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Calculation of Lamb waves dispersion curves in multi-layered planar structures

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Introduction

Ultrasonic nondestructive inspection plays important role in multilayered structures testing, especially in aeroindustry, where composites are replacing metallic parts. Conventional ultrasonic techniques do not always enable to detect defects like delamination, kissing bonds or to measure thickness of thin layers, so more advanced methods must be used. The Lamb waves provide one of the possible solutions for those problems, but this technique is much more complicated in application compared with conventional ultrasonic testing. The inspection technique based on Lamb waves requires the study of wave propagation and relies strongly on the predictive modeling tools to enable the best inspection strategies to be identified and their sensitivities to be evaluated [1].

The objective of this work is development of Lamb wave dispersion curves calculation method, suitable for investigation of multi-layered structures.

Overview of Lamb waves modeling methods

The Lamb waves in single-layered isotropic structure embedded in vacuum can be described by two transcendental equations, solution of which describes symmetric and asymmetric wave modes [2]. The propagation of Lamb waves in multi-layered structures can not be described analytically and requires the numerical approach.

Often for such tasks the numerical elements or matrix methods are used. Rose has developed hybrid boundary element method (HBEM) [3]. Cawley uses a finite element method [4]. Hayashi uses strip element method (SEM) for delamination analysis [5]. All those methods are almost unlimited for any structure configuration and waves generation sources and receivers, but are time consuming and stability of the solutions very depends on the product $f\times d$, where f is the frequency of the Lamb waves and d is the total thickness of the analyzed multi-layered structure.

The matrix methods require less of a calculation time, but enables analysis of the limited configuration of the structures. The Lamb waves dispersion curves can be calculated by the transfer matrix method, but this standard method isn't numerically stable, so it is applicable for limited frequency range and number of layers. There are modifications of the transfer matrix method, which are more stable, but computation speed is slow [6]. In 2001 Wang and Rokhlin published a new reformulated transfer matrix recursive algorithm by introducing the layer stiffness matrix [7]. The new modification of the algorithm is relatively stable in the case of multiple layers.

Global matrix method is other way for Lamb waves dispersion curves calculation in multi-layered structures [1]. It is numerically stable and is not sensitive to the product $f\times d$ value. The disadvantage is that the global matrix may be large and the solution may be slow. Nevertheless, this method enables simulation of multi-layer structures taking into account delamination and kissing bond cases. Because of that, this method was selected for further analysis and our version of it is presented in the next paragraphs.

The multi-layered structures definition by the global matrix method

The multi-layered structure for Lamb waves can be described by set of 4(n-1) equations, where n is total number of layers [1]. The equations are written in single matrix form, which is called the global matrix **G**:

$$\mathbf{G}=\begin{bmatrix}\mathbf{D}_{1hb} & \mathbf{D}_{2t} & & & & \\ & \mathbf{D}_{2b} & \mathbf{D}_{3t} & & & \\ & & \mathbf{D}_{3b} & \cdots & & \\ & & & \cdots & \mathbf{D}_{(n-1)t} & \\ & & & & \mathbf{D}_{(n-1)b} & \mathbf{D}_{htn}\end{bmatrix}, \quad (1)$$

where $\mathbf{D}_{1hb}$ - bottom matrix of half space, $\mathbf{D}_{htn}$ - top matrix of half space, $\mathbf{D}_{lt}$, $\mathbf{D}_{lb}$ - top and bottom matrix of l-th layer, $l=1\div(n-1)$. The sub-matrixes $\mathbf{D}_{lt}$, $\mathbf{D}_{lb}$ in general defines reflection and transmission conditions of the top and bottom boundaries of the layer. The sub-matrixes $\mathbf{D}_{1hb}$, $\mathbf{D}_{htn}$ describe acoustic loading conditions from both sides of the analyzed multi-layered structure. This enables simulation of different immersion techniques. The top and bottom sub-matrixes can be defined by Eq.2a and Eq.2b, where k is the Lamb wave wave-number, ω is the angular frequency, c_l and c_s are respectively the longitudinal and shear wave velocities in the layer, ρ is the density and d is the thickness of the layer.

