zr}}{\partial r} dr\right) \cdot \\ \cdot (r+dr)d\Theta dz = -\rho rd\Theta drdz \dfrac{\partial^2 w}{\partial t^2}, \\ -\sigma_r rd\Theta dz + (\sigma_r + \dfrac{\partial \sigma_r}{\partial r} dr)(r+dr)d\Theta dz + \\ + \dfrac{\partial \tau_{rz}}{\partial r} drrd\Theta dz - \sigma_\Theta drdzd\Theta = \rho rd\Theta drdz \dfrac{\partial^2 u}{\partial t^2}, \end{cases}$$

or

$$\begin{cases} r\dfrac{\partial \sigma_z}{\partial z} + r\dfrac{\partial \tau_{zr}}{\partial r} + \tau_{zr} = \rho r \dfrac{\partial^2 w}{\partial t^2}, \\ r\dfrac{\partial \sigma_r}{\partial r} + \sigma_r + r\dfrac{\partial \tau_{rz}}{\partial z} - \sigma_\Theta = \rho r \dfrac{\partial^2 u}{\partial t^2}. \end{cases} \quad (4)$$

The following dynamic differential equations of steering column element equilibrium are obtained:

$$\begin{cases} r\dfrac{\partial \sigma_Z}{\partial z} + \dfrac{\partial (r\tau_{rz})}{\partial r} = \rho r \dfrac{\partial^2 w}{\partial t^2}, \\ \dfrac{\partial (r\sigma_r)}{\partial r} - \sigma_\Theta + r\dfrac{\partial \tau_{rz}}{\partial z} = \rho r \dfrac{\partial^2 u}{\partial t^2}. \end{cases} \quad (5)$$

For the thin-walled steering column ($\sigma_r \approx 0, \quad \tau_{rz} \approx 0$):

$$\begin{cases} \dfrac{\partial \sigma_Z}{\partial z} = \rho \dfrac{\partial^2 w}{\partial t^2}, \\ -\dfrac{\sigma_\Theta}{r} = \rho \dfrac{\partial^2 u}{\partial t^2}. \end{cases} \quad (6)$$

Having put Eq.3 of stresses into the equations of dynamic equilibrium (6), the system of two differential equations of partial derivatives is obtained to for calculation of the movements *u* and *w*:

$$\begin{cases} (r-R)\dfrac{\partial^3 u}{\partial z^3} + \dfrac{\mu}{R}\dfrac{\partial u}{\partial z} = \rho \dfrac{(1-\mu^2)}{E}\dfrac{\partial^2 w}{\partial t^2}, \\ \dfrac{1}{R}\dfrac{u}{r} + \mu\left(1 - \dfrac{R}{r}\right)\dfrac{\partial^2 u}{\partial z^2} = -\rho \dfrac{(1-\mu^2)}{E}\dfrac{\partial^2 u}{\partial t^2}, \end{cases} \quad (7)$$

here $w = (r-R)\frac{\partial u}{\partial z}$.

The first equation of this system is integrated with respect to *z* in the interval [0;z]. The following equality is obtained:

$$(r-R)\frac{\partial^2 u}{\partial z^2} + \mu \frac{u}{R} = \rho \frac{(1-\mu^2)}{E}(r-R)\frac{\partial^2 u}{\partial t^2}. \quad (8)$$

The following expression may be obtained from the second equation of Eq.7:

$$\frac{u}{R} = -\mu(r-R)\frac{\partial^2 u}{\partial z^2} - r\rho \frac{(1-\mu^2)}{E}\frac{\partial^2 u}{\partial t^2}. \quad (9)$$

