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Low-frequency sound insulation of lift cabins in residential houses

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Introduction

It is known that lift cabins in spite of their quiet movement in the elevation shaft still cause certain a low-frequency noise. The emission of vibrations due to lift constructions in the partitions of buildings are transferred to adjacent premises and radiates the undesirable noise. Theoretical and experimental research conducted by our and previous authors showed that constructions of a cylindrical shape insulate well a low-frequency noise propagating from inside [1, 2, 3, 4]. The application of cylindrical shells and housings in the designing of residential and public buildings open the opportunity for insulation of an unpleasant noise that is radiated by the modern technical facilities used, for example, lift for elevating people and cargo.

In this paper, we shall deal with theoretical aspects that under certain conditions would provide an opportunity to evaluate the efficiency of the recommended construction in terms of noise reduction. In dependence on the situation where the recommended construction will be implemented, several different methods of calculation may be applied. In multi-apartment houses, lift shafts may be of several tenths meters, and our recommended calculation methods may be applied according to the sound excitation principle and its propagation in the cylindrical housing.

Theory of sound insulation of a cylindrical housing

Vibration of lift shaft walls may be represented as a sum of normal waves propagating through it. These waves emit a noise into the surrounding space in a nonuniform manner: for the same amplitudes, the emission declines quickly with the growth of the azimuth number *n.* This is related to the fact that with an increase in *n* the distance *d* between adjacent wall sections oscillating in counterphase is decreasing. If *d* is significantly less than the wavelength λ_c in the surrounding medium, than the emission of those sections is mutually compensatory, since they oscillate in opposed phases, and the phase overlap between them due to the difference in beam course is small. If, however, $d >> \lambda_c$, then those sections are good emitters of noise. In other words, small-scale ($d < \lambda_c$) wall oscillations are significantly worse emitters of a noise than the large-scale ($d > \lambda_c$) ones. At $n = 0$, all the points of the cabin located on a circumference vibrate in phase. The nature of noise emission in that case is determined by the lengths of the waves propagating along the pipe. As a rule, they are emitted into the surrounding space rather well. At $n = 1$ all the points located at the ends of a diameter have radial velocities of opposite phases. Therefore, the nature of emission is determined by the cross sectional dimensions of the pipe. At $n = 1$ the number of cabin sections along the circumference, which vibrate in opposite phases, grows with *n* and, therefore, effectiveness of a noise emission by those waves is low. In practice, the emission becomes noticeable either at high frequencies or in large diameter shells.

Soundproofing of covers for radial-symmetrical vibrations of pipelines was described in our works [5]. Let us examine here the case when $n = 1$. The presented problem can be solved by the same means as for the case when $n = 0$, with similar results. However, the expression for the cover impedance has rather complex, what complicates the analysis. Let us approach the problem in a different way.

It was demonstrated in the paper [6] that a system of equations of motion for a cylindrical shell at $n = 1$ can be reduced to a single equation of the same type as the equation for the transverse waves, which significantly exceeds the radius of the shell, then the displacements of the points of the shell cross section *y(z,t)* in the direction perpendicular to the axis are described by the equation

$$B\frac{\partial^4 y}{\partial z^4} - \rho_M S r_0^2 \frac{\partial^4 y}{\partial z^4 \partial t^2} + \rho_M S \frac{\partial^2 y}{\partial t^2} = F(z,t), \qquad (1)$$

where $B = ESr_0^2$ is the flexural rigidity of the shell, E and ρ_M are Young's modulus and the density of shell material; $S = 2\pi a h$ is its cross sectional area; a and h are the radius and the thickness of the shell; $r_0 = a/\sqrt{2}$ is the radius of inertia of the cross section; $F(z,t)$ are external forces applied to the shell which cause its displacement in the direction *y.*

Expression for the harmonic force is $F(z,t) = F_0 e^{i(kz-\omega t)}$. The solution of Eq.1 is sought in the form $y(z,t) = y_0 e^{i(kz-\omega t)}$. Substituting it into the equation, we obtain

$$y_0 = \frac{F_0}{Bk^4 - \omega^2 k^2 \rho S r_0^2 - \omega^2 \rho S} = \frac{F_0}{\omega^2 \rho S\left(\frac{k^4}{k_H^4} - k^2 r_0^2 - 1\right)}, \qquad (2)$$

where $k_H^4 = \frac{\omega^2 \rho S}{B} = \frac{\omega^2}{c_п^2 r_0^2}$.

