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Sound insulation of technological pipelines in premises

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Introduction

It frequently occurs in practice that technological pipelines run through silent premises from one wall to another. If a certain medium (air or any liquid) is moving in pipelines, their walls propagate noise into silent premises. In addition, sound may also propagate in pipelines from the noise source, which is located quite far away, at the end of the pipeline (e.g., a ventilator or compressor). Also, a depending on the velocity of the medium moving in a pipeline, various noise appear in the medium itself.

As it was proven [1], sound in the pipeline propagates quite far away, radiating a considerable part of sound power through pipe walls.

For abatement of noise caused by such pipelines, it is recommended to apply cylindrical shells. The general characteristics of sound insulation of the cylindrical shells were described in article [2]. The efficiency of sound insulation of cylindrical shells depends greatly on the reinforcement of the shell and its contact with the pipeline itself. For that purpose it is possible to propose many ways for fastening of cylindrical shells. In this paper we shall analyze one of these methods.

Sound insulation of limited shell in rigid screen

Let us consider a model of sound-insulated arrangement (see Fig. 1) an elastic shell of the length l and radius a_k make a part of infinite rigid cylinder the axis of which coincides with the axis z of cylindrical coordinate system (Fig. 2). Radiator is an infinite cylinder with the radius $a_0 < a_k$, coaxially located with the shell. Along the radiator length l an arbitrary distribution of radial velocities $\dot{w}$ is set (Fig. 1). Inside and outside the shell there is a medium which is characterized by density ρ_b and sound distribution c_b. Side walls at clearance between cylinder and shell by $z = 0$ and $z = l$ are to be rigid; consequently, eigenvalue functions along the axis z are $\cos k_m z$, where $k_m = m\pi / l$ and $m = 0, 1, 2, ...$ – the whole number.

It is supposed that at the shell edges by $z = 0$ and $z = l$ fixation conditions being carried out are analogous [3]. Then, the radial velocity of shell oscillations $\dot{w}$ may be written as

$$\dot{w} = (\varphi, z) = \sum_{n=-\infty}^{\infty} e^{in\varphi} \sum_{m=\infty}^{\infty} V_{mn} \cos k_m z \,. \qquad (1)$$

The sound field p_2, radiated by a shell to the surrounding space, is sought by Fourier series:

$$p_2(r, \varphi, z) = \sum_{n=-\infty}^{\infty} e^{in\varphi} \int_{-\infty}^{\infty} \bar{p}_{2n}(k) H_n^{(1)}(k, r) e^{ikz} dk \,, \qquad (2)$$

where $H_n^{(1)}$ is the Hankel function of the first order n, $k_b = \omega / c_b$ is the medium wave number, $k_r = \sqrt{k_b^2 - k^2}$ is the radial wave number.

At the boundary by $r = a_k$ boundary conditions

$$\left. \frac{1}{i\rho_b \omega} \frac{\partial p_2}{\partial r} \right|_{r=a_k} = \dot{w} \,, \qquad (3)$$

expressing equality both the shell radial velocity and the medium particles must be fullfilled. Taking derivative by r to Eq. 2 and substituting the given meaning to Eq. 3 we get

$$\int_{-\infty}^{\infty} \bar{p}_{2n}(k) k_r \dot{H}_n^{(1)}(k_r a_k) e^{ikz} dk = i\rho_b \omega \sum_m V_{mn} \cos k_m z \,.$$

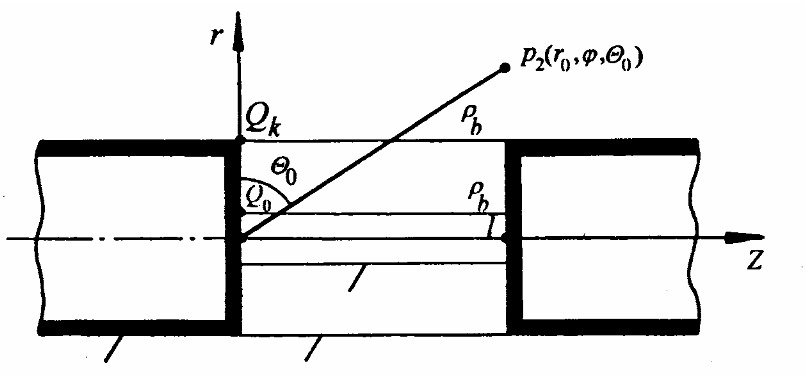

Fig. 1. **Coordinate axis and problem designation by sound insulation of limited shell**

