Modelling of ultrasound velocity measurement errors in a chamber with a buffer waveguide

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Introduction

Ultrasound velocity measurements in medicine and biology usually are performed using relatively small chambers. When the pulse echo method is used [1, 2], the presence of the measurement chamber close to an ultrasonic transducer can cause essential diffraction errors. This problem can be solved using an additional buffer rod as waveguide between the transducer and the measurement chamber and in such a way to shift measurement zone in a far acoustic field. But in some cases, such as the ultrasound velocity measurements during blood clotting process, when very high accuracy is needed, the influence of diffraction errors can be still essential. In this case due to a very high resolution required, diffraction errors become of a major importance limiting a potential accuracy of the measurement method.

Influence of diffraction phenomena on accuracy of ultrasonic measurements has been investigated by a number of researchers [3-8]. Most of the publications are devoted to analysis of a radiation coupling of disk shape transducers. However, most of the authors analysed the case of a direct coupling of two disk shape transducers, e.g., when there no waveguides between the transducers and fluid in which the measurements are performed. The objective of the presented work was analysis of possible ultrasound velocity errors caused by diffraction in measurement chambers with buffer rods.
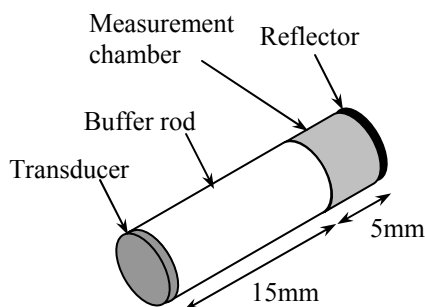
Task description

Fig. 1. **Geometry of the analysed measurement chamber.**

The geometry of selected for the analysis measurement chamber is presented in Fig.1 The diffraction errors are caused by deviation of the actual front of the ultrasonic wave from a plane wave. Evaluation of the diffraction errors in our case is complicated by the fact that measurements are carried out exploiting reflected ultrasonic waves, which are radiated and received through the waveguide.

Calculation of the acoustic field

An acoustic pressure at an arbitrary point *x,y* on the reflector surface is found as convolution of the driving pulse $u(t)$ and the spatial impulse response $h(x,z,t)$:

$$p_a(x,z,t) = k_c \int_0^\infty u(t) \cdot h(x,z,t-\tau) dt, \qquad (1)$$

where $h(t)$ is the impulse response of a circular transducer and the waveguide. The impulse response $h(t)$ is found by means of the mixed analytic-numeric procedure presented in [9,10]. This approach enables to simulate ultrasonic field in two media separated by planar interface. The input parameters for this model are the ultrasound velocities c_1, c_2 and the densities ρ_1, ρ_2 corresponding to the first and second medium, the distance to the interface and the transducer diameter.

Due to the reciprocity principle the spatial responses in the transmitting and receiving modes are the same except the constant factor. Therefore, the signal acting on the surface of the transducer in the receiving mode and caused by a point type reflector located at *x,y* is given by

$$p(x,z,t) = \int_0^\infty p_a(x,z,t) \cdot h(x,z,t-\tau) dt. \qquad (2)$$

The driving signal was approximated by

$$u(t) = e^{a(t-b)^2} \sin(2\pi f t), \qquad (3)$$

where $a = k_a f \sqrt{\dfrac{-2\ln 0.1}{p_s}}$; $b = \dfrac{2p_s}{3f}$, p_s is the number of periods, k_a is the asymmetry factor, f is the frequency. Such a signal has a shape of a radio pulse with the Gaussian envelope (Fig.2). Steepness of the front and back slopes of the pulse can be set separately selecting corresponding value of k_a.

The $p(x,z,t)$ can be called the transducer response in a pulse-echo mode for a selected excitation signal. The calculated field is presented in Fig.3. The length of the buffer rod was 15mm, the ultrasound velocity in it $c_r = 2000 \text{m/s}$, the density $\rho_r = 1.27 \cdot 10^3 \text{kg/m}^3$. The blood was selected as the liquid under investigation with the ultrasound velocity $c_b = 1580 \text{m/s}$ and the density