The velocity amplitude is $v = -i\omega y_0$. The ratio of the force $F(z,t)$ to the velocity v is the impedance of the cylindrical shell Z_δ for the beam-type vibration mode

$$Z_\delta = -i\omega M\left(1 + \frac{k_2 a^2}{2} - \frac{k}{k_и^4}\right), \quad M = \rho_M S = 2\pi\rho_M ah. \tag{3}$$

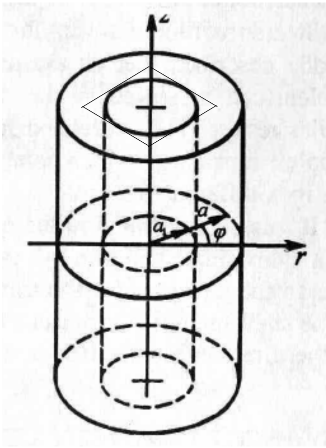

Fig. 1. **Scheme of housing disposition**

Let us now examine the problem of reducing noise in the pipeline by using a cylindrical cover located coaxially with the pipe. Let us assume that the pipeline and the cover are both of an infinite length. Such design diagram is applicable for describing the measurement of sound by extended pipelines when their length significantly exceeds the length of the transverse wave $\lambda_и$ and the wavelength in the medium (air) λ_c.

Let us line up the axis z of a cylindrical coordinate system r, φ, z with the axis of the pipeline (Fig. 1). Assume that the displacement $y(z,t)$ takes place in the direction $\varphi = 0$. The radii of the pipeline and the cover can be denoted by a_l and $a_2 (a_2 > a_1)$, the velocity of sound propagation in air and density by c_0 and ρ_c. Let there be a wave $V = V_1 e^{i(kz-\omega t)}$ propagating along the pipeline.

All the expressions for the pressure and the velocity in air and the displacements of the pipeline and the cover will contain $\exp i(kz - \omega t)$ as a factor, which cancels out in equations. For the sake of simplicity it will be omitted from here on.

The sound pressures in air between the pipeline and the cover p_1 and outside the cover p_2 can be written in the form

$$p_1(\varphi, z) = \left[AH_1^{(1)}(\mu, r) + BH_1^{(2)}(\mu_1, r)\right]\cos\varphi,$$

$$p_2(\varphi, z) = DH_1^{(1)}(\mu, r)\cos\varphi. \tag{4}$$

Here $\mu = \sqrt{k_c^2 - k^2}$ is the radial wave number; $k_c = \omega / e_c H_1^{(1)}$ and $H_1^{(2)}$ are the Hankel functions of the first and second kind.

The force acting on the cover from both internal and external media in the direction $\varphi = 0$ equals $F = F_1 - F_2$, where

$$\left.\begin{aligned} F_1(a_2) &= \int_0^{2\pi} p_1 \cos\varphi a_2 d\varphi = \pi a_2\left[AH_1^{(1)}(\mu a_2) + BH_1^{(2)}(\mu a_2)\right], \\ F_2(a_2) &= \int_0^{2\pi} p_2 \cos\varphi a_2 d\varphi = \pi a_2 DH_1^{(1)}(\mu a_2). \end{aligned}\right\} \tag{5}$$

For this shell at $r = a_1$ and $r = a_2$ the boundary conditions must hold which express the quality between the radial velocities of the walls of the pipeline and the cover v_1 and v_2 the radial velocities of air v_{1r} and v_{2r}. The radial wall velocities v_1 and v_2 are related to the particle velocities V_1, and V_2 by

$v_1 = V_1 \cos\varphi$ and $v_2 = V_2 \cos\varphi$.

