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Analysis of undulation film dynamics evaluating boundary interaction with oscillating fluid

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Introduction

The dynamics of an undulation plane on the surface of an oscillating fluid is a complex non-linear problem. Direct experimental analysis of the system is also complicated due to relatively small dimensions of the system and small amplitude of travelling waves occurring in the undulation plane which itself serves as an transportable organ for conveyance of small delicate elements in sterile environments. Application of whole field time average holography could be of practical interest in determining optimal parameters of the system, but the interpretation of the measurement results would be extremely difficult due to the complex interactions between the film and the oscillating fluid.

An attempt to build a mathematical model of the interacting elements is undertaken in this paper. Application of virtual environments for building numerical interferograms is the first step in visualization and interpretation of complex experimental results.

The dynamics of the layer of fluid is analyzed first. The finite element with three degrees of freedom per node (the horizontal displacements of the layer of the fluid and the vertical displacement of the surface of the fluid) is developed for the analysis of the described system.

Further, the film layer is analyzed as a structural element by taking into account the interaction with the layer of fluid. The finite element with five degrees of freedom per node (the deflection and the two rotations of the plate and the horizontal displacements of the fluid) is developed.

The experimental analysis of a vibration based transporter is presented in Fig.1. The specific feature of this assembly is characterized by the interaction of undulatory plate and a layer of liquid. The effective excitation of coupled vibrations requires knowledge of the resonant shapes and frequencies of the coupled system what is the primary goal of this paper.

Numerical model of the layer of the fluid

The nodal variables are the displacement of the fluid in the direction of the x axis u, the displacement of the fluid in the direction of the y axis v and the displacement of the surface of the fluid in the direction of the z axis w.

It is assumed that the vertical displacement of the fluid w_f is zero at the bottom wall [1, 2]. The assumption of linear variation of w_f leads to the following approximation:

$$w_f(x, y, z, t) = w(x, y, t)\frac{z}{H}, \qquad (1)$$

where H is the thickness of the layer of fluid.

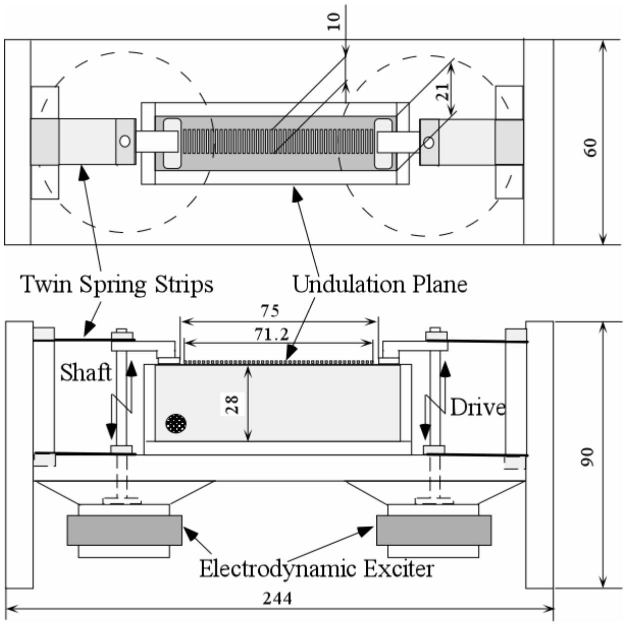


Fig. 1. **The principal scheme of the transporter consisting of coupled plate and liquid layer**

Then the volumetric strain in the fluid is [3, 4]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{w}{H} = [B]\{\delta\}, \qquad (2)$$

where:

$$[B] = \begin{bmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_1}{\partial y} & \frac{N_1}{H} & \ldots \end{bmatrix}, \qquad (3)$$

and $\{\delta\}$ is the vector of generalised displacements.