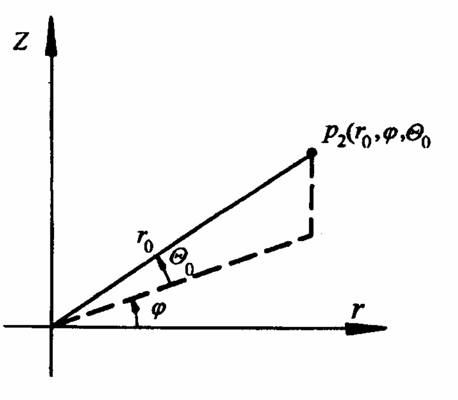

Fig. 2. **The chosen system of coordinates**

Let us apply the Fourier inverse transformation to this equation. Then

$$\overline{p}_{2n}(k)=\frac{i\rho_b\omega}{2\pi k_r \dot{H}_n(k_r a_k)}\sum_m V_{mn}\int_0^l \cos k_m z e^{-ikz}dz .$$

Integral in this expression equals to

$$\frac{ik\left[1-(-1)^m e^{-ikl}\right]}{k_m^2-k^2} .$$

Substituting the obtained expressions for $\overline{p}_{2n}(k)$ to (2), we get a sound field being radiated by the shell in a rigid screen at the known distribution of radial velocities:

$$p_2(r,\varphi,z)=\frac{\rho_b\omega}{2\pi}\sum_n e^{in\varphi}\sum_m V_{mn}$$
$$\int_{-\infty}^{\infty}\frac{k\left[(-1)^m e^{-ikl}-1\right]H_n^{(1)}(k_r r)e^{ikz}dk}{\left(k_m^2-k^2\right)k_r \dot{H}_n^{(1)}(k_r a_k)} . \qquad (4)$$

Integral I in Eq. 4 may be assessed by the crossing method [4]. Integral assessment technique of this kind is given a detail analysis in [5], therefore final result is presented.

It must be noted that the subintegral function F is featureless. By $k=k_m$ both the denominator and the numerator turn into zero. The function F by $k\to k_m$ is final. That is why by continuous deformation of integral contour to the crossing path Γ_1 special points of F are not touched and the main meaning I is defined by crossing the contour Γ_1. By $k_b r_0 >> 1$

$$p_2(r_0,\varphi,\theta_0)\approx\frac{\rho_b\omega}{\pi i}\frac{e^{ik_b r_0}}{r_0}\sum_{n=-\infty}^{\infty}\frac{e^{in\varphi}e^{-i\frac{n\pi}{2}}}{k_b\cos\theta_0 \dot{H}_n^{(1)}(k_b a_k\cos\theta_0)}\times$$
$$\sum_{m=0}^{\infty}V_{mn}\left[\frac{k(-1)^m e^{-ikl}-1}{k_m^2-k^2}\right]_{k=k_b\sin\theta_0} . \qquad (5)$$

Here r_0 is the distance from the beginning of the origin of coordinates to the observation point, θ_0 is the angle between directions r_0 and the plane $z=0$. In the expression given in square brackets $k=k_b\sin\theta_0$. Its peculiarities should be mentioned: by $\theta_0=0$ it turns into zero for all m, except $m=0$. The expression $[\ ]=il$ and sound pressure p_2 at $\theta_0=0$ is written as

$$p_2(r_0,\varphi,0)=\frac{\rho_b c_b l}{\pi}\frac{e^{ik_b r_0}}{r_0}\sum_{n=-\infty}^{\infty}\frac{V_{on}e^{in\varphi}e^{-i\frac{n\pi}{2}}}{\dot{H}_n^{(1)}(k_b a_k)} .$$

