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The effect of transverse vibrations of boundary to the flow of fluid type substances

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Introduction

The problem of vibro-transportation of fluids is important in the process of design of various engineering devices and optimization of processes of transportation [1, 2, 4]. The effect of longitudinal vibrations of boundary to the flow of non-Newtonian fluid was analyzed in [1]. In practice the generation of one-directional vibrations is quite complicated therefore it is important to define the dynamical effects of transverse vibrations to the flow of substances.

The results obtained in [1] provide the basis for the design of vibrotransportation devices, but in many practical applications vibrotransportation is performed not by longitudinal, but by transverse vibrations. It is understood that for the full analysis of such problems three dimensional models are required. But the analysis of such models and the interpretations of the results would be quite complicated. In this paper a two dimensional model with some similarities to the one presented in [1] is developed. The main feature of the model is that the vibrational excitation is performed through the boundary velocities, which are given in the convective acceleration terms of the equation of dynamic equilibrium. The prescribed velocities are assumed constant in the cross section (they are functions of time only). This model enables the investigation of the influence of vibrations through the convective inertia terms to the flow of the fluid.

Mathematical model of the system

It is assumed that the velocity of a laminar fluid flow in the direction of the flow is the function of the coordinates of the cross section and time, while the components of velocity in the plane of the cross section are given functions of time only, that is,

$$u = u(t),\ v = v(t),\ w = w(x, y, t). \qquad (1)$$

It is assumed that the cross section of the tube does not vary with the *z* coordinate. Here *u*, *v*, *w* denote the velocity components in the direction of the Cartesian orthogonal axes of coordinates, *t* – time (Fig. 1). The condition of the incompressibility of the fluid is identically satisfied. The stresses are:

$$\begin{aligned} &\sigma_x = \sigma_y = \sigma_z = -p, \\ &\sigma_{xy} = 0, \\ &\sigma_{yz} = \mu w_y, \\ &\sigma_{zx} = \mu w_x, \end{aligned} \qquad (2)$$

where *p* denotes the pressure, μ is the viscosity of the fluid, the subscripts denote partial derivatives.

The dynamic equilibrium equation in the direction of the *z* axis taking into account the full derivative of *w* takes the form

$$(\mu w_x)_x + (\mu w_y)_y - p_z + \rho g = \rho w_t + \rho u w_x + \rho v w_y, \qquad (3)$$

where ρ is the density of the fluid, *g* is the acceleration of gravity, p_z is the gradient of the pressure in the direction of the *z* axis.

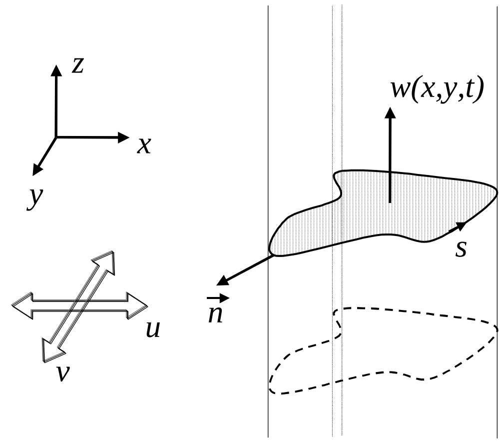


Fig. 1. **The principal model of the system**

The boundary condition takes the form:

$$-\mu w_n = \alpha w, \qquad (4)$$

where n is the outward normal vector to the boundary of the cross section of the flow, α is the coefficient of slippage (sliding friction between the fluid and the surface of the tube) [1, 2].

It is assumed that the boundary performs harmonic one-directional oscillations in the plane of the cross section. The appropriate components of the vibration vector are expressed like:

$$\begin{aligned} &u = a\sin(\omega t), \\ &v = b\sin(\omega t), \end{aligned} \qquad (5)$$

where *a*, *b* and ω denote the amplitudes and frequency of oscillations. This assumption of independence of the velocity components *u* and *v* on the coordinates *x* and *y* cannot be considered acceptable at very high frequencies, but it is essential in providing the simplified model and the method of analysis described further.

The cross section of the flow is meshed using the finite element approximation. The resulting matrixes take the form:

$$[C]=\iint [N]^T \rho[N]dxdy,$$
$$[K]=\iint [B]^T \mu[B]dxdy+\oint [M]^T \alpha[M]dx,$$
$$\{F\}=\iint [N]^T (\rho g-p_z)dxdy-$$
$$-\iint [N]^T \rho[u;v][B]\{\delta\}dxdy, \qquad (6)$$

where $\{\delta\}$is the vector of nodal velocities, s is the boundary line of the cross section of the flow; the matrixes [*N*], [*B*] and [*M*] are the same as defined in [1].

