Investigation of the elements of the printing device

K. Ragulskis[1], E. Kibirkštis[1], A. Bubulis[1], J. Ragulskienė[2], R. Maskeliūnas[3]

[1]*Kaunas University of Technology*

[2]*Kaunas College*

[3]*Vilnius Gediminas Technical University*

Introduction

Analysis of the dynamics of a contact pair of impression and blanket cylinders in a printing device is a complicated and multi-scale problem.

The dynamics of a plane circular composite beam is analyzed in this paper. The beam is considered to have two external layers of the beam type and an internal layer of the type of an elastic body. The finite element of this beam is obtained from the contributions of the three sub-elements: two of them (the lower and the upper ones) of beam type and the third (the internal one) of the type of an elastic layer. The resulting finite element has six degrees of freedom per node. The eigenmodes are calculated and it is evident that the multiple eigenmodes enable the excitation of wave motion in this system. The analysis is based on [1,2].

The steady state incompressible viscous flow in a narrow gap described by the Reynolds equation [3] is analyzed. From the solution the shear strain rates may be determined. The averaged in the thickness direction intensity of the shear strain rates may be obtained in the investigations of photoelastic type. The quantity proportional to it is obtained numerically and represented by intensity mapping. The described analysis is based on fundamental results in [3, 4, 5], but requires substantial adaptation for the development of adequate mathematical model describing the complex interactions.

Numerical model of the sub-element of the beam type

The sub-element is a modification of the beam element presented in [1].

The nodal variables are the transverse deflection of the layer of beam type v_{12}, the tangential deflection of the lower surface of the layer of beam type u_1 and the tangential deflection of the upper surface of the layer of beam type u_2.

Then:

$$u_1 = u + b\Theta,\quad u_2 = u - b\Theta,\quad u = \frac{u_1 + u_2}{2},\quad \Theta = \frac{u_1 - u_2}{2b}, \tag{1}$$

where u is the tangential displacement of the middle surface of the layer of beam type, Θ is the angle of rotation of the normal to the middle surface of the layer of beam type, b is the half thickness of the layer of beam type. This gives the following expression:

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{Bmatrix} r_x \\ r_y \end{Bmatrix} \frac{u_1 + u_2}{2} + \begin{Bmatrix} t_x \\ t_y \end{Bmatrix} v_{12}, \tag{2}$$

where r_x and r_y are the components of the unit tangential vector of the composite beam, t_x and t_y are the components of the unit normal vector of the composite beam, u and v are the displacements of the middle surface of the layer of beam type in the directions of the x and y axes of the orthogonal Cartesian system of co-ordinates. Here:

$$\begin{Bmatrix} t_x \\ t_y \end{Bmatrix} = \begin{Bmatrix} -r_y \\ r_x \end{Bmatrix}. \tag{3}$$

Then:

$$\begin{bmatrix} [N_u] \\ [N_v] \end{bmatrix} = \begin{bmatrix} x_{,s} & y_{,s} \\ -y_{,s} & x_{,s} \end{bmatrix} \cdot \left[\begin{Bmatrix} t_x \\ t_y \end{Bmatrix}_1 N_1 \quad \begin{Bmatrix} r_x \\ r_y \end{Bmatrix}_1 \frac{1}{2} N_1 \quad \begin{Bmatrix} r_x \\ r_y \end{Bmatrix}_1 \frac{1}{2} N_1 \quad \ldots \right], \tag{4}$$

where N_1, … are the shape functions of the finite element. The subscript 1, … after the {} denotes that the corresponding quantities are taken at node 1, … ; s denotes the longitudinal coordinate of the axis of the composite beam; comma denotes differentiation with respect to the quantity following after it; $[N_u]$ and $[N_v]$ are the row vectors for interpolation of the tangential and normal displacements of the middle surface of the layer of beam type. Also:

$$[N_\Theta] = \left[0 \quad \frac{1}{2b} N_1 \quad -\frac{1}{2b} N_1 \quad \ldots \right], \tag{5}$$

where $[N_\Theta]$ is the row vector for interpolation of the angular rotation of the normal to the middle surface of the layer of beam type.