The rotation is expressed as:

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = [\tilde{B}]\{\delta\}, \qquad (4)$$

where:

$$[\tilde{B}] = \begin{bmatrix} -\frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & 0 & \ldots \end{bmatrix}. \qquad (5)$$

The displacement of the surface of the fluid is expressed as:

$$w = [\overline{N}]\{\delta\}, \qquad (6)$$

where:

$$[\overline{N}] = [0 \quad 0 \quad N_1 \quad ...]. \tag{7}$$

The stiffness matrix takes the form:

$$[K] = \iint \begin{pmatrix} [B]^T \rho c^2 H [B] + [\widetilde{B}]^T \lambda H [\widetilde{B}] + \\ + [\overline{N}]^T \rho g [\overline{N}] \end{pmatrix} dxdy, \tag{8}$$

where ρ is the density of the fluid, c is the velocity of sound, λ is the penalty parameter for the introduction of the condition of irrotationality, g is the acceleration of gravity of the Earth.

The velocities in the fluid are $\begin{bmatrix} \frac{\partial u}{\partial t} & \frac{\partial v}{\partial t} & \frac{\partial w}{\partial t}\frac{z}{H} \end{bmatrix}^T$.

Assuming that:

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = [N]\{\delta\}, \tag{9}$$

where

$$[N] = \begin{bmatrix} N_1 & 0 & 0 & ... \\ 0 & N_1 & 0 & ... \end{bmatrix}, \tag{10}$$

and by taking into account that:

$$\int_0^H \left(\frac{z}{H}\right)^2 dz = \frac{H}{3}, \tag{11}$$

the kinetic energy of the fluid T takes the form:

$$T = \frac{1}{2}\left\{\frac{d\{\delta\}}{dt}\right\}^T \iint \begin{pmatrix} [N]^T \rho H \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} [N] + \\ + [\overline{N}]^T \rho \frac{H}{3} [\overline{N}] \end{pmatrix} dxdy \left\{\frac{d\{\delta\}}{dt}\right\}. \tag{12}$$

The mass matrix takes the form:

$$[M] = \iint \begin{pmatrix} [N]^T \rho H \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} [N] + \\ + [\overline{N}]^T \rho \frac{H}{3} [\overline{N}] \end{pmatrix} dxdy. \tag{13}$$

Numerical investigation of the layer of the fluid

A rectangular domain is analyzed and the eigenmodes are calculated.

For the representation of results the two plots for a single eigenmode are produced:

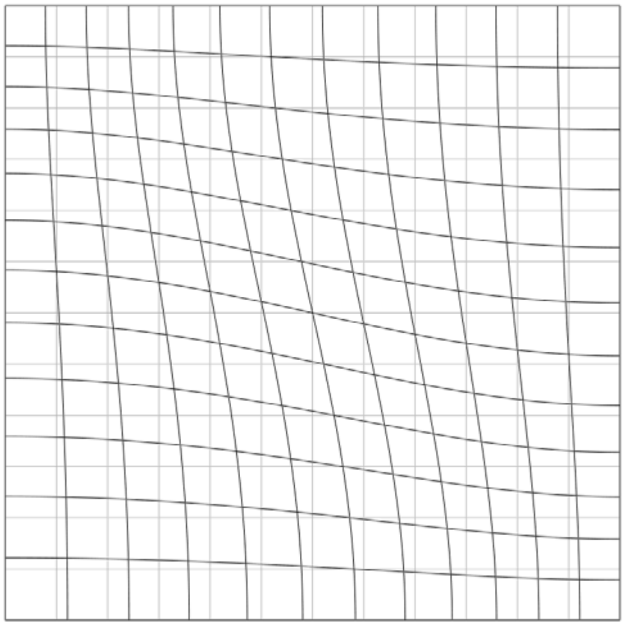
1) the displacements of the fluid u and v;
2) the displacement of the surface of the fluid w using the cavalier projection [5, 6] with the angle $\pi/4$.

The third eigenmode is shown in Fig. 2.