Numerical determination of mass flow rates

The averaged transverse velocities and the mass flow rates are determined by solving the four sequential problems. The numerical procedure is to some extent similar to the one described in [1], though the different mathematical formulations require alternation of the strategy of calculations.

Static solution. It is assumed that the term $-p_z+\rho g$ is defined as constant in time and in-plane velocities are negligible (u = v = 0, $\{\delta\}$ = 0). The static load is formed as:

$$\{F_0\}=\iint [N]^T (\rho g-p_z)dxdy \qquad (7)$$

leading to the system of linear algebraic equations:

$$[K]\{\delta_0\}=\{F_0\}. \qquad (8)$$

Steady harmonic solution. The sinusoidal variation of the in-plane velocities is assumed at $-p_z+\rho g=0$ taking into account $\{\delta_0\}$ obtained in the previous step what leads to the following system of linear algebraic equations:

$$\begin{bmatrix}[K] & -\omega[C]\\ \omega[C] & [K]\end{bmatrix}\cdot\begin{Bmatrix}\{\delta_s\}\\ \{\delta_c\}\end{Bmatrix}=\begin{Bmatrix}\{F_s\}\\ 0\end{Bmatrix}, \qquad (9)$$

where

$$\{F\}=\{F_s\}\sin\omega t,$$
$$\{F_s\}=-\iint [N]^T \rho[a;b][B]\{\delta_0\}dxdy, \qquad (10)$$

$\{\delta_s\}$ and $\{\delta_c\}$ are the vectors of the sine and cosine components of the steady state motion. The total approximate velocities up to this point are:

$$\{\delta(t)\}=\{\delta_0\}+\{\delta_s\}\sin(\omega t)+\{\delta_c\}\cos(\omega t). \qquad (11)$$

Averaged transverse velocities. The final load vector is obtained in the process of assembly of the following loads:

- The load occurring from the constant term $-p_z+\rho g$;
- The non-linear term due to the static solution and the harmonic motion including the prescribed sinusoidal variation of the velocities u and v.

The load is obtained by averaging the loads at a number of discrete time instants in a period of steady state harmonic motion:

$$\{F_1\}=$$
$$=\iint [N]^T (\rho g-p_z)dxdy-\overline{\iint [N]^T \rho[u;v][B]\{\delta\}dxdy}=$$
$$=\iint [N]^T (\rho g-p_z)dxdy-$$
$$-\frac{1}{m}\sum_{i=1}^{m}\left(\iint\left(\begin{matrix}[N]^T\rho[a;b]\sin\left(\frac{2\pi}{m}(i-1)\right)[B]\cdot\\ \cdot\begin{Bmatrix}\{\delta_0\}+\{\delta_s\}\sin\left(\frac{2\pi}{m}(i-1)\right)+\\ +\{\delta_c\}\cos\left(\frac{2\pi}{m}(i-1)\right)\end{Bmatrix}\end{matrix}\right)dxdy\right) \qquad (12)$$

where m is the number of discrete time instants within a period. Satisfactory approximation is achieved when m = 16. Finally, the velocities w are calculated from

$$[K]\{\delta_1\}=\{F_1\}. \qquad (13)$$

Mass flow rate. The average mass flow rate is found by integrating over the cross sectional area:

$$Q=\iint \rho w(x,y)dxdy\ , \qquad (14)$$

where $w(x,y)$ are the averaged transverse velocities calculated from $\{\delta_1\}$ by using the shape functions of the appropriate finite elements. It is assumed that the averaging interval is much longer than the period of oscillations.

By the way, this model has certain limitations at very high frequencies. The second set of equations from Eq.(9) leads to the condition $\{\delta_s\}\approx\{0\}$, so the influence of transverse vibrations to the flow diminishes when $\omega\to\infty$.