$$\mathbf{D}_t=\begin{bmatrix}-k & -k & -\sqrt{\omega^2/c_s^2-k^2} & \sqrt{\omega^2/c_s^2-k^2}\\ -\sqrt{\omega^2/c_l^2-k^2} & \sqrt{\omega^2/c_l^2-k^2} & k & k\\ -i\rho(\omega^2-c_s^2k^2) & -i\rho(\omega^2-c_s^2k^2) & i\rho c_s^2k\sqrt{\omega^2/c_s^2-k^2} & -i\rho c_s^2k\sqrt{\omega^2/c_s^2-k^2}\\ -i\rho c_s^2k\sqrt{\omega^2/c_l^2-k^2} & i\rho c_s^2k\sqrt{\omega^2/c_l^2-k^2} & -i\rho(\omega^2-c_s^2k^2) & -i\rho(\omega^2-c_s^2k^2)\end{bmatrix}, \quad (2a)$$

$$\mathbf{D}_b = \begin{bmatrix} ke^{id\sqrt{\omega^2/c_l^2-k^2}} & \dfrac{k}{e^{id\sqrt{\omega^2/c_l^2-k^2}}} & \dfrac{\sqrt{\omega^2/c_s^2-k^2}}{e^{-id\sqrt{\omega^2/c_s^2-k^2}}} & -\dfrac{\sqrt{\omega^2/c_s^2-k^2}}{e^{id\sqrt{\omega^2/c_s^2-k^2}}} \\ \dfrac{\sqrt{\omega^2/c_l^2-k^2}}{e^{-id\sqrt{\omega^2/c_l^2-k^2}}} & -\dfrac{\sqrt{\omega^2/c_l^2-k^2}}{e^{id\sqrt{\omega^2/c_l^2-k^2}}} & -ke^{id\sqrt{\omega^2/c_s^2-k^2}} & -\dfrac{k}{e^{id\sqrt{\omega^2/c_s^2-k^2}}} \\ \dfrac{i\rho(\omega^2-c_s^2k^2)}{e^{-id\sqrt{\omega^2/c_l^2-k^2}}} & \dfrac{i\rho(\omega^2-c_s^2k^2)}{e^{id\sqrt{\omega^2/c_l^2-k^2}}} & \dfrac{-2i\rho kc_s^2\sqrt{\omega^2/c_s^2-k^2}}{e^{-id\sqrt{\omega^2/c_s^2-k^2}}} & \dfrac{2i\rho c_s^2k\sqrt{\omega^2/c_s^2-k^2}}{e^{id\sqrt{\omega^2/c_s^2-k^2}}} \\ \dfrac{2i\rho c_s^2k\sqrt{\omega^2/c_l^2-k^2}}{e^{-id\sqrt{\omega^2/c_l^2-k^2}}} & -\dfrac{2i\rho c_s^2k\sqrt{\omega^2/c_l^2-k^2}}{e^{id\sqrt{\omega^2/c_l^2-k^2}}} & \dfrac{i\rho(\omega^2-c_s^2k^2)}{e^{-id\sqrt{\omega^2/c_s^2-k^2}}} & \dfrac{i\rho(\omega^2-c_s^2k^2)}{e^{id\sqrt{\omega^2/c_s^2-k^2}}} \end{bmatrix}, \qquad (2b)$$

The first and the third columns in matrices $\mathbf{D}_{lt}$, $\mathbf{D}_{lb}$ represent incident longitudinal and shear (vertical polarization) waves into the layer respectively. The second and fourth columns represent transmission of corresponding waves from the layer. The first two rows describe displacements in the layer and the second two rows describe the stresses.

In general, energy equilibrium principle for the acoustic weaves in the multi-layered structure can be defined using the global matrix **G**:

$$\mathbf{AG} = 0\,, \qquad (3)$$

where the vector **A** represents the amplitudes of the waves displacements and stresses on the boundaries of the different layers. In general, the solved values of the vector **A** correspond to some mode m of a propagating Lamb wave with the phase velocity c_m at the angular frequency ω. The dispersion curves can be expressed as the set of the curves $c_m=f(\omega)$, where $m=1\div M$ and M is the number of wave modes under analysis. So, for the dispersion curves determination it is necessary to determine velocities of different Lamb waves in the frequency range (ω_{min}, ω_{max}). The valid modal solution of Eq.3 at fixed angular frequency occurs when complex determinant

$$\det \mathbf{G} = 0\,. \qquad (4)$$

This solution gives the value of the corresponding Lamb wave mode velocity c_m. The problem is that for one fixed frequency point f_i, there may be a number of roots c_1, c_2,…, c_m, corresponding to different modes and for determination of each of them it is necessary to solve this nonlinear matrix equation. This usually takes a long time. In the next paragraph the relatively fast and reliable dispersion curves calculation algorithm is presented.