Model of the plate interacting with the fluid film

The model of the analysed system is presented in Fig.3. The element developed here is a modification of the plate element presented in [2].

The nodal variables are the deflection of the plate w, the rotation of the plate about the x axis Θ_x, the rotation of the plate about the y axis Θ_y, the displacement of the fluid in the direction of the x axis u and the displacement of the fluid in the direction of the y axis v. The displacement of the ideal fluid normal to the boundary of the region covered by the plate is assumed equal to zero.



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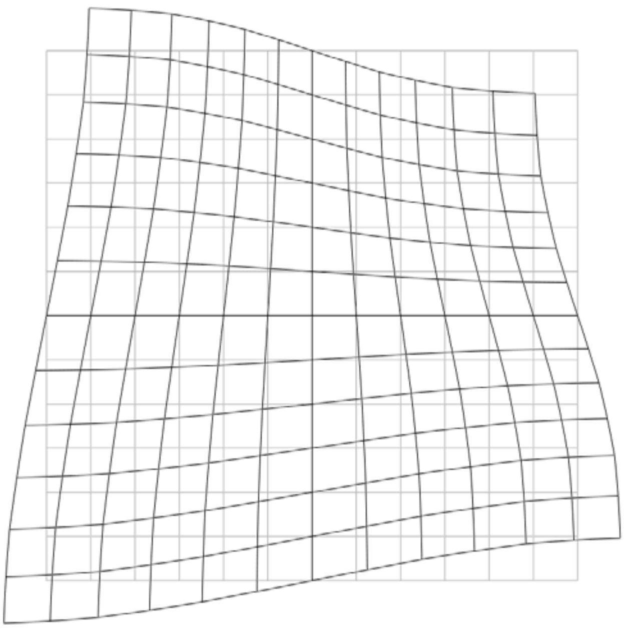


b)

Fig. 2. **The third eigenmode: a) the plane motion of the fluid, b) the motion of the surface of the fluid**

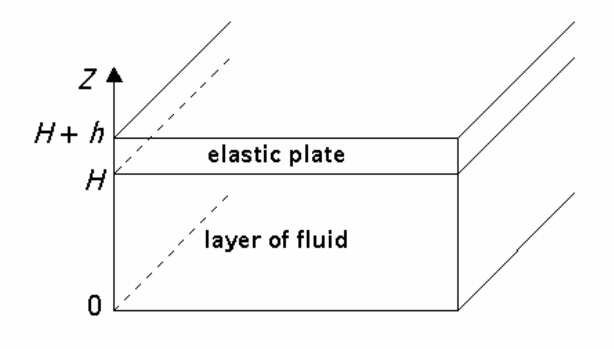


Fig. 3. **The model of the plate interacting with the fluid film**

It is assumed that the vertical displacement of the fluid w_f is zero at the bottom wall and coincides with the displacement of the plate at the top surface of the fluid. The assumption of linear variation of w_f leads to the following approximation:

$$w_f(x,y,z,t)=w(x,y,t)\frac{z}{H}, \quad (14)$$

where H is the thickness of the fluid film.

Then the volumetric strain in the fluid is:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{w}{H}=[B_f]\{\delta\}, \quad (15)$$

where:

$$[B_f]=\begin{bmatrix}\frac{N_1}{H} & 0 & 0 & \frac{\partial N_1}{\partial x} & \frac{\partial N_1}{\partial y} & \cdots\end{bmatrix}. \quad (16)$$

The rotation is expressed as:

$$\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}=[\tilde{B}]\{\delta\}, \quad (17)$$

where:

$$[\tilde{B}]=\begin{bmatrix}0 & 0 & 0 & -\frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & \cdots\end{bmatrix}, \quad (18)$$