The results of simulations

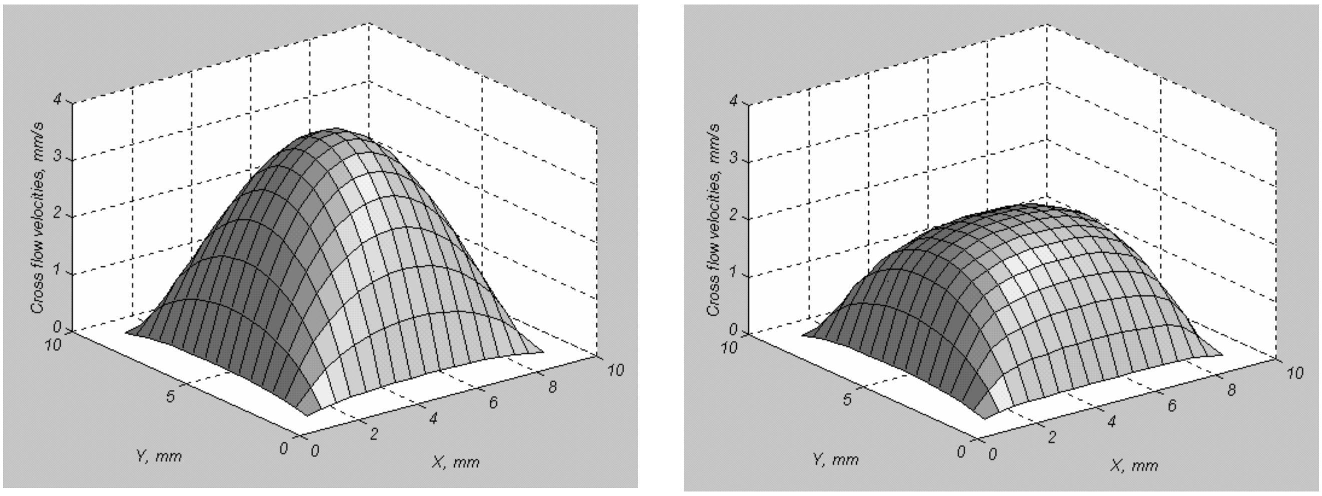
The cross-section of the tube is assumed to be a rectangle. The characteristics of the fluid are assumed to represent a liquid type suspension.

The velocity profiles (averaged surfaces of cross sectional velocities) with no external dynamic excitation and with the excitation in the direction of the x axis are presented in Fig. 2.

The shape of the surfaces of cross flow velocities shows that the high-speed flow is concentrated around the relatively small middle part of the cross section. Better interpretation of the distribution of the velocity in the cross section is provided by the corresponding contour lines in Fig. 3.

The contour lines in Fig. 3 clearly represent the change of the profile of velocities at appropriate transverse excitation. It can be noted that the amplitudes a and b in Fig. 3d are selected in a way that the length of the excitation vector would coincide with appropriate excitation vectors in Fig. 3b and Fig. 3c. Though the relatively high velocity region is expanded in the direction of the excitation, the module values are decreased, resulting in lower mass flow rates.

External transverse excitation decreases the flow velocities. This is a rather surprising effect keeping in mind the results from [1] where the longitudinal vibrations

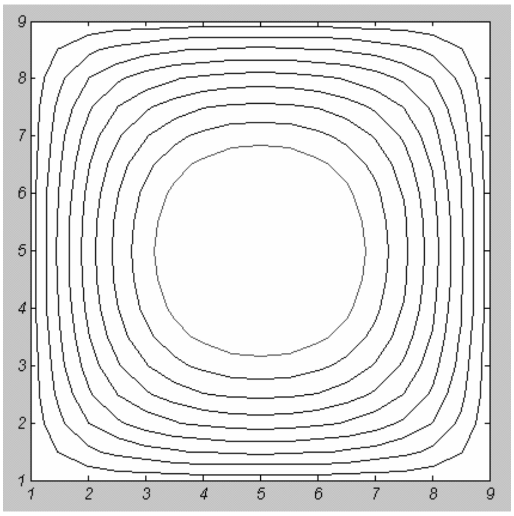


a

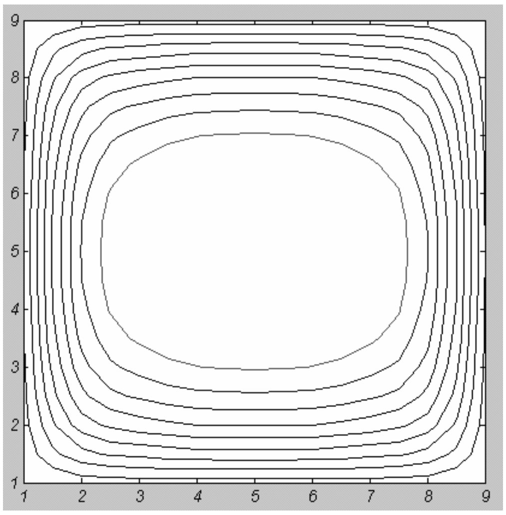
b

Fig. 2. **Cross flow velocities: a) without vibrational excitation; b) with the excitation in the direction of the *x* axis**