Dispersion curves calculation algorithm

There are two main factors, which can reduce the calculation time: the first is the effectiveness of root calculation method and the second is the reduction of the points where the solution of matrix equation is performed. Dealing with the first factor, Lowe gives overview of different methods such as bisection, Newton-Raphson, Monte Carlo and etc. used by other researchers [1]. The secant method is chosen in this work because it does not need the analytical expression of function derivatives and has convergence similar to the Newton-Raphson method, based on analytic function derivatives.

The big number of points where it is necessary to find the test solution is caused by the fact that at each fixed frequency exist many modes of propagating Lamb waves and as consequence the big number of roots. For the reliable detection of each of them the search intervals must be relatively small and this leads to the big number of calculation points. For reduction of number of these points the dispersion curves calculations algorithm was divided into two stages. The objective of the first stage is to find one, new (not calculated) dispersion curve at the fixed velocity. The objective of the second stage is to follow along the detected dispersion curve and in such a way to calculate the phase velocity values of the detected mode in the interested range of frequencies.

During the first stage the some initial value of the Lamb wave velocity c_{0m} is selected and the step by step searching for the root of Eq.4 in the frequency range under the interest (ω_{min}, ω_{max}) is performed. Such a scanning is performed until the first root is found, that is, the frequency ω_{0m} at which the detected mode m have velocity c_{0m}.. This means that one point (c_{0m}, ω_{0m}) of the dispersion curve is known.

In the second stage the tracing along the detected curve should be performed. In the initial part of this stage the additional four points of the curve are calculated using small step in frequency domain, that is (c_{0m}, ω_{0m}), (c_{1m}, $\omega_{0m}+d\omega$), (c_{2m}, $\omega_{0m}+2\,d\omega$), (c_{3m}, $\omega_{0m}+3\,d\omega$), (c_{4m}, $\omega_{0m}+4\,d\omega$). The selection of small step enables to reach required accuracy very fast, but application of such an approach for tracing of the complete curve will require a big number of steps. Because of that, in next steps the fourth order extrapolation is used for setting of the search interval for a new point using essentially bigger step $\Delta\omega$ in the frequency domain.

So, the dispersion curves calculation algorithm consists of the next general steps:

I. Construction of multi-layered structures using global matrix approach and corresponding properties of each layer. Also, ranges of frequency and Lamb waves phase velocities ranges are selected.
II. Searching for the first point of a new dispersion curve:
    1. Select initial velocity value c_0;
    2. Select initial angular frequency value ω_0;
    3. Set the root search interval in the frequency domain ($\omega_0-\Delta\omega_r$, $\omega_0+\Delta\omega_r$);
    4. Solve the matrix Eq.4 by secant method. Result is the frequency of the Lamb wave with the velocity c_0.;

5. If the root does not exist, increase the frequency value ω_0 and repeat steps the 3-5 again till the root will be found;
6. The calculated point is selected as the initial point (c_{0m}, ω_{0m}) for the dispersion curve tracing;

III. Tracing of the dispersion curve:

1. Calculate Lamb wave velocities at the frequencies $\omega_{0m}+d\omega$, $\omega_{0m}+2d\omega$, $\omega_{0m}+3d\omega$, $\omega_{0m}+4d\omega$;
2. Calculate extrapolation coefficients of the fourth order polynomial using the last 5 points of the dispersion curve;
3. Calculate approximate Lamb wave velocity c'_{new} at the frequency, $\omega_{0m}+\Delta\omega_{\text{big}}$
4. Set the root search interval in the velocities domain ($c'_{\text{new}} - \Delta c_r$, $c'_{\text{new}} + \Delta c_r$);
5. Solve the matrix Eq.4 by secant method. The result is the Lamb wave velocity c_{new} at the frequency $\omega_{0m}+\Delta\omega_{\text{big}}$;
6. Repeat the steps 2-5 until the frequency range limit will be reached;

IV. Store the calculated dispersion curve data;

Repeat steps II-IV for calculation of other dispersion curves.