Similarly it is possible to define the sound pressure p_0 which is set up by the source in the case the shell is absent

$$p_0(r_0,\varphi,\theta_0)\approx\frac{\rho_b\omega}{\pi i}\frac{e^{ik_b r_0}}{r_0}\sum_n\frac{e^{in\varphi}e^{-i\frac{n\pi}{2}}}{k_r\cdot\dot{H}_n^{(1)}(k_r a_0)}\times$$
$$\sum_m V_{0mn}\left\{\frac{k\left[(-1)^m e^{-ikl}-1\right]}{k_m^2-k^2}\right\}_{k=k_b\sin\theta_0} \qquad (6)$$

Here V_{0mn} is the amplitude in harmonic series scanning of the radial velocity of the source $\dot{\omega}_0$ by functions $e^{-in\varphi}\cos k_m\cdot z_0$:

$$V_{0mn}=\frac{1}{\pi l}\int_0^{2\pi}e^{-in\varphi}d\varphi\int_0^l \dot{\omega}_0\cos k_m z\,dz \qquad (7)$$

In some cases the expression for p_0 gets considerably simplified, if not to scan $\dot{\omega}_0$ in series by $\cos k_m z$ and to present it in the form of the Fourier integral. Then

$$\dot{w}_0=\sum_n e^{in\varphi}\int_{-\infty}^{\infty}\tilde{\dot{w}}_{0n}(k)e^{ikz}dk ,$$

where

$$\tilde{\dot{w}}_{0n}(k)=\frac{1}{4\pi^2}\int_0^{2\pi}e^{in\varphi}d\varphi\int_0^l \dot{w}_0 e^{-ikz}dz . \qquad (8)$$

By using boundary conditions Eq. 3 at $r=a_0$ we define $\overline{p}_0(k)$ and the pressure p_0

$$p_0=i\rho_b\omega\sum e^{in\varphi}\int_{-\infty}^{\infty}\frac{\tilde{\dot{w}}_{0n}(k)H_n^{(1)}(k_r r)e^{ikz}dk}{k_r\dot{H}_n^{(1)}(k_r a_0)}$$

Applying the crossing method, expression for the sound field source is obtained:

$$p_0(r_0,\varphi,\theta_0)=\frac{2\rho_b\omega}{r_0}e^{ik_b r_0}\sum_{n=-\infty}^{\infty}e^{in\varphi}e^{-i\frac{n\pi}{2}}$$
$$\left[\frac{\tilde{\dot{w}}_{0n}(k)}{k_r\dot{H}_n^{(1)}(k_r a_0)}\right]_{k=k_b\sin\theta_0} \qquad (9)$$

In Eq. 9 summation is performed only by n. In the case of axial symmetry excitation, only the term with $n=0$ is left and the expression becomes simplified.

In order to define shell sound insulation it is necessary to find its radial velocity $\dot{w}(\varphi,z)$ or spectrum amplitudes V_{mn}. It is not easy, because the pressure p_2 included into the equation of shell movement is expressed by the radial velocity. Thus $\dot{w}$ is defined whether by solution of the Fredholm's integral equation or by infinite system of algebraic equations. Both approaches lead to complex expressions, and in space medium case we will not get appreciable results. The essence may be explained in the following way: shell movement equation in the form of impedance may be written as

$$Z_{mn}V_{mn} = p_{1mn} - p_{2mn}, \qquad (10)$$

where p_{1mn} and p_{2mn} are the pressure amplitudes of normal waves, m, n are the numbers inside and outside the shell and Z_{mn} is the impedance for this wave. It follows from Eq. 10 that at large values of the impedance Z_{mn} the sound pressure p_{1mn} will be significantly greater than p_{2mn} and outside medium reaction may be neglected. Mathematically, this condition may be written as

$$Z_{mn} >> Z_{изл},$$

where $Z_{изл} = p_{2mn} / V_{mn}$ is the radiation impedance for infinite cylindrical shell or limited shell considered in [3]:

$$Z_{изл} = \frac{i\rho_0 \omega H_n^{(1)}(k_{rm}a_k)}{k_{rm}\dot{H}_n^{(1)}(k_{rm}a_k)} \qquad (11)$$