From the Euler's equation follows

$$v_{1r} = \frac{1}{i\rho\omega}\frac{\partial p_1(a_1)}{\partial r} \text{ and } v_{2r} = \frac{1}{i\rho\omega}\frac{\partial p_2(a_2)}{\partial r}.$$

Substituting Eq. 5 into Eq. 1 and considering the boundary conditions, we obtain the system of linear equations for amplitudes A, B, D and v_r:

$$\left.\begin{aligned} &A\dot{H}_1^{(1)}(\mu a_1) + B\dot{H}_1^{(2)}(\mu a_1) = Zv_1; \\ &A\dot{H}_1^{(1)}(\mu a_2) + B\dot{H}_1^{(2)}(\mu a_2) = D\dot{H}_1^{(1)}(\mu a_2) = Zv_2; \\ &AH_1^{(1)}(\mu a_2) + BH_1^{(2)}(\mu a_2) - DH_1^{(1)}(\mu a_2) = \frac{Z_2}{\pi a_2} v_r; \\ &Z = \frac{i\rho\omega}{\mu}, \; Z_2 = -i\omega M\left(1 + \frac{k^2 a_2}{2} - \frac{k^4}{k_{и2}^4}\right). \end{aligned}\right\} \tag{6}$$

Here $\dot{H}$ is a derivative of the Hankel function with respect to its argument; Z_2 is the cover impedance $k_{и2}^4 = 2\omega^2 / c_п^2 a_2^2$. From here on we will omit the order in the Hankel functions and the argument (μa_2).

From the second Eq.6 let us express v_r and substitute the obtained value into the third equation. Then

$$\left.\begin{aligned} &A\dot{H}^{(1)} + B\dot{H}^{(2)}(a_1) = Zv_1, \; A\dot{H}^{(1)} + B\dot{H}^{(2)} - D\dot{H}^{(1)} = 0, \\ &AH^{(1)} + BH^{(2)} - D\left[H^{(1)} + \frac{Z_2\dot{H}^{(1)}}{\pi a_2 Z}\right] = 0. \end{aligned}\right\}$$

Solving this system of equations for D, we obtain

$$D = Zv_1\left[\dot{H}^{(1)}H^{(2)} - \dot{H}^{(2)}H^{(1)}\right] /$$
$$/ \dot{H}^{(1)}\left[\dot{H}^{(1)}(a_1)H^{(2)} - \dot{H}^{(2)}(a_1)H^{(1)}\right] -$$
$$-\left[H^{(1)} + \frac{Z_2\dot{H}^{(1)}}{Z\pi a_2}\right]\left[\dot{H}^{(1)}(a_1)\dot{H}^{(2)} - \dot{H}^{(2)}(a_1)\dot{H}^{(1)}\right]$$

To simplify the notation the following abbreviation is introduced

$$\left[\dot{H}^{(1)}(a_1)H^{(2)} - \dot{H}^{(2)}(a_1)H^{(1)}\right] = [2].$$

The denominator D can be represented in the form

$$\dot{H}^{(1)}\left[\dot{H}^{(1)}(a_1)\dot{H}^{(2)} - \dot{H}^{(2)}(a_1)\dot{H}^{(1)}\right] - H^{(1)}[2] - \frac{Z_2\dot{H}^{(1)}}{Z\pi a_2}[2],$$

where [7] is the last multiplier.

Subtracting the second term from the first, we obtain

$$D = Zv_1\left[\dot{H}^{(1)}H^{(2)} - \dot{H}^{(2)}H^{(1)}\right]/ \\ /\dot{H}^{(1)}(a_1)\left[\dot{H}^{(1)}H^{(2)} - \dot{H}^{(2)}H^{(1)}\right] - \\ -\frac{Z_2\dot{H}^{(1)}}{Z\pi a_2}\left[\dot{H}^{(1)}(a_1)\dot{H}^{(2)} - \dot{H}^{(2)}(a_1)\dot{H}^{(1)}\right] \quad (7)$$

Let us define the sound insulation of the cover as the difference in the levels of the sound pressure generated by the pipeline at some r with and without the cover:

$$R = L_{20} - L_{10} = 20\lg\left|\frac{p_{20}(r)}{p_2(r)}\right|. \quad (8)$$

In the pipeline without a cover the sound pressure is D_0. The amplitude of D_0 is determined from the boundary conditions at $r = a_1$

$$D_0\dot{H}_1^{(1)}(\mu a_1)/Z = v_1, \quad D_0 = Zv_1/\dot{H}_1^{(1)}(\mu a_1)$$

and

$$p_{20}(r) = Zv_1H_1^{(1)}(\mu r)/\dot{H}_1^{(1)}(\mu a_1).$$

Substituting p_{20} and $p_2 = DH_1^{(1)}(\mu r)$ into Eq. 8, we obtain

$$R = 20\lg\left|1 - \frac{Z_2\dot{H}^{(1)}\left[\dot{H}^{(1)}(a_1)\dot{H}^{(2)} - \dot{H}^{(2)}(a_1)\dot{H}^{(1)}\right]}{Z\pi a_2\dot{H}^{(1)}(a_1)\left[\dot{H}^{(1)}H^{(2)} - \dot{H}^{(2)}\dot{H}^{(1)}\right]}\right|. \quad (9)$$

