ISSN 1392-2114 ULTRAGARSAS, Nr.4(49). 2003.

Investigation of the oscillatory dynamics of the interacting plates

J. Ragulskiene[1], J. Maciulevicius[1], R. Maskeliunas[2], L. Zubavicius[2]

[1]*Kaunas Technical College*

[2]*Vilnius Gediminas Technical University*

Introduction

This paper deals with the analysis of dynamics of two interacting plates. The simplified model of the interaction of the plates through the layer of material consists of the distributed springs performing vertical motion and without mutual interaction. This is a model similiar to the elastic foundation of Winkler type known in applied elasticity [1, 4, 9]. The finite element with six degrees of freedom per node (the deflection and the two rotations of the first plate and the deflection and the two rotations of the second plate) is developed for the analysis of the described system. The eigenmodes are calculated and each of them provides the two surfaces: the first one describing the bending of the first plate and the other one describing the bending of the second plate.

In the model of the plate the deformation in the direction of its thickness is not taken into account. The layer of the springs takes into account this deformation in the direction of the thickness. So the presented model is applicable for the analysis of composite plates, the lower and upper layers being considered as non-deformable in the direction of the thickness, while the internal layer takes only the deformation in the direction of the thickness into account.

Such a model of two interacting plates with a layer of material between them is an approximation of the dynamics of ink film between the blanket and impression cylinders in a printing machine (Fig. 1).

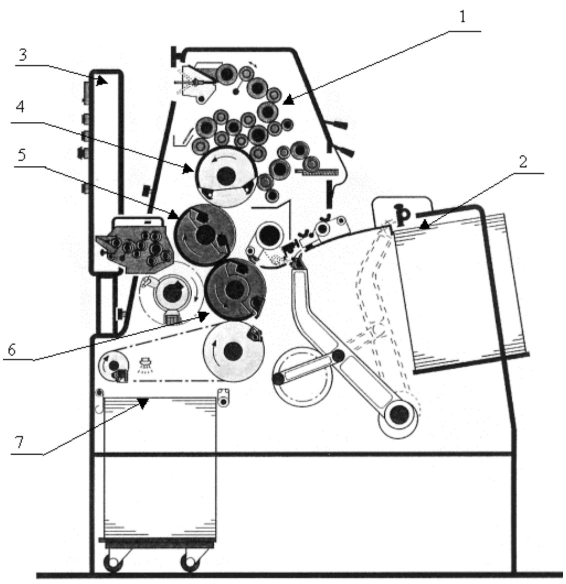

Fig. 1. **The schematic diagram of a printing machine: 1- ink apparatus; 2 – paper feeding mechanism; 3 – control block; 4 - plate cylinder; 5 - blanket cylinder; 6 - impression cylinder; 7 – printed materials sorting mechanism**

An important problem is the evaluation of the process ink film distribution between the blanket and impression cylinders, especially the dynamic layer formation in the process of print image generation (Fig. 2).

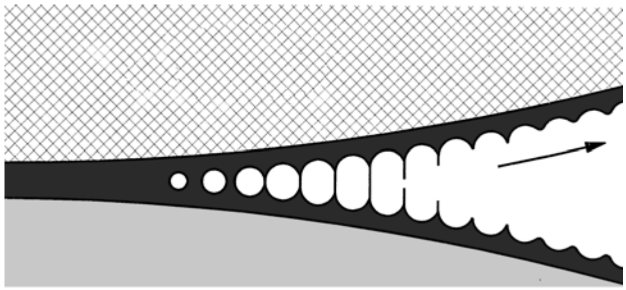

Fig. 2. **The schematic diagram of ink film distribution between the blanket and impression cylinders**

Such a complex dynamical problem requires development of appropriate mathematical and numerical strategy of analysis, what is the object of this paper.

Numerical model of the interacting plates

The model of the analysed system is presented in Fig.3. Numerical model of the described system requires the development of a finite element which is a modification of the plate element presented in [2, 7, 8].