Then the Eq.9 substituted into Eq.8:

$$(r-R)\frac{\partial^2 u}{\partial z^2}-\mu^2(r-R)\frac{\partial^2 u}{\partial z^2}-r\rho\mu\frac{(1-\mu^2)}{E}\frac{\partial^2 u}{\partial t^2}=$$
$$\rho\frac{(1-\mu^2)}{E}(r-R)\frac{\partial^2 u}{\partial t^2},$$

or

$$(1-\mu^2)\frac{\partial^2 u}{\partial z^2}=\rho\frac{(1-\mu^2)}{E}\left(1+\mu\frac{r}{r-R}\right)\frac{\partial^2 u}{\partial t^2},$$

i. e.,

$$\frac{\partial^2 u}{\partial t^2}=A_1^2\frac{\partial^2 u}{\partial z^2},$$

where

$$A_1^2=\frac{E}{\rho\left(1+\mu\frac{r}{r-R}\right)},\qquad(10)$$

h is the thickness of steering column wall, $R-\frac{h}{2}\le r\le R+\frac{h}{2}$, $0\le z\le l$.

Longitudinal vibrations are calculated using the formula $w=(r-R)\frac{\partial u}{\partial z}$. Eq.10 is solved if the beginning of steering column is fixed rigidly and the end – freely. Then the following boundary and initial conditions are given by:

$$u\big|_{z=0}=0,\quad \frac{\partial u}{\partial z}\bigg|_{z=0}=0,\quad \frac{\partial^2 u}{\partial z^2}\bigg|_{z=l}=0,\qquad(11)$$

$$u\big|_{t=0}=u_0(z),\quad \frac{\partial u}{\partial t}\bigg|_{t=0}=u_1(z).\qquad(10)$$

According to [2], the curved pivot or pipe, which fulfils the boundary conditions at the initial time instant t=0, according to resilience theory has the following equation for transversal bend:

$$u_0(z)=(1-\cos kz)\cdot f\qquad(12)$$

The solution of Eq.10 is given by:

$$u(z,t)=u_0(z)T(t),\ T(0)=1\ \text{and}\ T'(0)=T_1.\qquad(13)$$

As $u_0''(z)\big|_{z=l}=0$, then $\cos kl=0$, $k=\frac{\pi(2n+1)}{2l}$, and $u_0(z)=f\cdot\left(1-\cos\frac{\pi(2n+1)}{2l}z\right),\quad(n\in N)$.

Substituting Eq.13 into Eq.10 we shall obtain:

$$\left(1-\cos\frac{\pi(2n+1)}{2l}z\right)T''(t)=\frac{A_1^2\pi^2(2n+1)^2}{4l^2}\cdot$$
$$\cdot\cos\frac{\pi(2n+1)}{2l}zT(t).\qquad(14)$$

The approximate Bubnov – Galiorkin method is used to find the function $T(t)$. For this purpose terms of Eq.14 are multiplied by $u_0(z)$ and the obtained equality is integrated in the region $z\in[0;l]$, that is

$$T''\int_0^l\left(1-\cos\frac{\pi(2n+1)}{2l}z\right)^2dz=$$
$$=\frac{A_1^2\pi^2(2n+1)^2}{4l^2}T\int_0^l\cos\frac{\pi(2n+1)}{2l}z\left(1-\cos\frac{\pi(2n+1)}{2l}z\right)dz\qquad(15)$$

Having integrated these integrals the following differential equation is obtained for calculation of the function $T(t)$:

$$T''+B_n^2T=0,$$

where

$$B_n=\frac{\pi A_1(2n+1)}{2l}\sqrt{\frac{\pi(2n+1)-4(-1)^n}{3\pi(2n+1)-8(-1)^n}}.\qquad(16)$$

The general solution of Eq.16 given by:

$$T(t)=C_1\cos B_nt+C_2\sin B_nt.$$

As $T(0)=1$, then $C_1=1$. Having used other initial condition $T'(0)=T_1$, the following is obtained:

$$T'(t)=-C_1B_n\sin B_nt+C_2B_n\cos B_nt,$$

$$C_2B_n=T_1\ \text{and}\ C_2=\frac{T_1}{B_n}.$$

Then $T(t)=\cos B_nt+\frac{T_1}{B_n}\sin B_nt$.