The stiffness matrix takes the form:

$$[K]=\iint\begin{pmatrix}[B]^T[D][B]+[\overline{B}]^T[\overline{D}][\overline{B}]+\\+[B_f]^T\rho_f c^2 H[B_f]+[\tilde{B}]^T\lambda H[\tilde{B}]\end{pmatrix}dxdy, \quad (19)$$

where:

$$[D]=\frac{h^3}{12}\begin{bmatrix}\frac{E}{1-\nu^2} & \frac{E\nu}{1-\nu^2} & 0\\ \frac{E\nu}{1-\nu^2} & \frac{E}{1-\nu^2} & 0\\ 0 & 0 & \frac{E}{2(1+\nu)}\end{bmatrix},$$

$$[\overline{D}]=\frac{Eh}{2(1+\nu)k_s}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},$$

$$[B]=\begin{bmatrix}0 & 0 & \frac{\partial N_1}{\partial x} & 0 & 0 & \cdots\\ 0 & -\frac{\partial N_1}{\partial y} & 0 & 0 & 0 & \cdots\\ 0 & -\frac{\partial N_1}{\partial x} & \frac{\partial N_1}{\partial y} & 0 & 0 & \cdots\end{bmatrix},$$

$$[\overline{B}]=\begin{bmatrix}\frac{\partial N_1}{\partial y} & -N_1 & 0 & 0 & 0 & \cdots\\ \frac{\partial N_1}{\partial x} & 0 & N_1 & 0 & 0 & \cdots\end{bmatrix}, \quad (20)$$

and E is the modulus of elasticity, ν is the Poisson's ratio, h – the thickness of the plate, k_s is the shear correction factor assumed equal to 1.2, ρ_f is the density of the fluid, λ is the penalty parameter for the introduction of the condition of irrotationality.

The velocities in the fluid are $\begin{bmatrix}\frac{\partial u}{\partial t} & \frac{\partial v}{\partial t} & \frac{\partial w}{\partial t}\frac{z}{H}\end{bmatrix}^T$.

Assuming that:

$$\begin{Bmatrix}u\\ v\\ w\end{Bmatrix}=[N_f]\{\delta\}, \quad (21)$$

where:

$$[N_f]=\begin{bmatrix}0 & 0 & 0 & N_1 & 0 & \cdots\\ 0 & 0 & 0 & 0 & N_1 & \cdots\\ N_1 & 0 & 0 & 0 & 0 & \cdots\end{bmatrix}, \quad (22)$$

and by taking into account that:

$$\int_0^H \rho_f\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & \left(\frac{z}{H}\right)^2\end{bmatrix}dz=\rho_f\begin{bmatrix}H & 0 & 0\\ 0 & H & 0\\ 0 & 0 & \frac{H}{3}\end{bmatrix}, \quad (23)$$

the kinetic energy of the fluid T takes the form:

$$T=\frac{1}{2}\left\{\frac{d\{\delta\}}{dt}\right\}^T\cdot$$

$$\cdot\iint[N_f]^T\rho_f\begin{bmatrix}H & 0 & 0\\ 0 & H & 0\\ 0 & 0 & \frac{H}{3}\end{bmatrix}[N_f]dxdy\left\{\frac{d\{\delta\}}{dt}\right\}. \quad (24)$$

The mass matrix takes the form:

$$[M]=\iint\begin{pmatrix}[N]^T\begin{bmatrix}\rho h & 0 & 0\\ 0 & \frac{\rho h^3}{12} & 0\\ 0 & 0 & \frac{\rho h^3}{12}\end{bmatrix}[N]+\\ +[N_f]^T\rho_f\begin{bmatrix}H & 0 & 0\\ 0 & H & 0\\ 0 & 0 & \frac{H}{3}\end{bmatrix}[N_f]\end{pmatrix}dxdy, \quad (25)$$