by small sizes a_k, when $k_{rm}a_k << n$, $Z_{изл} \approx -i\rho_b \omega a_k / n$. By large sizes a_k, when $k_{rm}a_k >> n$, $Z_{изл} = \rho_b c_b / \sqrt{1 - k_m^2 / k_b^2}$, i.e., is striving for a normal medium impedance. Characteristically air impedance is not large, that is why condition $Z_{mn} >> Z_{изл}$ is fulfilled everywhere, except low frequency areas, where $Z_{mn} \sim 0$. That is why neglection of medium reaction by definition V_{mn} will not lead to big errors. However, the first approximation it is easier to take into account outside medium reaction towards shell oscillations, supposing that it does not differ significantly from medium reaction of the problem [6]. Then it is possible to use the results obtained and to write the radial speed amplitude

$$\left.\begin{aligned} V_{mn} &= \frac{V_{0mn}\dot{H}_n^{(1)}(a_k)}{\dot{H}_n^{(1)}(a_0)\gamma_{mn}}, \\ \gamma_{mn} &= 1 - \frac{Z_{mn}\pi a_k k_{rm}^2 \dot{H}_n^{(1)}}{4\rho_b \omega \dot{H}_n^{(1)}(a_0)}\left[\dot{H}_n^{(1)}\dot{H}_n^{(2)}(a_0) - \dot{H}_n^{(2)}\dot{H}_n^{(1)}(a_0)\right] \end{aligned}\right\} \qquad (12)$$

where $\dot{H}_n^{(1,2)}(a_k) = \dot{H}_n^{(1,2)}(k_{rm}a_k)$.

By substituting V_{mn} from Eq. 12 to Eq. 5, we get expression for the sound pressure radiated by the shell.

Let us define the sound insulation as

$$R = -10\lg\left|p_2(r_0, \varphi, \theta_0) / p_0(r_0, \varphi, \theta_0)\right|^2.$$

By using Eq. 5, Eq. 6 and Eq. 12 we find

$$R = -10\lg\left|\frac{\sum_{n=-\infty}^{\infty} \frac{e^{in\varphi}e^{-i\frac{n\pi}{2}}}{\dot{H}_n^{(1)}(k_b a_k \cos\theta_0)} \sum_{m=0}^{\infty} \frac{V_{0mn}\dot{H}_n^{(1)}(a_k)}{\dot{H}_n^{(1)}(a_0)\gamma_{mn}} F_m}{\sum_{n=-\infty}^{\infty} \frac{e^{in\varphi}e^{-i\frac{n\pi}{2}}}{\dot{H}_n^{(1)}(k_b a_0 \cos\theta_0)} \sum_{m=0}^{\infty} V_{0mn}F_m}\right| . \qquad (13)$$

Here

$$F_m = \frac{\left[(-1)^m e^{-ik_b l \sin\theta} - 1\right]}{\left(k_m^2 - k_b^2 \sin^2\theta_0\right)}. \qquad (14)$$

The quantity F_m is limited by $k_b \sin\theta_0 \to k_m$, $F_m \to il / 2k_m$. In the case of axial symmetric excitation, summation by m disappears, as only the member with $m = 0$ remains.

Then

$$R = -10\lg\left|\frac{H_1^{(1)}(k_b a_0 \cos\theta_0)\sum_{m=0}^{\infty} F_m \frac{V_{0m0}H_1^{(1)}(k_{rm}a_k)}{\gamma_{m0}H_1^{(1)}(k_{rm}a_0)}}{H_1^{(1)}(k_b a_k \cos\theta_0)\sum_{m=0}^{\infty} V_{0m0}F_m}\right|^2 . \qquad (15)$$

In the case of excitation by a pulsating cylinder the source of the zero order from sum by m one member $m = 0$ remains, so as $V_{0m0} = 0$, when $m \neq 0$ and $V_{000} = V_0 const$, when $m = 0$. Then

$$R_0 = 10\lg\left|\gamma_{00}\frac{H_1(k_b a_0)H_1(k_b a_k \cos\theta_0)}{H_1(k_b a_k)H_1(k_b a_0 \cos\theta_0)}\right|^2 . \qquad (16)$$

