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Modelling of spatial and frequency responses of the ultrasonic interferometer used for displacement measurements

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Introduction

Ultrasonic methods may be successfully applied for measurement of very short distances and displacements. All these methods may be divided into two main groups. The first group exploits pulse methods which are based on measurement of time of flight of ultrasonic waves between ultrasonic sensors and the object. In the second group of methods interference of ultrasonic waves between the source of ultrasonic waves and the object under investigation is exploited.

The pulse methods can be used for measurement of distances in the range from a few millimeters up to tens of meters [1-8]. However, there are at least two serious limitations of the pulse methods in the case of measurement of small displacements. The first one is due to very small variations of time of the flight, which is necessary to measure. These variations of the time of flight are in the range of a few nanoseconds, when displacements are of order of one micrometer. The second limitation is called “death zone”, which is caused by residual vibrations of the ultrasonic transducer after excitation. It means that there is some time interval during which the measurement system is not able to pick-up the reflected signals. Hence, during this interval measurements are not feasible.

Therefore, when it is necessary to measure small displacements in the range of a few micrometers at an absolute distance between ultrasonic transducer and vibrating surface of a few millimeters, then the pulse methods are not suitable.

The second method is based on exploitation of an interference phenomenon of ultrasonic waves in an acoustic cavity resonator. This method possesses higher sensitivity and accuracy than the pulse method. The cavity resonator was very successfully used for measurement of small ultrasound velocity changes in biologic fluids [9], but it was not used for distance measurements in air.

The ultrasonic interferometric technique based on phase-locked loop is the most suitable for measurements of small displacements. This method ensures the highest sensitivity and accuracy [10-12]. The acoustic resonator, which consists of two coaxial planar circular transducers placed in air, was analyzed earlier [12-14]. In this analysis losses of an ultrasonic signal in air were not taken in account

The objective of this investigation is estimation of influence of finite dimensions, diffraction and attenuation in air on a performance of the acoustic resonator filled with air, consisting of two coaxial planar circular transducers.

Model

It was proposed in previous our publication [14] to use disk shape transducer diffraction model for investigation of the performance of the ultrasonic interferometric technique. Application of such an approach enables to take into account the finite dimensions of the transducer and diffraction phenomena. According to the proposed method multiple reflections in the gap between ultrasonic transducers were simulated as additional virtual receivers placed at the distance 3*d* for the first time reflected signal in the gap, 5*d* for the second reflection and etc. Here *d* is the distance between transducers or in other words, the gap thickness. In such a way any desired number of multiple reflections can be taken into account. The disadvantage of the method proposed is that it does not take into account attenuation of ultrasonic waves what in the case of air filled gap can be very essential.

The attenuation usually is expressed by the exponential dependency

$$K_{\text{att}} = e^{-\alpha x}, \qquad (1)$$

where x is the distance, α is the attenuation coefficient. The attenuation coefficient in the case of air is the function of the temperature, humidity and frequency and can be expressed by

$$\alpha(T, f, h) = \frac{(33+2T)f^2}{2\cdot 10^{12}} + \frac{\left(\dfrac{f^2}{6.8\cdot 10^9\cdot h^{1.3}}\right)}{2\cdot\left(1+\left(\dfrac{f}{8.5\cdot 10^4\cdot h^{1.3}}\right)\right)} \qquad (2)$$

where T is the absolute air temperature, h is the relative humidity, f is the frequency of ultrasonic wave.

The problem is how to include this attenuation into the model. In the case of one dimensional approach it is included simply by multiplying a transfer function at fixed distance by the coefficient K_{att} . In the diffraction approach when the field is calculated at short distances it is impossible to estimate attenuation in such a simple way, because the transfer function due to Huygens principle is “formed” by signal arriving from different points on the transmitter surface and, consequently, differently attenuated due to difference in propagation distances.