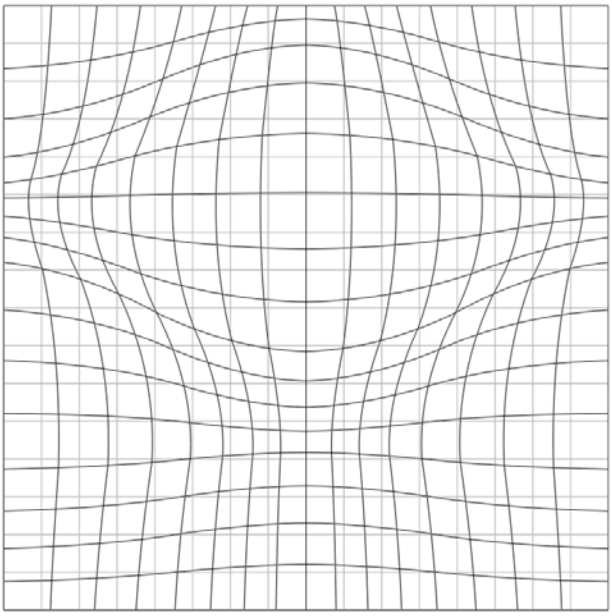
where:

$$[N] = \begin{bmatrix} N_1 & 0 & 0 & 0 & 0 & \cdots \\ 0 & N_1 & 0 & 0 & 0 & \cdots \\ 0 & 0 & N_1 & 0 & 0 & \cdots \end{bmatrix}, \qquad (26)$$

and ρ is the density of the material of the plate.

Numerical investigation of the plate interacting with the fluid film

The analyzed object is a rectangular elastic plate interacting with the fluid film with a fastened edge of the plate.



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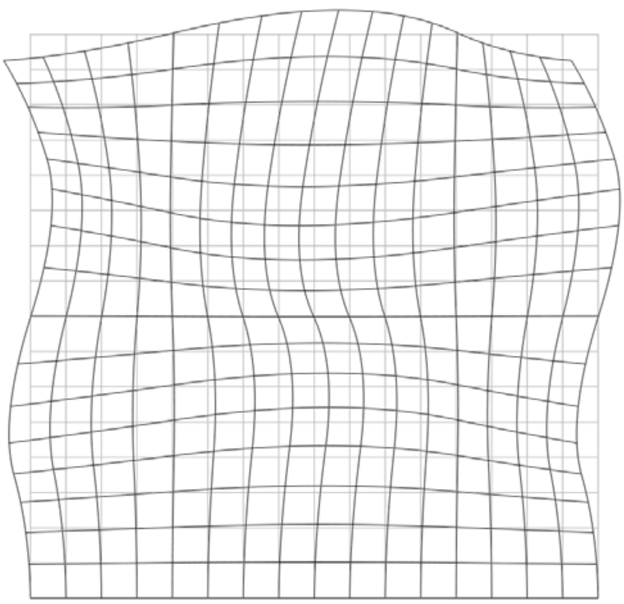


b)

Fig. 4. **The tenth eigenmode: a) the plane motion of the fluid, b) the motion of the plate**

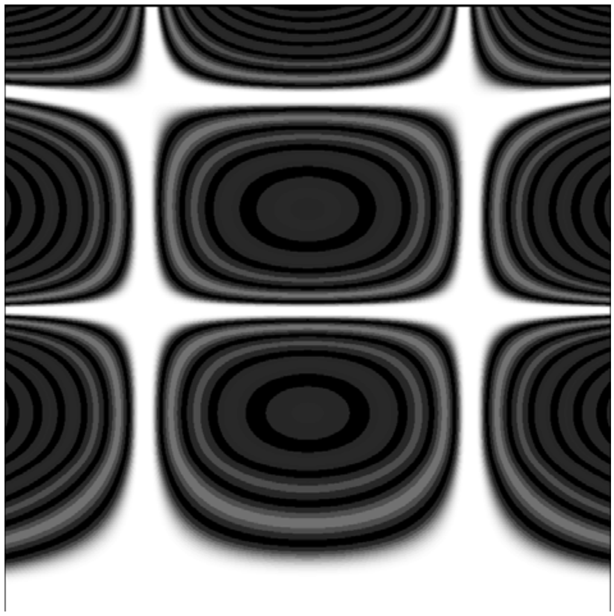
For the representation of results the two plots for a single eigenmode are produced:

1) the displacements of the fluid u and v;
2) the displacement of the plate w using the cavalier projection [5, 6] with an angle $\pi/4$.