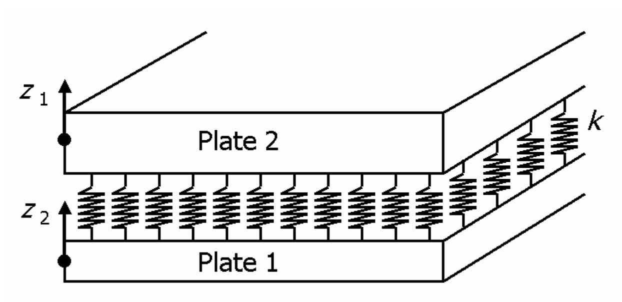

Fig. 3. **The model of the interacting plates**

The nodal variables are the deflection of the first plate w_1, the rotation of the first plate about the x axis Θ_{x1}, the rotation of the first plate about the y axis Θ_{y1}, the deflection of the second plate w_2, the rotation of the second plate about the x axis Θ_{x2}, the rotation of the second plate about the y axis Θ_{y2} (it is assumed that $v_i = -z_i\Theta_{xi}$, $u_i = z_i\Theta_{yi}$, $i=1,2$, here u_i and v_i are the displacements of the plate i in the x and y directions, z_i are the z coordinates measured from the midsurface of the corresponding plate).

The potential energy of the layer of the distributed springs is:

$$\Pi = \frac{1}{2}\iint k(w_1 - w_2)^2 dxdy = \frac{1}{2}\iint k\left(\lfloor \overline{N} \rfloor \{\delta\}\right)^2 dxdy = \\ = \frac{1}{2}\{\delta\}^T \iint \lfloor \overline{N} \rfloor^T k \lfloor \overline{N} \rfloor dxdy \{\delta\}, \tag{1}$$

where:

$$\lfloor \overline{N} \rfloor = [N_1 \quad 0 \quad 0 \quad -N_1 \quad 0 \quad 0 \quad \vdots \quad \cdots], \tag{2}$$

and k is the distributed stiffness of the layer, $\{\delta\}$ is the vector of generalised displacements. So the stiffness matrix takes the form:

$$[K] = \iint \begin{pmatrix} [B_1]^T [D_1][B_1] + [\overline{B}_1]^T [\overline{D}_1][\overline{B}_1] + \\ + [B_2]^T [D_2][B_2] + [\overline{B}_2]^T [\overline{D}_2][\overline{B}_2] + \\ + \lfloor \overline{N} \rfloor^T k \lfloor \overline{N} \rfloor \end{pmatrix} dxdy, \tag{3}$$

where:

$$[D_i] = \frac{h_i^3}{12}\begin{bmatrix} \frac{E_i}{1-\nu_i^2} & \frac{E_i \nu_i}{1-\nu_i^2} & 0 \\ \frac{E_i \nu_i}{1-\nu_i^2} & \frac{E_i}{1-\nu_i^2} & 0 \\ 0 & 0 & \frac{E_i}{2(1+\nu_i)} \end{bmatrix},$$

$$[\overline{D}_i] = \frac{E_i h_i}{2(1+\nu_i)k_s}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$[B_1] = \begin{bmatrix} 0 & 0 & \frac{\partial N_1}{\partial x} & 0 & 0 & 0 & \vdots & \cdots \\ 0 & -\frac{\partial N_1}{\partial y} & 0 & 0 & 0 & 0 & \vdots & \cdots \\ 0 & -\frac{\partial N_1}{\partial x} & \frac{\partial N_1}{\partial y} & 0 & 0 & 0 & \vdots & \cdots \end{bmatrix},$$

$$[B_2] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \frac{\partial N_1}{\partial x} & \vdots & \cdots \\ 0 & 0 & 0 & 0 & -\frac{\partial N_1}{\partial y} & 0 & \vdots & \cdots \\ 0 & 0 & 0 & 0 & -\frac{\partial N_1}{\partial x} & \frac{\partial N_1}{\partial y} & \vdots & \cdots \end{bmatrix},$$

$$[\overline{B}_1] = \begin{bmatrix} \frac{\partial N_1}{\partial y} & -N_1 & 0 & 0 & 0 & 0 & \vdots & \cdots \\ \frac{\partial N_1}{\partial x} & 0 & N_1 & 0 & 0 & 0 & \vdots & \cdots \end{bmatrix},$$

$$[\overline{B}_2] = \begin{bmatrix} 0 & 0 & 0 & \frac{\partial N_1}{\partial y} & -N_1 & 0 & \vdots & \cdots \\ 0 & 0 & 0 & \frac{\partial N_1}{\partial x} & 0 & N_1 & \vdots & \cdots \end{bmatrix}, \tag{4}$$

and E_i is the modulus of elasticity of the plate i, ν_i is the Poisson's ratio of the plate i, h_i is the thickness of the plate i, k_s is the shear correction factor assumed equal to 1.2.