So, the mass matrix takes the form:

$$[M] = \int \left(\begin{matrix} [N_u]^T \rho F [N_u] + [N_v]^T \rho F [N_v] + \\ + [N_\Theta]^T \rho I [N_\Theta] \end{matrix} \right) ds, \tag{6}$$

where:

$$\rho F = \rho 4ab,\quad \rho I = \rho \frac{4ab^3}{3}, \tag{7}$$

and ρ is the density of the material of the layer of beam type, a is the half width of the composite beam.

The stiffness matrix takes the form:

$$[K]=\int\left(\begin{matrix}[N_u']^T EF[N_u']+[N_\Theta']^T EI[N_\Theta']+\\ +([N_\Theta]-[N_v'])^T S([N_\Theta]-[N_v'])\end{matrix}\right)ds, \quad (8)$$

where:

$$EF=E4ab,$$
$$EI=E\frac{4ab^3}{3}, \quad (9)$$
$$S=\frac{G4ab}{k_s},$$

and E is the modulus of elasticity of the layer of beam type, G is the shear modulus of the layer of beam type, k_s is the shear correction factor assumed equal to 1.2, the prime denotes differentiation with respect to the longitudinal axis of the composite beam.

Numerical model of the sub-element of the elastic type

The sub-element of the elastic type is a modification of the plain stress element presented in [2].

The nodal variables are the tangential displacement of the lower surface of the elastic layer u_1, the transverse displacement of the lower surface of the elastic layer v_1, the tangential displacement of the upper surface of the elastic layer u_2, the transverse displacement of the upper surface of the elastic layer v_2.

Then:

$$\begin{Bmatrix}u\\v\end{Bmatrix}=\begin{Bmatrix}r_x\\r_y\end{Bmatrix}u_i+\begin{Bmatrix}t_x\\t_y\end{Bmatrix}v_i, \quad (10)$$

where u and v are the displacements in the directions of the x and y axes of the orthogonal Cartesian system of co-ordinates for the lower surface of the elastic layer when $i=1$ and for the upper surface of the elastic layer when $i=2$.

Then:

$$\left[\overline{N_1}\right]=\begin{bmatrix}[N_{1u}]\\ [N_{1v}]\end{bmatrix}=\begin{bmatrix}x_{,s} & y_{,s}\\ -y_{,s} & x_{,s}\end{bmatrix}\cdot$$
$$\cdot\begin{bmatrix}\begin{Bmatrix}r_x\\r_y\end{Bmatrix}_1 N_1 & \begin{Bmatrix}t_x\\t_y\end{Bmatrix}_1 N_1 & \begin{matrix}0 & 0 & \dots\\ 0 & 0 & \dots\end{matrix}\end{bmatrix},$$
$$\left[\overline{N_2}\right]=\begin{bmatrix}[N_{2u}]\\ [N_{2v}]\end{bmatrix}=\begin{bmatrix}x_{,s} & y_{,s}\\ -y_{,s} & x_{,s}\end{bmatrix}\cdot \quad (11)$$
$$\cdot\begin{bmatrix}\begin{matrix}0 & 0\\ 0 & 0\end{matrix} & \begin{Bmatrix}r_x\\r_y\end{Bmatrix}_1 N_1 & \begin{Bmatrix}t_x\\t_y\end{Bmatrix}_1 N_1 & \begin{matrix}\dots\\ \dots\end{matrix}\end{bmatrix},$$

where $[N_{1u}]$ and $[N_{1v}]$ are the row vectors for interpolation of the tangential and normal displacements of the lower surface of the elastic layer, $[N_{2u}]$ and $[N_{2v}]$ are the row vectors for interpolation of the tangential and normal displacements of the upper surface of the elastic layer.

The interpolation of the displacements in the transverse direction of the elastic layer is given by:

$$\frac{H-t}{H}\begin{Bmatrix}u_1\\v_1\end{Bmatrix}+\frac{t}{H}\begin{Bmatrix}u_2\\v_2\end{Bmatrix}, \quad (12)$$

where $H=2b$ is the thickness of the elastic layer, $t\in[0, H]$ is the transverse co-ordinate of the elastic layer.