When the source is of the small radius a_0, by $k_0 a_0 << 1$ and $k_m a_0 << 1$:

$$\left.\begin{aligned} R_0 &= 10\lg|\gamma_{00}|^2 + 10\lg\left|\frac{\cos\theta_0 H_1^{(1)}(k_b a_k \cos\theta_0)}{H_1^{(1)}(k_b a_k)}\right|^2 = R_1 + R_2, \\ \gamma_{00} &= 1 + i\frac{\pi m_0 c_0^2}{2\rho_b c_b a_k}\left(k_0^2 a_k^2 - 1\right)H_1^{(1)}(k_b a_k)J_1(k_b a_k). \end{aligned}\right\} \qquad (17)$$

Thus, sound insulation is composed of two parts – R_1 and R_2. R_1 represents the already known sound insulation quantity of an infinite shell [7], producing radial oscillations. Zeros of R_1 will be on air resonances inside the shell when $J_1(k_b a_k) = 0$ and on the shell resonance when $k_0 a_k = 1$, e.g., at the frequency $f_0 = c_0 / 2\pi a_k$, where c_0 is the propagation velocity of a longitudinal wave in the plate. R_2 is dependent on the sound insulation angle θ_0.

Analysis of the results obtained

The formulas obtained for calculation of insulation of cylindrical shells is somewhat complex. However, if the high precision of calculation is not required, we can simplify them by introducing certain corrections.

It should be noted that R_2 includes ratio of the function $xH_1(x)$, so that it could be written: $R_2 =$ $= 10\lg\left|k_b a_k \cos\theta_0 H_1(k_b a_k \cos\theta_0)\right|^2 - 10\lg\left|k_b a_k H_1(k_b a_k)\right|^2$

Diagram of the function $F = 10\lg|xH_1(x)|^2$ is given in Fig.3.

At small values $x \to 0 F \to 10\lg\left(4/\pi^2\right) = -3{,}9\text{dB} \approx -4\text{dB}$.

At $x >> 1$, $F \approx 10\lg(0{,}64x) = (10\lg x - 2)\text{dB}$.

By means of the diagram F it is possible to calculate correction of R_2 to sound insulation of an infinite shell. For this we may write: $k_b a_k = f / f_0$, where $f_0 = c_b / 2\pi a_k$ is the frequency at which one wave length fits in air on the shell circumference. At the chosen frequencies f and at the angle θ_0 values $\alpha = f / f_0$ and $\cos\theta_0$ are located. By means of the diagram values $R_{21} = F(\alpha\cos\theta_0)$ and $R_{22} = F(\alpha)$ and later $R_2 = R_{21} - R_{22}$.

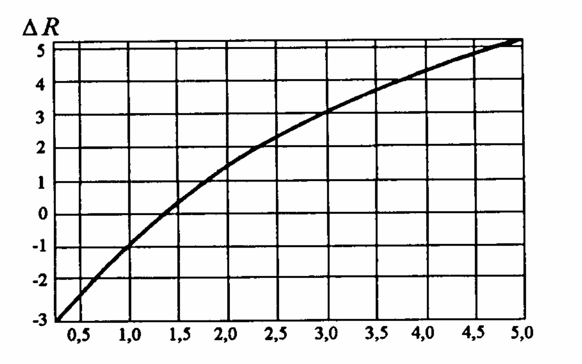

Fig.3. **Frequency characteristic of limited shell sound insulation correction**

It follows from Eq. 17 that at $\theta_0 = 0$, $R_2 = 0$, e. g. R coincides with the sound insulation of the shell. Under different angles the sound insulation is lower, as F is monotonously increasing function, and $\cos\theta_0 \le 1$. In consequence of this $R_2 = F(k_b a_b \cos\theta_0) - F(k_b a_b) \le 0$, where R_2 increases by absolute value with the increase of the angle θ_0.