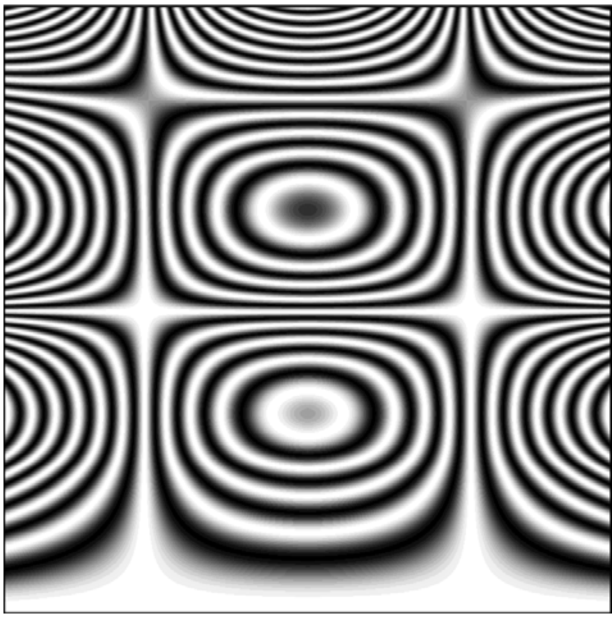
This representation enables to see the correspondence of the motions of the fluid and the plate in the eigenmode.

The tenth eigenmode is shown in Fig. 4.

The holographic images of the tenth eigenmode of the plate are shown in Fig. 5.



a)



b)

Fig. 5. **The holographic images of the tenth eigenmode of the plate: a) the time averaged image, b) the stroboscopic image**

So, the procedure of experimental and numerical analysis of the coupled system consists of the following stages:

a) determination of the holographic image of the analyzed eigenmode of the plate;
b) performance of the numerical calculations for the first eigenmodes as described previously;
c) determination of the correspondence of the experimental eigenmode to the calculated one;
d) analysis of the numerical results of the vibrations of the fluid for this eigenmode.

Conclusions

The eigenmodes of the layer of the fluid are determined. The obtained results provide the basis for the investigation of vibrational devices incorporating the layer of the fluid.

The eigenmodes of the plate interacting with the fluid film are determined. The results of calculations describe the conditions when the motions of the plate and of the fluid film correspond to each another in appropriate eigenmodes.

The procedure of experimental and numerical analysis of the system incorporating the determination of the holographic image of the plate is proposed.

The obtained results provide the basis for the investigation of hybrid vibrational transportation devices.

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Banguojančios plėvelės dinamikos analizė įvertinant paviršinę sąveiką su virpančiu skysčiu

Reziumė

Sudarytas skysčio sluoksnio baigtinių elementų modelis. Sava forma vaizduojama dviem piešiniais: skysčio judesio plokštumoje ir skysčio paviršiaus judesio vertikaliąja kryptimi laisvoje projekcijoje.

Sudarytas plokštelės tipo transportavimo organo, sąveikaujančio su skysčio plėvele, matematinis modelis. Gautos pirmosios savos formos parodo, kad plokštelės ir skysčio judesiai yra tarpusavyje suderinti. Pasiūlytas formos vaizdavimas dviem piešiniais: skysčio judesio plokštumoje ir plokštelės laisvoje projekcijoje. Sistemai tirti eksperimentiniu ir skaitmeniniu būdais pasiūlyta plokštelės holografiniu vaizdu paremta metodika.

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