By taking into account that:

$$\int_0^H \frac{H-t}{H}\frac{t}{H}dt=\frac{H}{6},$$
$$\int_0^H\left(\frac{H-t}{H}\right)^2 dt=\int_0^H\left(\frac{t}{H}\right)^2 dt=\frac{H}{3}, \quad (13)$$

the mass matrix takes the form:

$$[M]=\int\left(\begin{matrix}\left[\overline{N_1}\right]^T(2a\rho)\frac{2b}{6}\left[\overline{N_2}\right]+\\ +\left[\overline{N_2}\right]^T(2a\rho)\frac{2b}{6}\left[\overline{N_1}\right]+\\ +\left[\overline{N_1}\right]^T(2a\rho)\frac{2b}{3}\left[\overline{N_1}\right]+\\ +\left[\overline{N_2}\right]^T(2a\rho)\frac{2b}{3}\left[\overline{N_2}\right]\end{matrix}\right)ds, \quad (14)$$

where ρ is the density of the material of the elastic layer.

The expression for the strains in the elastic layer is given by:

$$\frac{H-t}{H}\begin{Bmatrix}u_{1r}\\0\\v_{1r}\end{Bmatrix}+\frac{t}{H}\begin{Bmatrix}u_{2r}\\0\\v_{2r}\end{Bmatrix}+\frac{1}{H}\begin{Bmatrix}0\\v_2-v_1\\u_2-u_1\end{Bmatrix}, \quad (15)$$

where the subscript r denotes differentiation in the longitudinal direction of the composite beam. So, the following matrixes are introduced:

$$[B]=\begin{bmatrix}[0]\\ [N_{2v}]-[N_{1v}]\\ [N_{2u}]-[N_{1u}]\end{bmatrix},$$
$$[B_1]=\begin{bmatrix}[N_{1u}']\\ [0]\\ [N_{1v}']\end{bmatrix}, \quad (16)$$
$$[B_2]=\begin{bmatrix}[N_{2u}']\\ [0]\\ [N_{2v}']\end{bmatrix}.$$

By taking into account that:

$$\int_0^H\frac{H-t}{H}\frac{1}{H}dt=\int_0^H\frac{t}{H}\frac{1}{H}dt=\frac{1}{2},$$
$$\int_0^H\left(\frac{1}{H}\right)^2 dt=\frac{1}{H}, \quad (17)$$

the stiffness matrix takes the form:

$$[K]=\int\begin{pmatrix}[B]^T[D]\frac{1}{2}[B_1]+[B_1]^T[D]\frac{1}{2}[B]+\\+[B]^T[D]\frac{1}{2}[B_2]+[B_2]^T[D]\frac{1}{2}[B]+\\+[B_1]^T[D]\frac{2b}{6}[B_2]+[B_2]^T[D]\frac{2b}{6}[B_1]+\\+[B_1]^T[D]\frac{2b}{3}[B_1]+[B_2]^T[D]\frac{2b}{3}[B_2]+\\+[B]^T[D]\frac{1}{2b}[B]\end{pmatrix}ds, \quad (18)$$

where:

$$[D]=2a\begin{bmatrix}\frac{E}{1-\nu^2} & \frac{E\nu}{1-\nu^2} & 0\\ \frac{E\nu}{1-\nu^2} & \frac{E}{1-\nu^2} & 0\\ 0 & 0 & \frac{E}{2(1+\nu)k_s}\end{bmatrix}, \quad (19)$$

and E is the modulus of elasticity of the elastic layer, ν is the Poisson's ratio of the elastic layer, k_s is the shear correction factor assumed equal to 1.2.

Numerical model of the element of the composite beam

The nodal variables are (Fig. 1) the transverse displacement of the lower layer of beam type v_{12}, the tangential displacement of the lower surface of the lower layer of beam type u_1, the tangential displacement of the upper surface of the lower layer of beam type u_2, the transverse displacement of the upper layer of beam type v_{34}, the tangential displacement of the lower surface of the upper layer of beam type u_3, the tangential displacement of the upper surface of the upper layer of beam type u_4.