In case of excitation by a fixed source not directed at the angle φ, the radial velocity $\dot{\omega}_0 = V_0 \delta(z - z_0)$ (point source located at the point $z - z_0$). The amplitude

$$V_{0m0} = 2V_0 \cos k_m z_0 / l$$

$$V_{m0} = \frac{2V_0 \cos k_m z_0 H_1(k_{rm} a_k)}{l \gamma_{m0} H_1(k_{rm} a_k)}.$$

The sound pressure produced by the shell is given by

$$p_2 = \frac{2\rho_b \omega V_0 \sum\limits_{m=0}^{\infty} \dfrac{\cos k_m z_0 H_1(k_{rm} a_k)}{\gamma_{m0} H_1^{(1)}(k_{rm} a_0)}}{i\pi l \cos\theta_0 H_1^{(1)}(k_b a_k \cos\theta_0)} \frac{e^{ik_b r_0}}{r_0}.$$

The sound pressure produced by the source may be defined by Eq. 9

$$p_0 = \frac{V_0 \rho_b \omega e^{-ik_b z_0 \sin\theta_0}}{\pi k_b \cos\theta_0 H_1(k_b a_0 \cos\theta_0)} \frac{e^{ik_b r_0}}{r_0}.$$

The sound insulation is given by

$$R = -10\lg\left|\frac{p_2}{p_0}\right| =$$

$$-10\lg\left|\frac{2k_b \sin\theta_0 H_1(k_b a_0 \cos\theta_0)}{l H_1(k_b a_k \cos\theta_0)} \sum_{m=0}^{\infty} \frac{\cos k_m z_0 H_1(k_{rm} a_k)}{\gamma_{m0} H_1(k_{rm} a_0)} F_m\right|^2 \tag{18}$$

$$F_m = \left[\frac{(-1)^m e^{-ikl} - 1}{k_m^2 - k^2}\right]_{k = k_b \sin\theta_0}.$$

For the source of a small radius a_0, when $k_{rm} a_0 << 1$ and $k_b a_0 << 1$,

$$R = -10\lg\left|\frac{2\sin\theta_0}{l \cos\theta_0 H_1(k_b a_k \cos\theta_0)} \sum_{m=0}^{\infty} \frac{\cos k_m z_0 k_{rm} H_1^{(1)}(k_b a_k \cos\theta_0)_k}{\gamma_{m0}} F_m\right|^2 \tag{19}$$

It should be noted that

$k \to k_b \sin\theta \to k_m \dfrac{2k}{il} F_m \to 1$. At $\theta_0 = 0$, e. g., in the plane $m = 0$, from sum by m only one member with $m = 0$ remains. Then

$$R = 10\lg\left|\frac{\gamma_{00}}{2}\right|^2 = 10\lg|\gamma_{00}|^2 - 6\ \text{(dB)} \tag{20}$$

Addend in (20) is already known sound insulation of an infinite shell, producing radial oscillations. Total sound insulation by concentrated excitation on axis when $\theta_0 = 0$ at 6 dB is lower than the sound insulation of an infinite shell, but it remains rather high.

Conclusions

In the case under consideration the cylindrical noise source, placed between the two rigid screens, may be insulated by a cylindrical shell, the efficiency of noise reduction of which is evaluated theoretically. When identifying the sound insulation of the cylindrical shell, many factors that have an effect on the determination of insulation properties have been evaluated. Here it was established that sound insulation consists of two parts. The first R_1 represents the indicated value of sound insulation of an infinite shell. The second part R_2 contains the dependence of sound insulation on the angle between the direction Θ and the plane $z = 0$.

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Technologinių vamzdynų garso izoliacija patalpose

Reziumė

Straipsnyje pateikiama teorija, kaip galima sumažinti triukšmą, sklindantį patalpoje nuo vamzdynų sienelių. Patalpoje išdėstytų vamzdynų sukeliamą triukšmą galima izoliuoti cilindriniais gaubtais, kurių parametrai apskaičiuojami naudojantis pateikta teorija. Šiame straipsnyje nurodomi veiksniai, kurie turi įtakos garso izoliavimo cilindriniais gaubtais efektyvumui. Nustatyta, kad tyrinėjamo cilindrinio gaubto garso izoliacija susideda iš dviejų dalių - R_1+R_2. Pirmoji dalis (R_1) parodo begalinio cilindrinio kevalo garso izoliacijos dydį, antroji (R_2) - garso izoliacijos priklausomybę nuo kampo Θ tarp krypties rodiklio r_0 ir plokštumos z=0. Straipsnyje pateikiama supaprastinta teorija, kaip, darant tam tikras pataisas, preciziškai tiksliai apskaičiuoti garso izoliaciją.

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