$\rho_b = 1.0 \cdot 10^3 \, \text{kg} / \text{m}^3$. Due to difference in acoustic impedances ($Z_r = 2.54 \cdot 10^6 \, \dfrac{\text{kg}}{\text{m}^2 \text{s}}$, $Z_b = 1.58 \cdot 10^6 \, \dfrac{\text{kg}}{\text{m}^2 \text{s}}$) the transmission coefficient through the boundary between
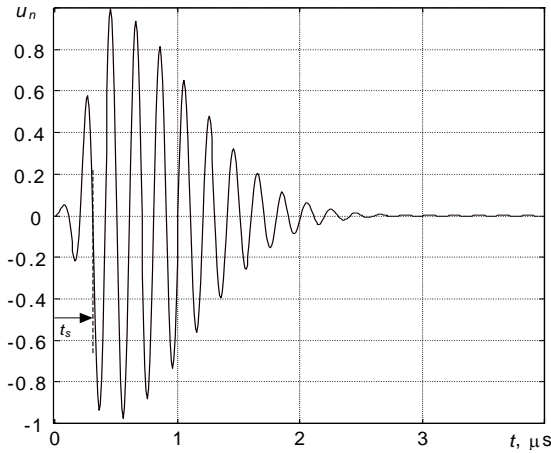

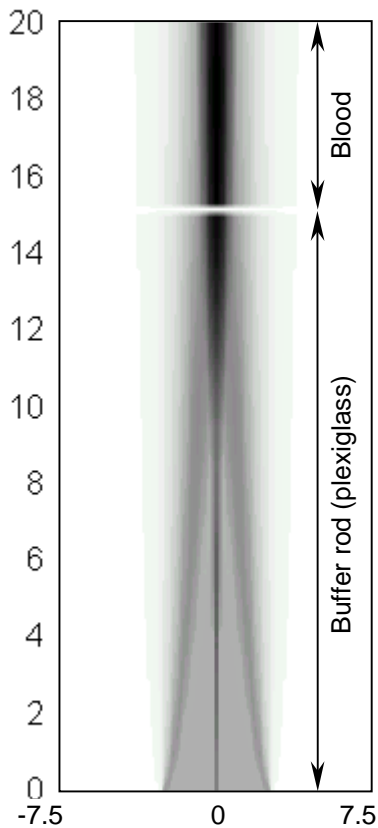Fig. 2. **Ultrasonic pulse radiated by the circular ultrasonic transducer.**

Fig. 3. **Transducer field in transmission – reflection mode in the case of two different media separated by a planar interface**
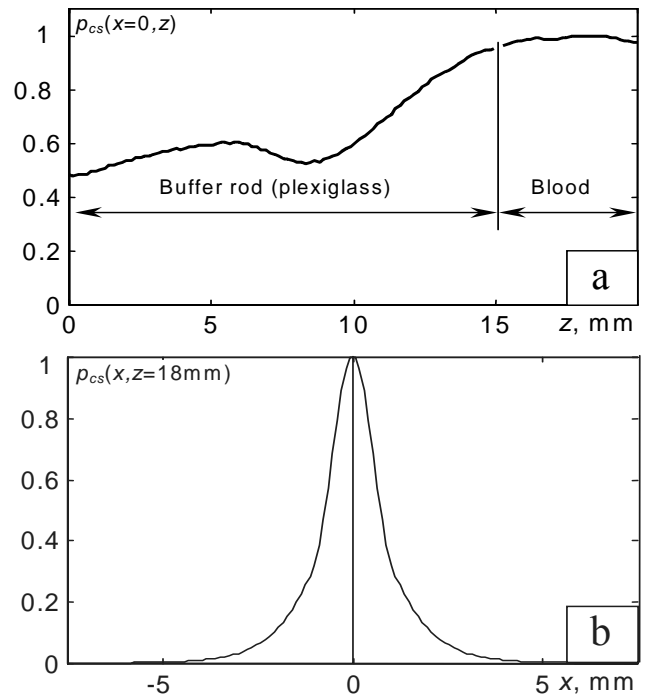

Fig. 4. **Crossection of ultrasonic field along acoustic axis (a) and across the beam (b)**

two media will be $K_T = \dfrac{4 Z_r Z_b}{(Z_r + Z_b)^2} = 0.9457$ and the absolute values of the acoustic pressure in adjacent media will be different. The field is presented as $p_{cs}(x,z) = \max_t |p(x,z,t)|$. For better understanding the field presented is normalised with respect to the maximum value of the pressure in the second medium. Therefore, there is no step in the field on the boundary between two media.
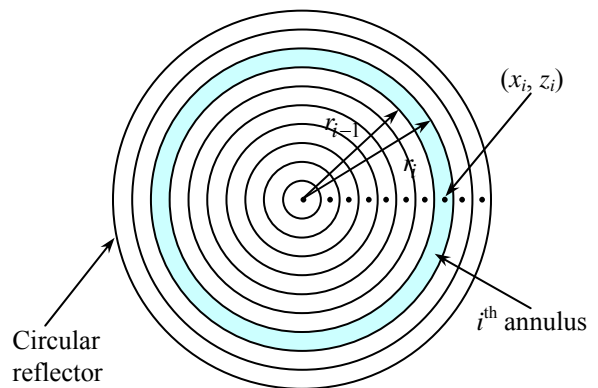
Calculation of reflected signals