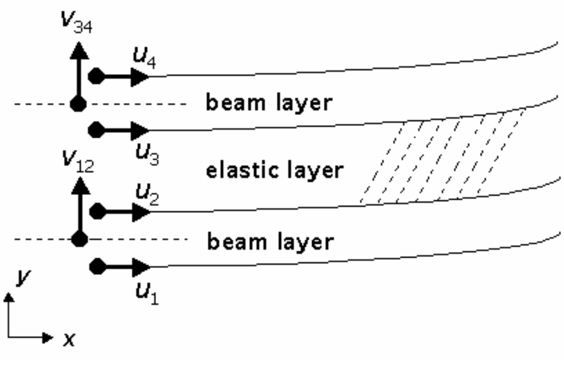


Fig. 1. **The nodal variables of the composite beam**

The finite element is constructed by summation from the previously described sub-elements by taking the correspondence of the degrees of freedom into account.

Numerical investigation of the circular elastic system

The circular composite beam is analyzed. The eighth eigenmode is shown in Fig. 2.

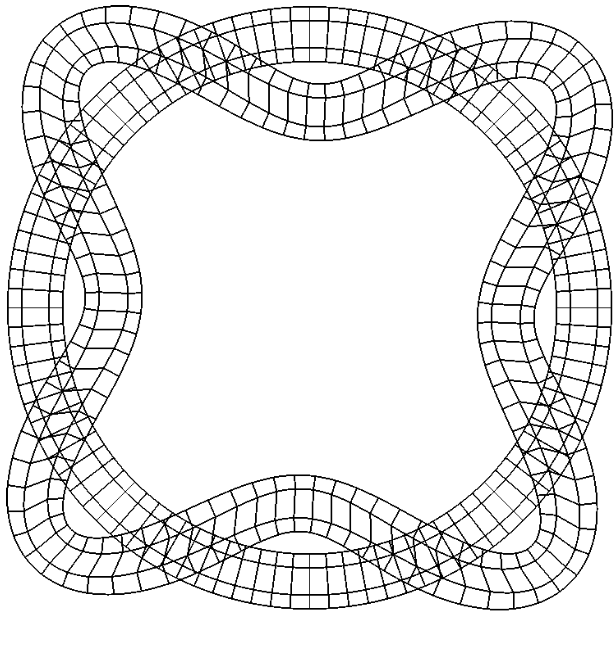


Fig. 2. **The eighth eigenmode of the composite beam (the structure in the status of equilibrium is gray, the eigenmode is black)**

The ninth eigenmode is shown in Fig. 3.

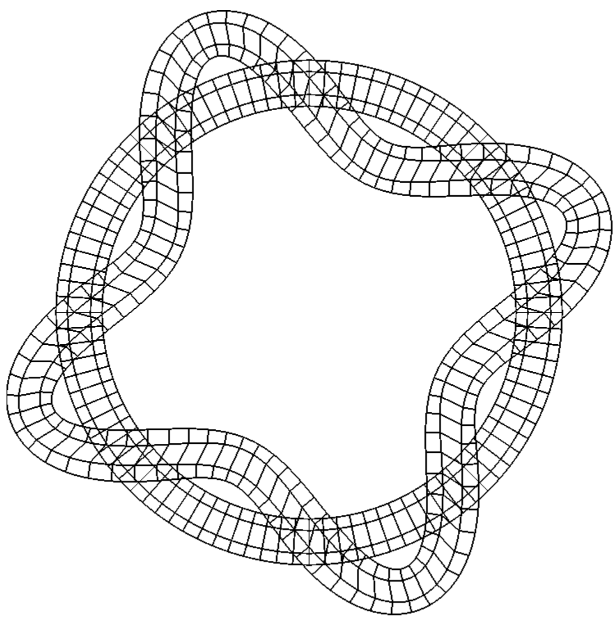


Fig. 3. **The ninth eigenmode of the composite beam (the structure in the status of equilibrium is gray, the eigenmode is black)**

From the presented figures it is evident that the wave motion may be excited on those two modes.