The mass matrix takes the form:

$$[M] = \iint \begin{pmatrix} [R_1]^T \begin{bmatrix} \rho_1 h_1 & 0 & 0 \\ 0 & \frac{\rho_1 h_1^3}{12} & 0 \\ 0 & 0 & \frac{\rho_1 h_1^3}{12} \end{bmatrix} [R_1] + \\ + [R_2]^T \begin{bmatrix} \rho_2 h_2 & 0 & 0 \\ 0 & \frac{\rho_2 h_2^3}{12} & 0 \\ 0 & 0 & \frac{\rho_2 h_2^3}{12} \end{bmatrix} [R_2] \end{pmatrix} dxdy, \tag{5}$$

where:

$$[R_1] = \begin{bmatrix} N_1 & 0 & 0 & 0 & 0 & 0 & \vdots & \cdots \\ 0 & N_1 & 0 & 0 & 0 & 0 & \vdots & \cdots \\ 0 & 0 & N_1 & 0 & 0 & 0 & \vdots & \cdots \end{bmatrix},$$

$$[R_2] = \begin{bmatrix} 0 & 0 & 0 & N_1 & 0 & 0 & \vdots & \cdots \\ 0 & 0 & 0 & 0 & N_1 & 0 & \vdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & N_1 & \vdots & \cdots \end{bmatrix}, \tag{6}$$

and ρ_i is the density of the material of the plate i.

Numerical investigation of the vibrations of the interacting plates

The analyzed object is a rectangular elastic composite plate of the previously described type with a fastened edge. The thickness of the second plate is twice bigger than the thickness of the first one, while the other parameters of the plates are the same. Such a model is of course the first approximation of ink film dynamics between the blanket and impression cylinders, but nevertheless can provide quite deep insight in the processes taking place in such contact type systems. The boundary conditions (the fastened edge) corresponds to the area of maximum compression between the two rotating cylinders, whereas the free surface areas of the interacting plates enable computational analysis of complex dynamic processes from the ink film is passing the zone of maximum compression.

For the representation of dynamic results the intensity mapping proposed in [3] is used.

The first eigenmode of the deflection of the first plate represented by the intensity mapping is shown in Fig. 4.

The first eigenmode of the deflection of the second plate despite the different thicknesses of the plates has no evident difference compared with the motion of the first plate.

The seventh eigenmode of the deflection of the first plate represented by the intensity mapping is shown in Fig.5.

The seventh eigenmode of the deflection of the second plate represented by the intensity mapping is shown in Fig.6.

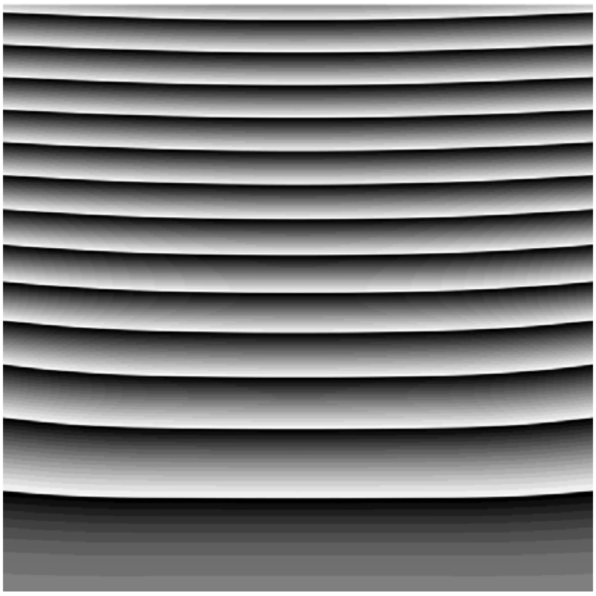

Fig. 4. **The first eigenmode of the deflection of the first plate**

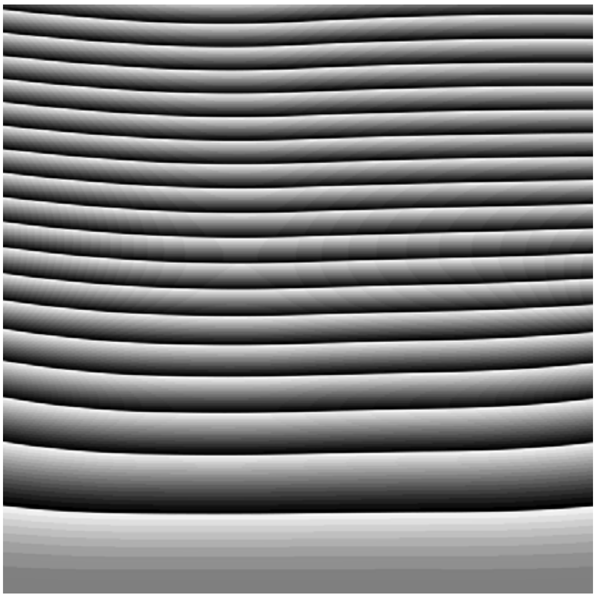

Fig. 5. **The seventh eigenmode of the deflection of the first plate**

From the two previous figures it is evident that the amplitude of vibrations of the thicker plate is smaller.