Fig. 5. **Division of the circular reflector into annuli**

In the case of a circular planar reflector the pressure on the surface of the transducer is found as the weighted sum of the acoustic pulses reflected by annuli of different diameters (Fig.5):

$$p_s(t) = \pi \sum_{i=1}^{M} \left(r_i^2 - r_{i-1}^2 \right) p_i(t, x_i, z_i). \qquad (4)$$

where r_{i-1} and r_i are the inner and outer radii of the i-th annulus

$$r_i = \left| x_i + \frac{\Delta x}{2} \right| - x_{k0}$$
$$r_{i-1} = \left| x_i - \frac{\Delta x}{2} \right| - x_{k0}$$
(5)

x_i is the coordinate of the centre line of the annulus and Δx is the width of the annulus, $p_i(t)$ is the signal caused by a point type reflector located at the centre line of the annulus, $i = 1..M$, M is the number of annulus on the reflector surface.
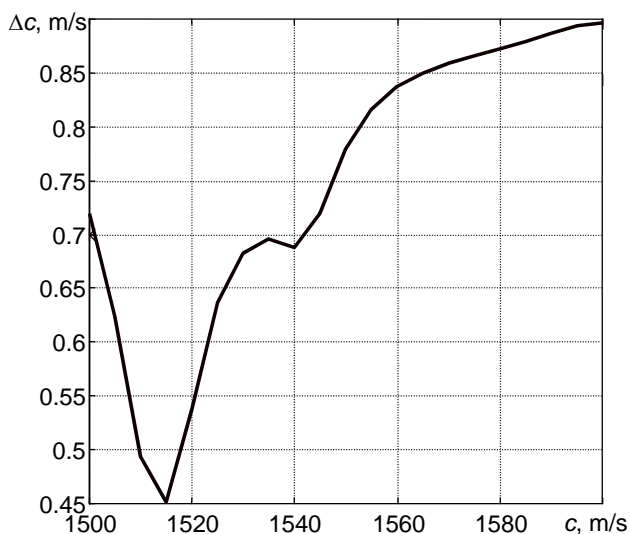
Estimation of diffraction errors

Fig. 6. **Diffraction error as a function of the ultrasound velocity**

The measured ultrasound velocity was obtained from the delay times t_{s1}, t_{s2} of the ultrasonic signals reflected from the end of the waveguide and the reflector, respectively. These times were estimated as the instant, when the signal at the selected period crosses the zero level. The estimated ultrasound velocity was found using the differential measurement algorithm:

$$\hat{c}_b = \frac{2(z_r - z_w)}{t_{s2} - t_{s1}}.$$
(6)

Here z_w and z_r are the distances from the transducer till the end of the waveguide and the reflector, respectively.

The diffraction error Δc_d was found as a difference between the ultrasound velocity value used in calculations c_b and the estimated value $\hat{c}_b$:

$$\Delta c_d = c_b - \hat{c}_b.$$
(7)

The results of simulation are shown in Fig.6. The results presented indicate that for the given geometry the diffraction errors in the range of ultrasound velocities (1500-1600m/s), characteristic for a clotting blood, are about 0.4m/s (Fig.6).

Conclusions

The evaluation of diffraction errors showed that for some ultrasound velocity range their absolute values can reach the level up to 0.4m/s and can essentially influence the results of measurements.

From the other hand, the optimal selection of lengths of the rod and the measurement chamber can essentially reduce diffraction errors.

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Ultragarso greičio matavimo kameroje su bangolaidžiu difrakcinių paklaidų modeliavimas

Reziumė

Išnagrinėtos ultragarso greičio matavimo impulsiniu atspindžio metodu paklaidos, atsirandančios matavimo kameroje su bangolaidžiu. Kompiuterinis modeliavimas leido nustatyti, kad, taikant diferencinį matavimo metodą, šios paklaidos skysčiuose neviršija 1%.

Paklaidoms įvertinti sudarytas mišrus analitinis skaitmeninis kompiuterinis modelis, leidžiantis apskaičiuoti išspinduliuotų bei atspindėjusių ultragarsinių laukų struktūrą sistemoje su buferiniu bangolaidžiu tiek artimoje, tiek tolimoje zonoje.