According to the approach proposed in our previous publication [14], the pulse response of each receiver (real and virtual ones) was calculated as a sum of pulse responses of annulus forming disk shape of the receivers [14]:

$$h_r(t) = \sum_{l=1}^{L} h_l(t), \quad (3)$$

where $h_l(t)$ is the pulse response of l-th annulus, L is the number of annulus. According to the used numerical calculation method [14, 15] the pulse response of each annulus has been expressed as the set

$$h_l(t) = \{A_1(t_1),..., A_k(t_k),..., A_N(t_N)\}, \quad (4)$$

where A_k is the pulse response value at the time instance t_k, N is the total number of discrete points in calculated pulse response. The ultrasonic wave propagates the distance s_k during the time interval t_k. So, the attenuation of ultrasonic wave was obtained as sets of attenuation values at discreet time instants. Then the attenuation can be estimated including additional coefficient

$$h_{l,att}(t) = h_l(t)\cdot \mathrm{e}^{-\alpha c t} = \\ = \left\{A_1(t_1)\cdot \mathrm{e}^{-\alpha c t_1},..., A_k(t_k)\cdot \mathrm{e}^{-\alpha c t_k},..., A_N(t_N)\cdot \mathrm{e}^{-\alpha c t_N}\right\}, \quad (5)$$

where c is the ultrasound velocity in medium.

The complete complex transfer function now is given by

$$K(j\omega) = \sum_{m=1}^{M}\sum_{l=1}^{L} h_{l,m} e^{-\alpha c t_{l,m} + j\omega t_{l,m}}, \quad (6)$$

where ω is the angular frequency, t is the time, $h_{l,m}$ is the impulse response value of l-th annulus of m-th receiver at the time instant $t_{l,m}$, M is the total number of multiple reflections taken into account, c is the velocity of ultrasound in air (c=342 m/s, when T=20°C)...

The complex transfer function consists of real and imaginary parts, both of which are necessary for calculation of amplitude and phase frequency responses of the acoustic system. The magnitude of the complex transfer function describes the amplitude frequency response of the acoustic system:

$$|K| = \sqrt{(\mathrm{Re}(K(j\omega)))^2 + (\mathrm{Im}(K(j\omega)))^2} \quad (7)$$

where the real and the imaginary terms are correspondingly given by:

$$\mathrm{Re}(K(j\omega)) = \sum_{m=1}^{M}\sum_{l=1}^{L} h_{l,m} e^{-\alpha c t_{l,m}} \cos(\omega t_{l,m}), \quad (8)$$

$$\mathrm{Im}(K(j\omega)) = \sum_{m=1}^{M}\sum_{l=1}^{L} h_{l,m} e^{-\alpha c t_{l,m}} \sin(\omega t_{l,m}). \quad (9)$$

The amplitude and phase of the complex transfer function are expressed by:

$$A(\omega) = |K(j\omega)|, \quad (10)$$

$$\varphi(\omega) = \arctan\left(\frac{\mathrm{Re}(j\omega)}{\mathrm{Im}(j\omega)}\right). \quad (11)$$

So, using Eq.6-11 and diffraction model presented in [14] it is possible to calculate the transfer function of an acoustic resonator filled with air, consisting of two coaxial planar circular transducers.

Simulation results

For verification of the developed model calculations were performed for the resonator consisting of two transducers with the diameter 17mm and situated at the distance 3 mm one from another. The amplitude and phase responses were calculated in the frequency range from 200 kHz up to 500 kHz. Attenuation of ultrasonic waves in this frequency range for different distances is shown in Fig.1.

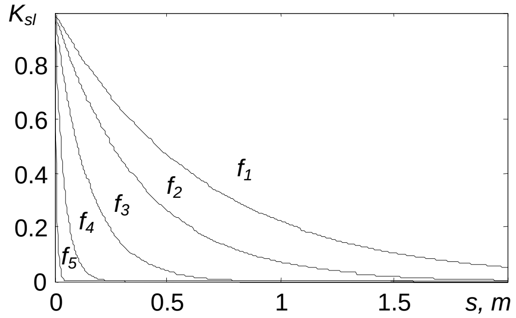

Fig. 1. **Amplitude of the reflected ultrasonic wave versus propagation distance at different frequencies: f_1=100kHz; f_2=200 kHz; f_3=300 kHz; f_4=400 kHz; f_5=500 kHz.**

The first question, which arises, is how many multiple reflections in the model should be taken into account during calculations. To clarify this question the calculations with different number of estimated multiple reflection were carried out. The results of these calculations are presented in Fig.2-4.