Then, the solution of Eq.10 is written in the following way:

$$u_n(z,t)=u_0(z)T(t)=$$
$$f\cdot\left(\left(1-\cos\frac{\pi(2n+1)}{2l}z\right)\left(T_0\cos B_nt+\frac{T_1}{B_n}\sin B_nt\right)\right)\qquad(17)$$

where f is the maximum bend of the column at the free end (f=((0,01-0,05)*l)).

The transversal bends are added up to get the general solution for transversal vibrations:

$$u(z,t)=\sum_1^\infty u_n(z,t)=$$
$$=f\cdot\sum_1^\infty\left(1-\cos\frac{\pi(2n+1)}{2l}z\right)(C_n\cos B_nt+D_n\sin B_nt)\qquad(18)$$

$$u\big|_{t=0}=u_0(z)=$$
$$=f\cdot\sum_1^\infty\left(1-\cos\frac{\pi(2n+1)}{2l}z\right)\cdot C_n\Rightarrow C_n=1\qquad(19)$$

$$u(z,t)=$$
$$=f\cdot\sum_1^\infty\left(1-\cos\frac{\pi(2n+1)}{2l}z\right)(\cos B_nt+D_n\sin B_nt)\qquad(20)$$

$$\frac{\partial u}{\partial t}=f\cdot\sum_1^\infty\left(1-\cos\frac{\pi(2n+1)}{2l}z\right)\cdot$$
$$\cdot(-B_n\sin B_nt+D_nB_n\cos B_nt)\qquad(21)$$

$$\left.\frac{\partial u}{\partial t}\right|_{t=0} = u_1(z) =$$
$$= f \cdot \sum_{n=1}^{\infty} T_n \left(1-\cos\frac{\pi(2n+1)}{2l} z\right) = \tag{22}$$
$$= f \cdot \sum_{n=1}^{\infty} \left(1-\cos\frac{\pi(2n+1)}{2l} z\right) D_n B_n ,$$

$$T_n = D_n B_n \Rightarrow D_n = \frac{T_n}{B_n},$$
$$u(z,t) =$$
$$= f \cdot \sum_{n=1}^{\infty} \left(1-\cos\frac{\pi(2n+1)}{2l} z\right)\left(\cos B_n t + \frac{T_n}{B_n}\sin B_n t\right). \tag{23}$$

The longitudinal vibrations of the steering column are calculated from the formula:

$$w(z,t) = (r-R)\frac{\partial u}{\partial z} =$$
$$= f \cdot \frac{\pi(r-R)}{2l} \sum_{n=1}^{\infty} (2n+1) \sin\frac{\pi(2n+1)}{2l} z \cdot . \tag{24}$$
$$\cdot \left(\cos B_n t + \frac{T_n}{B_n}\sin B_n t\right).$$

However, in real construction steering wheel the mass M is fixed by the pivot of l_2 length to the cylinder body of the mechanical system, described above, look Fig. 3.

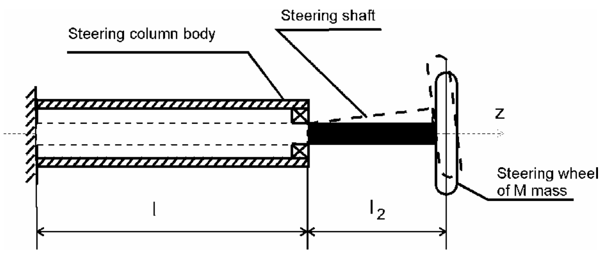


Fig. 3.**Mechanical scheme corresponding with the scheme of steering links – column, shaft and steering wheel connector**