Modeling results

The two types of layered structures were selected for analysis. The first structure consists of two layers bonded together. The first layer is made of steel and the second one of aluminum. Another structure under analysis was the five layer composite: three layers of aluminum and two layers of glass-fiber with bonding material between aluminum plates. Materials properties are presented in Table 1. It was assumed that all materials including glass-fiber with bonding material were isotropic. In all cases was assumed that the structure is placed in vacuum. Modeling results are presented in Fig.1 and Fig.2.

Table 1. **Material properties.**

Structure	Material	d, mm	ρ, kg/m³	c_l, km/s	c_s, km/s
Al - steel	Al	0.4	2700	6.32	3.13
	steel	1.2	7800	5.9	3.19
composite	Al	0.3	2770	6.32	3.15
	bonding	0.25	2500	3.15	1.72

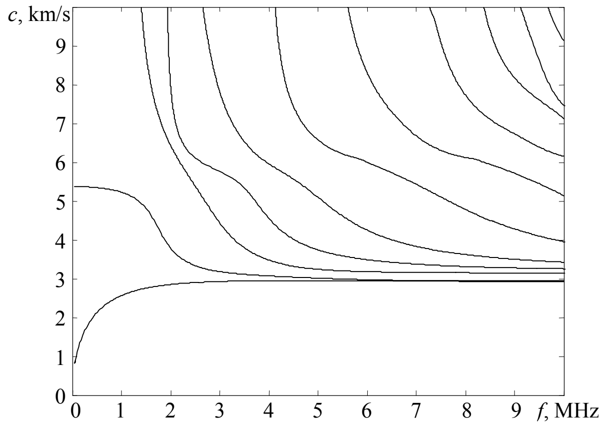

Fig. 1. **Lamb waves pahase velocity dispersion curves in aluminum-steel structure.**

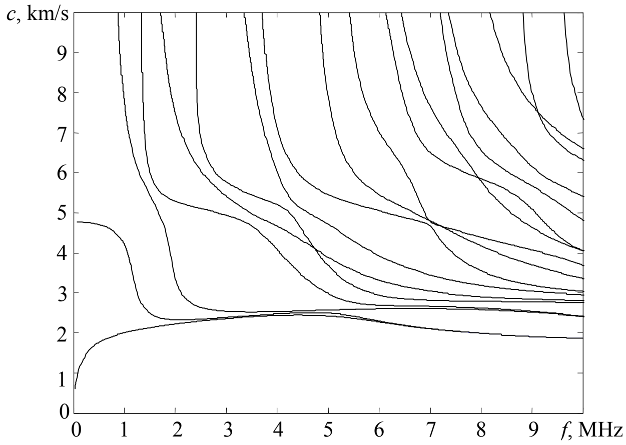

Fig. 2. **Lamb waves pahase velocity dispersion curves in composite.**

It can be seen the complicated dependencies of velocities, especially for case of the five layer composite structure.

Conclusions

The Lamb waves modeling methods were reviewed and the global matrix method was selected for calculation of Lamb waves dispersion curves in multi-layered plate-like structures. The robust and simple calculation algorithm of dispersion curves based on the global matrix method has been developed.

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Lembo bangų daugiasluoksnėse plokščiose struktūrose dispersinių kreivių apskaičiavimas

Referatas

Darbe apžvelgti Lembo bangų modeliavimo metodai. Atlikta globaliosios matricos metodo izotropinių medžiagų atveju analizė, pateiktos sluoksnius aprašančių matricų koeficientų analitinės išraiškos. Pasiūlytas algoritmas Lembo bangų dispersinėms kreivėms apskaičiuoti, pagrįstas globaliosios matricos metodu, ir išnagrinėti pagrindiniai šio algoritmo realizavimo etapai. Pateikti dispersinių kreivių daugiasluoksnėse lakštinėse struktūrose apskaičiavimo pavyzdžiai.

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