Numerical investigations of the Reynolds equation

The problem is described by the equation [3]:

$$\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}=0, \quad (20)$$

where x and y are orthogonal Cartesian coordinates and p is pressure. The gap is assumed to be in the direction of the

z axis of coordinates from $z=0$ to $z=h$. Then the velocity components u and v in the directions of the axes x and y are [3]:

$$u = \frac{1}{2\mu}\frac{\partial p}{\partial x} z(z-h), \quad v = \frac{1}{2\mu}\frac{\partial p}{\partial y} z(z-h), \tag{21}$$

where μ is the viscosity of the fluid.

The shear strain rates are obtained on the basis of equation (21):

$$\gamma_{xz} = \frac{\partial u}{\partial z} = \frac{1}{2\mu}\frac{\partial p}{\partial x}(2z-h), \quad \gamma_{yz} = \frac{\partial v}{\partial z} = \frac{1}{2\mu}\frac{\partial p}{\partial y}(2z-h). \tag{22}$$

So, the averaged in the through thickness direction intensity of the shear strain rates is:

$$\frac{1}{h}\int_0^h \sqrt{\gamma_{xz}^2 + \gamma_{yz}^2}\, dz = = \frac{1}{2h\mu}\int_0^h |2z-h| dz \sqrt{\left(\frac{\partial p}{\partial x}\right)^2 + \left(\frac{\partial p}{\partial y}\right)^2}. \tag{23}$$

This indicates that the intensity in the photoelastic image may be considered to be proportional to:

$$\sqrt{\left(\frac{\partial p}{\partial x}\right)^2 + \left(\frac{\partial p}{\partial y}\right)^2}. \tag{24}$$

The pressures are determined by solving the system of linear algebraic equations.

The derivatives of the pressures at the points of numerical integration of the finite element are calculated in the usual way:

$$\begin{Bmatrix} p_x \\ p_y \end{Bmatrix} = [B]\{\delta\}, \tag{25}$$

where $\{\delta\}$ is the vector of nodal pressures; p_x, p_y are the derivatives of pressure; $[B]$ is the matrix relating the pressure gradients with the nodal pressures:

$$[B] = \begin{bmatrix} \frac{\partial N_1}{\partial x} & \ldots \\ \frac{\partial N_1}{\partial y} & \ldots \end{bmatrix}, \tag{26}$$

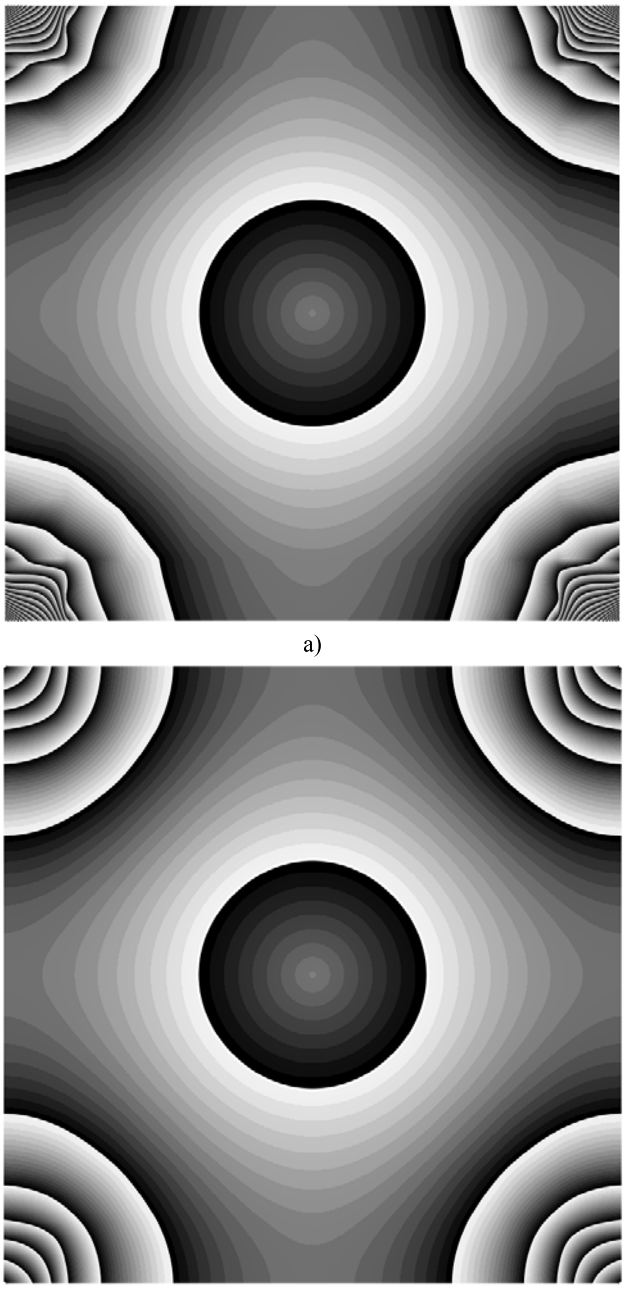
where N_i are the shape functions of the finite element. The pressures are continuous at interelement boundaries, but the calculated pressure gradients due to the operation of differentiation are discontinuous.