According to Eq.3 of steering shaft transversal vibrations the boundary conditions are the following:

$$u_1\big|_{z=l} = f \; ; \qquad \left.\frac{\partial u_1}{\partial z}\right|_{z=l} = \left.\frac{\partial u}{\partial z}\right|_{z=l},$$
$$\frac{\partial^2 u_1}{\partial t^2} = -a_1^2 \frac{\partial^4 u_1}{\partial z^4}, \; l \le z \le l + l_2 , \tag{25}$$

where $a_1 = \sqrt{\dfrac{E \cdot I}{m_r}}$, m_0 is the mass of the shaft length unit; m_r is the reduced mass:

$$m_r = \frac{4}{3} m_0 + M \cdot \left(1 + \frac{1}{2}\left(\frac{r_0}{l_2}\right)^2\right);$$

M is the steering wheel mass; r_0 is the radius of the mass.

Solution of Eq.12 can be found as:

$$u_1(z,t) = f \cdot \left(1 - \sin\frac{\pi(2n+1)}{2l_2}(z-l) \cdot T(t)\right), \tag{26}$$

As $u_1(l,0) = f \cdot T(0) = f$, then, $T(0) = 1$.

Eq.26 is substituted into Eq.25. As $\dfrac{\partial^2 u}{\partial t^2} = f \cdot \left(1 - \sin\dfrac{\pi(2n+1)}{2l_2}(z-l)\right) \cdot T''$ and

$$\frac{\partial^4 u_1}{\partial z^4} = -f \frac{\pi^4 (2n+1)^4}{16 l_2^4} \sin\frac{\pi(2n+1)}{2l_2}(z-l) \cdot T(t).$$

then

$$f \cdot \left(1 - \sin\frac{\pi(2n+1)}{2l_2}(z-l)\right) \cdot T'' =$$
$$= f a_1^2 \frac{\pi^4 (2n+1)^4}{16 l_2^4} \sin\frac{\pi(2n+1)}{2l_2}(z-l) \cdot T(t). \tag{27}$$

To find the function *T(t)* the Bubnov – Galiorkin method is used within the interval [l; l+l$_2$] for the Eq.27. For this purpose the Eq.27 is multiplied by Eq.26 *$u_1(z,t)$* and the obtained is integrated. Then

$$T'' \int_l^{l+l_2} \left(1 - \sin\frac{\pi(2n+1)}{2l_2}(z-l)\right)^2 dz = a_1^2 \frac{\pi^4 (2n+1)^4}{16 l_2^4} \cdot$$
$$\cdot T \int_l^{l+l_2} \sin\frac{\pi(2n+1)}{2l_2}(z-l)\left(1 - \sin\frac{\pi(2n+1)}{2l_2}(z-l)\right) dz. \tag{28}$$

Having integrated the Eq.28 to calculate the function *T(t)* the following differential equation for calculation of the function *T(t)* is obtained:

$$T'' + b_n^2 T = 0,$$

where

$$b_n = \frac{\pi^2 a_1 (2n+1)^2}{4 l_2^2} \sqrt{\frac{\pi(2n+1) - 4}{3\pi(2n+1) - 8}}. \tag{29}$$

The solution of the Eq.29 is $T(t) = \cos b_n t + d_n \sin b_n t$, as $T(0) = 1$. Then the following transversal bends of steering shaft is given by:

$$u_1(z,t) = f \cdot \left(1 - \sin\frac{\pi(2n+1)}{2l_2}(z-l)\right) \cdot$$
$$\cdot \left(\cos b_n t + d_n \sin b_n t\right). \tag{30}$$

These transversal movements of the steering shaft, described by Eq.30 at the initial time instant *t*=0 match the transversal bend of the steering column *u(z, t)* at the point *z*=1, where *u(z, t)* is given by the Eq.17:

$$u(z,t) = f \cdot \left(1 - \cos\frac{\pi(2n+1)}{2l} z\right)\left(\cos B_n t + D_n \sin B_n t\right).$$

Then, transversal vibrations of the steering column and the steering shaft *u(z, t)* are written in the following way:

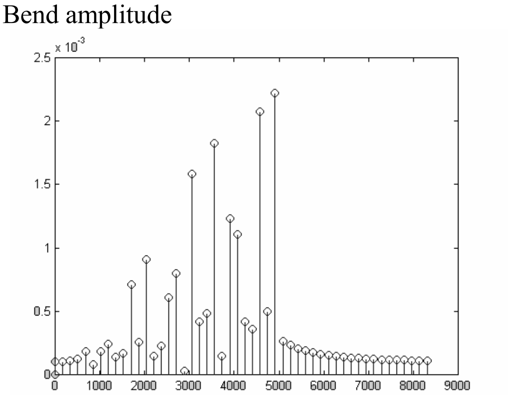
$$u(z,t)=\begin{cases} f\cdot\sum_{n=1}^{\infty}\left(1-\cos\frac{\pi(2n+1)}{2l}z\right)\cdot \\ \cdot\left(\cos B_n t + D_n\sin B_n t\right), when\ z\in[0;l] \\ f\cdot\sum_{n=1}^{\infty}\left(1-\sin\frac{\pi(2n+1)}{2l_2}(z-l)\right)\cdot \\ \cdot\left(\cos b_n t + d_n\sin b_n t\right), when\ z\in[l,l+l_2] \end{cases} \quad (31)$$

where B_n are the steering column vibration frequencies, b_n are the steering shaft vibration frequencies.

The solution, made by partial derivatives of differential equations, are modelled by means of computers. The Fourier transform of the Eq.30 is performed by the specialised programming packages "MathCAD 2000" and "MATLAB 5.2". These programs have been used to calculate frequency response of the steering system links amplitude of the low power tractor of T-25A modification. The results reflect the real vibroacustic characteristics of the tractor T-25A steering column, which have been obtained by the mechanical impedance method [4, 5, 6].

Results of modelling

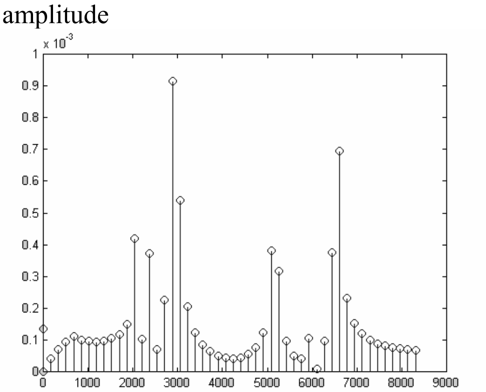
Bend amplitude



Hz

Fig. 4. **Modelling results. Column body length *l*=0,8 m, *E*=2*10[11]N/m[2], *ρ*=7800 kg/m[3] , *μ*=0,29, the wheel mass *m*=2,5 kg.**

Bend amplitude



Hz

Fig. 5**. Modelling results. Length of the protruded part of the shaft l_2=0,1 m, *E*=2*10[11] N/m[2], *ρ*=7800 kg/m[3], *μ*=0,29, the wheel mass *m*=2,5 kg**.

Conclusions

1. Differential equations of tractor steering mechanism links vibration have been obtained and the ways of their solution have been presented.
2. Equations of the movement of the steering mechanism elements and the results obtained are useful in the investigation of vibration processes and determining the influence of construction elements.
3. Frequency response of tractor steering mechanism links body and shaft have been presented.

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Mažos galios traktoriaus vairavimo mechanizmo grandies virpėjimo proceso matematinis modeliavimas

Reziumė

Straipsnyje nagrinėjami ratinių ž. ū. traktorių vairo mechanizmo grandžių virpesiai, jų matematinio modeliavimo galimybės.

Sudarytas matematinis modelis gali būti naudojamas praktiniams tikslams, modeliuojant vairo mechanizmo grandžių amplitudės dažninės charakteristikos priklausomybę nuo konstrukcinių parametrų.

Pateikta spaudai 2000 09 11