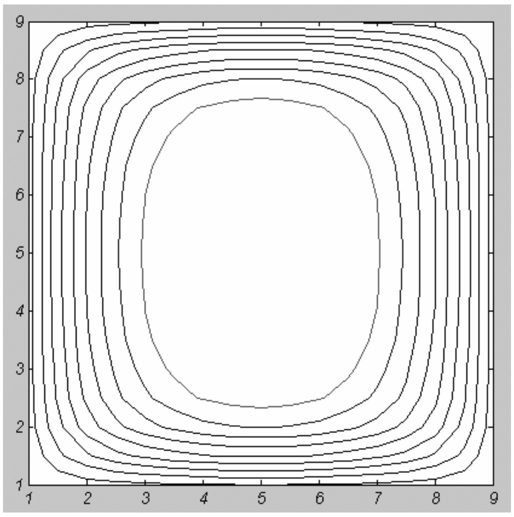
(a = 0,5 mm; b = 0 mm; ω = 300 Hz)



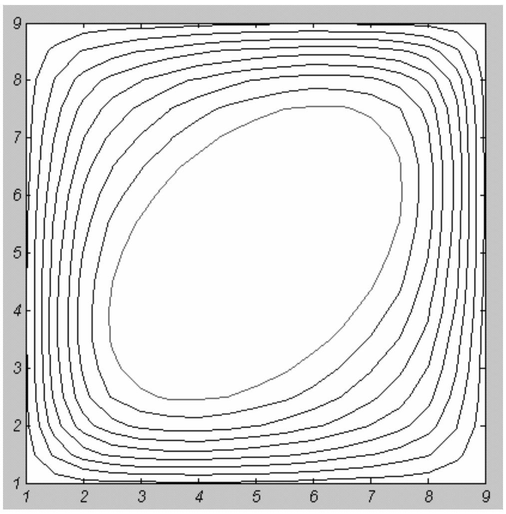
a) no external excitation



b) excitation in the direction of the *y* axis
(a = 0,5 mm; b = 0 mm; ω = 300 Hz)



c) excitation in the direction of the *x* axis
(a = 0 mm; b = 0,5 mm; ω = 300 Hz)



d) excitation in the direction of the bisector of *x-y* axes
(a = 0,35 mm; b = 0,35 mm; ω = 300 Hz)

Fig. 3. **The contour lines of cross flow velocities:**

of the boundary increased the total mass flow rate. Such an effect can be explained by the presence of convective inertia terms of the fluid in the governing equation of motion describing the flow at the transverse excitation of the boundary.

Relationship between mass flow rates and the frequency of excitation in presented in Fig. 4. As noted previously, this model should be considered invalid for very high frequencies, but the range of its applicability it provides the effect of convective inertia terms to the flow.

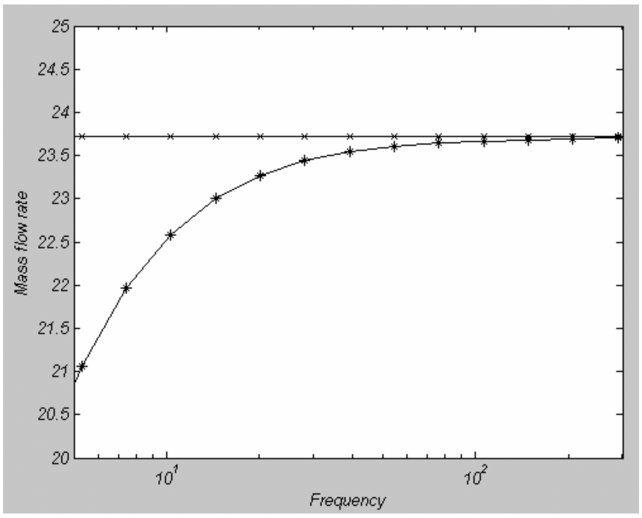


Fig. 4. **The relationship between the excitation frequency ω and the mass flow rate: x markers represent the amplitudes a = 0,35 mm; b = 0,35 mm (no external excitation); * markers represent the amplitudes a = 0,35 mm; b = 0,35 mm**

Conclusions

The mathematical model describing the motion of fluid in a tube performing transverse vibrations is developed. It must be noted that this model incorporates the excitation through the convective intertia terms and represents the dynamic behavior of the liquid suspension. The approximate method for the determination of the surface of the averaged transverse velocity is proposed. The obtained results of the modeling enable the selection of suitable frequencies and feasible levels of amplitudes of excitation for the control of mass flow rates. It is shown that the decrease of the flow rate can be controlled by the frequency of transverse vibrations. The results of the analysis can be applied in the design of vibrational spraying and dosing devices of various substances.

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Skersinių sienelės virpesių įtaka skysčių tekėjimui

Reziumė

Straipsnyje analizuojama skersinių vamzdelio virpesių įtaka skysčių tekėjimui. Sukurtas sistemos matematinis modelis bei netiesinės dinaminės sistemos skaitinės analizės strategija. Tyrimų rezultatai gali būti panaudoti vibraciniams purkštukams ir dozatoriams konstruoti.

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