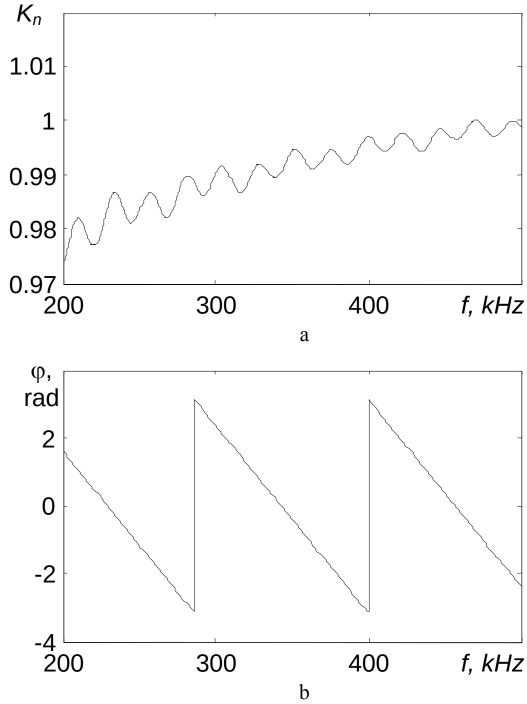

Fig. 2 .**The calculated amplitude (a) and phase (b) frequency responses of the acoustic resonator, when only one virtual reflector was taken account**

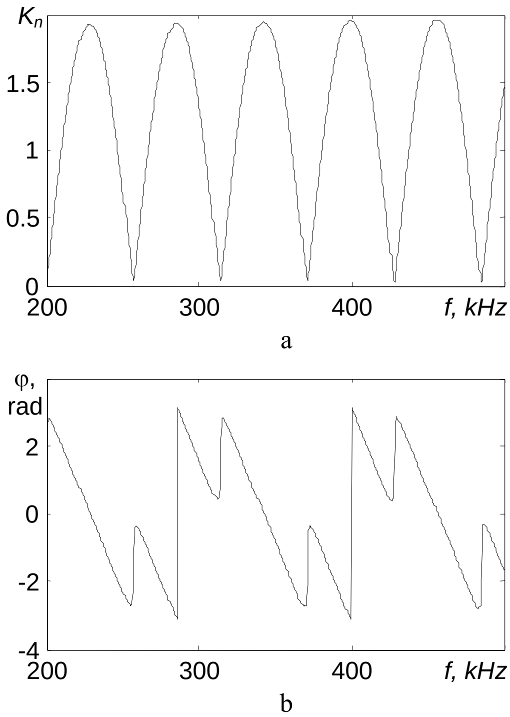

Fig. 3 . **The calculated amplitude (a) and phase (b) frequency responses of the acoustic resonator, when only one multiple reflection was taken into account**

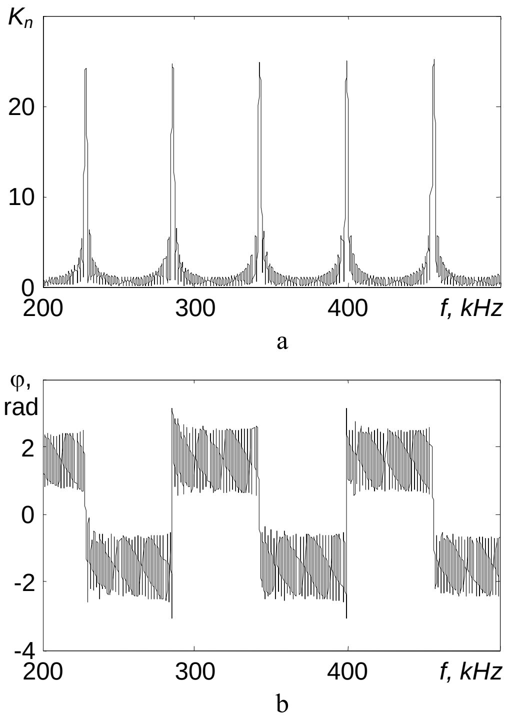

Fig. 4 . **The calculated amplitude (a) and phase (b) frequency responses of the acoustic resonator, when 30 multi-reflection were estimated**

It can be seen that an almost flat amplitude frequency response (no any resonance is expressed) is obtained when no multiple reflections were taken into account. (Fig. 2). When only one reflection is taken into account some resonance phenomena are observed in the analysed frequency response. (Fig.3). In the case when 30 multiple reflections were taken into account, sharp resonance spikes in the frequency response are obtained (Fig.4). At the same time the phase is almost constant in the frequency ranges between the resonances and has a sharp jump of 2π in the resonance region. The resonances occur when the condition

$$s_n = \frac{n}{2}\frac{c}{f_r} \qquad (9)$$

is fulfilled. There f_r is the fundamental resonance frequency of the air gap between transducers. It means that resonances occur at every half wavelength. This phenomenon is sharper if more multiple reflections are taken into account during calculations. Please note that in the frequency range between resonances high frequency ripples can be seen. They are caused by a discreet nature of the model, in which only a finite number of reflections is present.