The nodal values of pressure gradients are obtained by minimising the following errors:

$$\frac{1}{2}\iint\left(([N]\{\delta_x\} - p_x)^2 + \lambda\left(\left(\frac{\partial p_x}{\partial x}\right)^2 + \left(\frac{\partial p_x}{\partial y}\right)^2 \right)\right) dxdy = = \frac{1}{2}\iint\left(([N]\{\delta_x\} - p_x)^2 + \lambda\{\delta_x\}^T [B^*]^T [B^*]\{\delta_x\}\right) dxdy,$$

$$\frac{1}{2}\iint\left(([N]\{\delta_y\} - p_y)^2 + \lambda\{\delta_y\}^T [B^*]^T [B^*]\{\delta_y\}\right) dxdy, \tag{27}$$

where λ is the smoothing parameter; $\{\delta_x\}$ is the vector of nodal values of p_x; $\{\delta_y\}$ is the vector of nodal values of p_y; $[N]$ is the row of the shape functions of the finite element; $[B^*]$ is the matrix of the derivatives of the shape functions for this problem coinciding with the matrix $[B]$.



a)

b)

Fig. 4. **The calculated result represented by intensity mapping: a) without smoothing, b) with smoothing**

This leads to the following systems of linear algebraic equations for the determination of the nodal values of the gradients of pressure:

$$\iint\left([N]^T[N]+[B^*]^T\lambda[B^*]\right)dxdy\cdot\{\delta_x\}=$$
$$=\iint[N]^T p_x dxdy,$$
$$\iint\left([N]^T[N]+[B^*]^T\lambda[B^*]\right)dxdy\cdot\{\delta_y\}=$$
$$=\iint[N]^T p_y dxdy. \tag{28}$$

Then the required values given by Eq.24 are calculated and represented by intensity mapping of the type proposed in [6, 7].

A rectangular domain is analyzed and the values of pressure on the left and the right boundaries are assumed equal to 1, while on the remaining boundaries they are assumed equal to 0. The results of calculations are presented in Fig. 4.

Conclusions

The finite element of a composite beam is constructed from the lower and upper sub-elements of the beam type and an internal sub-element of an elastic layer type. It is shown that the multiple eigenmodes exist in a circular system. They are suitable for the excitation of wave motion in it.

The quantity proportional to the one obtained in the photoelastic investigations is calculated numerically and represented by the intensity mapping. The obtained results provide the basis for the investigation of devices incorporating the layer of the fluid when viscosity is important.

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Spausdinimo įtaiso elementų tyrimas

Reziumė

Sudėtinio strypo baigtinis elementas sudarytas iš apatinio ir viršutinio strypinių subelementų ir vidinio tampraus sluoksnio subelemento. Gautos apskritiminės konstrukcijos savos formos tinkamos banginiam judesiui žadinti.

Fototampriuose tyrimuose vaizdo intensyvumas gali būti laikomas proporcingu vidutiniam per storį šlyties deformacijų greičio intensyvumui. Jam proporcingas dydis gautas Reinoldso lygties skaitmeniniuose tyrimuose. Rezultatai taikytini tiriant įtaisus su klampaus skysčio sluoksneliu.

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