It can be noted that the representation by intensity mapping enables to see that the plates perform counterphase motions in this mode.

The seventh eigenmode and the first eigenmode would look similar when represented for example by the holographic interferograms of the plates, but this representation by intensity mapping enables to see the different character of motion in those modes.

Conclusions

The interaction of the plates through the simplified model of the layer of material is analysed. The layer is represented by the distributed springs performing vertical motion and without mutual interaction. Such a model is a relevant description of complex dynamic processes taking place when an ink film is fed through the contact zone between blanket and impression cylinders.

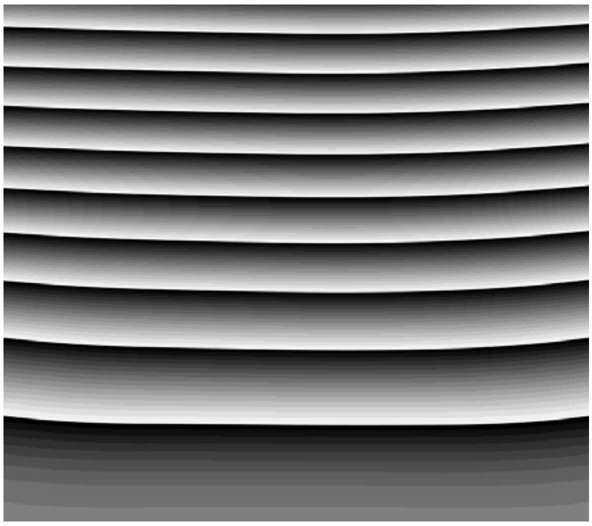

Fig. 6. **The seventh eigenmode of the deflection of the second plate**

It is shown that counterphase vibrations of the plates may be observed in the eigenmodes of this system. Such a dynamic mode of motion would be critical for the operation of the printing machine and could lead to dramatic losses in print quality. Therefore the selection of the working modes should account for such potential hazards and the discussed regimes of operation should be avoided.

References

1. **Timoshenko S. P., Goodier J. N**. Theory of elasticity. Moscow: Nauka. 1975. P. 576.
2. **Bathe K.-J.** Finite element procedures in engineering analysis. New Jersey: Prentice-Hall. 1982. P. 738.
3. **Ragulskis M., Kravčenkiene V.** Investigation of the oscillatory dynamics of the plate with the control layer of material. Ultragarsas. 2003. 1(46). P.7-11.
4. **Baker A. J.** Finite element computational fluid mechanics. Hemisphere PC. USA. 1983. P.510.
5. **Thomasset F.** Implementation of finite element methods for Navier-Stokes Equations. Springer Verlag. Germany. 1981. P.159.
6. **Ragulskis M., Palevicius A.** Theoretical and experimental studies of tubular valve dynamics. International Journal of Acoustics and Vibration. 2002. Vol. 7. No. 1. P.53-57.
7. **Huang Y., Hsu C.** Dynamic behaviour of tubes subjected to internal and external cross flows. J. Shock and Vibration. 1997. Vol.4. No.2. P.77-91.
8. **Morgan K., Peraire J.** Unstructured grid finite element methods for fluid dynamics. Reports on Progress in Physics. Institute of Physics. USA. 1998. Vol.61. No 6. P.569–638.
9. **Blawzdziewicz J., Feuillebois F.** Calculation of an effective slip in a settling suspension at a vertical wall. International Journal of Fluid Mechanics Research. 1995. Vol.22. No 1. P.164–169.

J. Ragulskienė, J. Maciulevičius, R. Maskeliūnas, L. Zubavičius

Sąveikaujančių plokštelių osciliacijos dinamikos tyrimas

Reziumė

Sudarytas dviejų plokštelių sąveikaujančių per išskirstytas spyruokles, judančias vertikalia kryptimi, modelis ir gautos baigtinio elemento matricų išraiškos. Kiekvienos plokštelės savosios formos atvaizduotos intensyvumu su permetimais parodo, kad sistemoje galimi priešfaziai plokštelių judesiai, kurie gali gerokai pakenkti atspaudo kokybei.

Pateikta spaudai 2003 05 26