In the examples given in Fig. 1-6 attenuation of ultrasonic wave in air was not estimated.

Due to attenuation the amplitude of the resonance peaks decreases at higher frequencies. The calculations were performed with a different number of multiple reflections taken into account in the model (Fig.5-6).

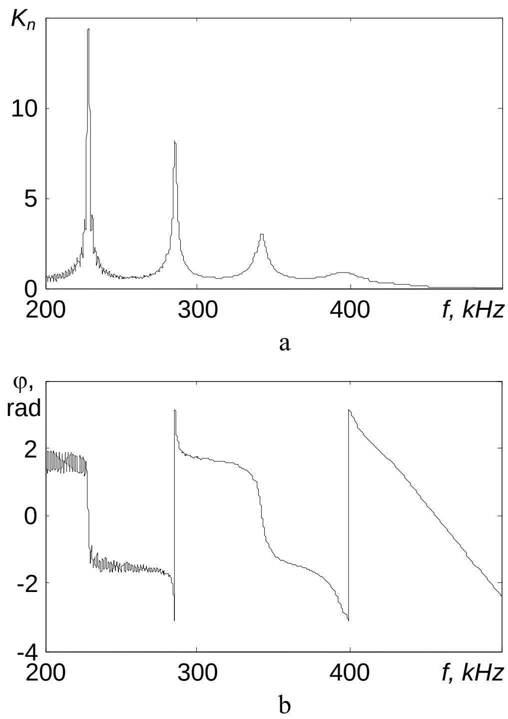

Fig. 5 . **The calculated amplitude (a) and phase (b) frequency responses of the acoustic resonator, when 30 multi-reflection and attenuation were taken into account**

Every next reflected wave propagates further and attenuation of this wave is proportional to the propagation distance.

Attenuation in air depends upon wave frequency *f* and propagation distance *s* (Fig. 1). Therefore, influence of reflections with a higher number in to formation of resonance peaks is smaller, especially at higher frequencies. For example, if the number of estimated reflections is increased from 30 to 50, difference is not very significant (Fig. 10, 11). Hence, the number of virtual reflectors in the model depends on a required accuracy. In order to obtain the same accuracy at different frequencies it is necessary to take into account different number of reflections. For example, at the 300 kHz frequency it is necessary to take into account about one hundred thirty reflections and for the 400 kHz frequency only thirty reflections, because all remaining reflections are very small. In both cases the obtained accuracy is the same.

We can see that frequency responses presented in Fig. 7 and 8 become very similar to the frequency response obtained using one dimensional approach [14]. The relative error of resonance peaks frequency is only 0.1-0.03%.

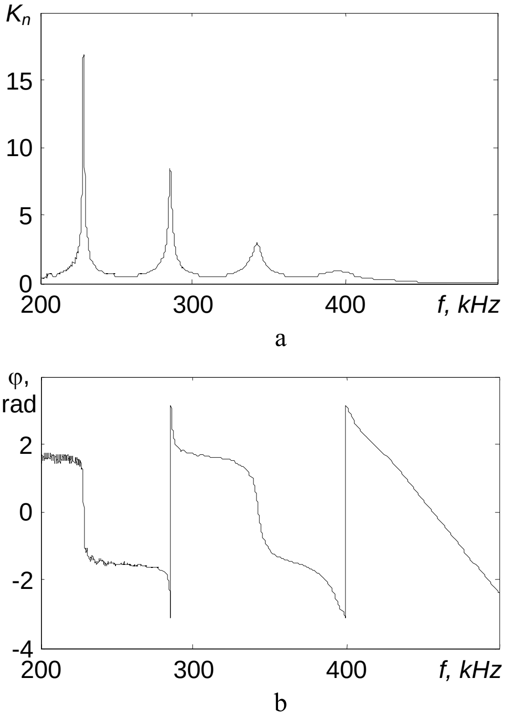

Fig. 6 . **The calculated amplitude (a) and phase (b) frequency responses of the acoustic resonator, when 50 multiple reflections and attenuation in air were taken into account**

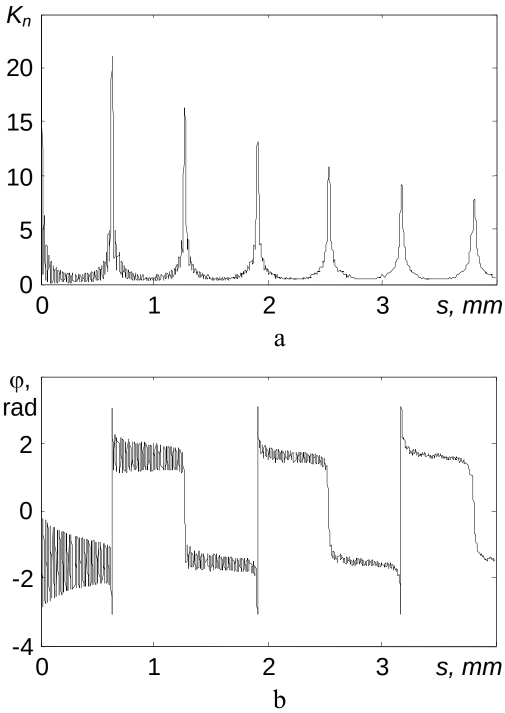

Fig. 7 **The amplitude (a) and phase (b) of the transfer function versus distance between transducers at the frequency *f*=270 kHz. The model of the acoustic resonator estimates 30 reflectors and attenuation in air**

Performance of the analyzed acoustic resonator as a displacement meter may be evaluated from a spatial transfer function, e.g., dependence of the complex transfer coefficient at the fixed frequency on a distance between ultrasonic transducers. The calculation algorithm is similar to the calculation algorithm of the frequency responses. The results of calculations carried out using Eq.10 and 11 at the frequency 270 kHz in the distance range from 0.01 to 4 mm are presented in Fig.7. The high frequency ripples, observed in these responses, especially at short distances, are caused by a discreet nature of the model.

The magnitude of the transfer function K_n is normalised with respect to the maximum value of the transfer function in the case when only one reflection is taken to account, e.g., when there is no standing wave between transducers and no resonance phenomena occur. In other words, the value of a peak magnitude equals to the quality factor of the acoustic resonator when signal losses and diffraction phenomenon are estimated. It is necessary to point out that difference between the spatial transfer functions obtained using one-dimensional and diffraction models is rather small [13]. The relative error of position of resonance peaks is 0.125 %.

Conclusions

The developed diffraction model of the acoustic resonator takes into account finite lateral dimensions of the ultrasonic transducers, attenuation of ultrasonic waves in air and enables to calculate with a high accuracy frequency and spatial responses. The number of virtual reflectors, simulating the number of multiple reflections between transducers, depends on the frequency of an ultrasonic wave and a required accuracy of calculations. For example, in order to get the same accuracy at the 300 kHz frequency it is necessary to take into account about 130 reflections and at the 400 kHz only 30 reflections, because other reflections due to frequency dependent attenuation in air are very small.

The frequency and spatial responses calculated by one dimensional and diffraction model with losses are rather similar. The relative errors of the frequencies of resonance peaks are 0.1-0.03 % and of the spatial positions is 0.125 %. The results presented illustrate that sharpness of the resonance peaks depends on the distance between transducers and the frequency of the signal. The results obtained allow select the number of resonance peak, frequency and the absolute distance between transducers most suitable for measurement of small displacements.

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Ultragarsinio interferometrinio poslinkio matavimo metodo modelis ir charakteristikų analizė

Reziumė

Straipsnyje pristatytas ultragarsinio interferometrinio mažų atstumų bei poslinkių matavimo metodo skaitmeninis modelis. Tiriama dviejų bendraašių ultragarsinių keitiklių sistema, įvertinant baigtinius skersinius jų matmenis. Metodas pagrįstas diskinio keitiklio difrakcinio modelio panaudojimu. Daugkartiniai atspindžiai įvertinami naudojant virtualius reflektorius. Pasiūlytas slopinimo panaudojimo difrakciniame modelyje metodas. Pateikta tikslių modelio parametrų parinkimo metodika ir pagrindimas bei palyginimas su žinomais vienmačiais modeliais. Naudojant pasiūlytą modelį apskaičiuotos amplitudinės ir fazinės perdavimo funkcijos priklausomybės nuo atstumo tarp keitiklių bei dažnio.

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