The denominator in the logarithmic expression can be simplified by using the relations

$$\dot{H}_1^{(1,2)}(x) = -\frac{1}{x}H_1^{(1,2)}(x) + H_0^{(1,2)}(x)$$

and

$$H_0^{(1)}H_1^{(2)} - H_0^{(2)}H_1^{(1)} = 4i/\pi x.$$

Then Eq. 9 can be written as

$$R = 20\lg\left|1 + \frac{Z_2\mu^2\dot{H}_1^{(1)}}{4\rho\omega\dot{H}_1^{(1)}(a_1)}\left[\dot{H}_1^{(1)}(a_1)\dot{H}_1^{(2)} - \dot{H}_1^{(2)}(a_1)\dot{H}_1^{(1)}\right]\right|. \quad (10)$$

Eq.10 includes derivatives of the Hankel functions, which in principle can be eliminated, but it would not lead to simpler expressions. It is more convenient to make use of the exponential form of notation

$$\dot{H}_1^{(1)}(x) = iC_1(x)e^{i\delta_1^1(x)} \text{ and } \dot{H}_1^{(2)}(x) = -iC_1^1(x)e^{i\delta_1^1(x)}.$$

The amplitudes C_1 and the phases $\delta_1^1(x)$ have been tabulated in [7]. Substituting these expressions into Eq. 10, we obtain

$$R = 20\lg\left|1 - \frac{\omega Mk_c\cos^2\Theta}{2\rho c}Z_{20}\cdot C_1^2(k_ca_2\cos\Theta)e^{i\Delta}\sin\Delta\right| \quad (11)$$

Here $Z_{20} = 1 + 0{,}5k_c^2a_2^2\sin^2\Theta - \frac{f^2}{f_k^2}\sin^4\Theta$;

$\Delta = \delta_1'(k_ca_2\cos\Theta) - \delta_1'(k_ca_1\cos\Theta)$; $f_k = c_c^2/\pi\sqrt{2}c_{\text{п}}a_2$

is the critical frequency at which the length of the transverse wave (without considering the inertia of the cross sectional rotation of the shell) equals the wavelength in air; Θ is the angle between the direction of the cylindrical wave propagation in air and the plane z = 0.

The components of the wave vector k and μ are relited to the wave vector $\vec{k}_c$ in air by relations $k = k_c\sin\Theta$ and $\mu = k_c\cos\Theta$, from which $\Theta = \arcsin(k/k_c)$. At $k > k_c$ the value of Θ becomes imaginary; no noise is emitted in the direction radial to the pipe. The sound wave propagates only along the pipe.

Let us examine the obtained expression (11). The square of the amplitude $C_1'^2$ never becomes zero. At high argument values, when $k_ca\cos\Theta >> 1$; $C_1'^2 = 2/\pi k_ca_2\cos\Theta$, then the phase $\delta_1'(a_2) \approx k_ca_2\cos\Theta - 3\pi/4$, and the phase difference $\Delta = k_c\cos\Theta(a_2 - a_1) = k_cd\cos\Theta$ ($d = a_2 - a_1$ is the clearance between the pipeline and the cover). At small argument values, when $k_ca\cos\Theta << 3$, $C_1'^2 \approx 2/\pi(k_ca_2\cos\Theta)^2$, then the phase

$$\delta_1'(a_2) \approx -\pi(k_ca_2\cos\Theta - 3\pi)^2/4,$$

and the phase difference

$$\Delta = -\pi k_c^2\cos^2\Theta\left(a_2^2 - a_1^2\right)/4.$$

In this way the magnitude of sound insulation is determined by the behavior of Z_{20} and $\sin\Delta$ in the second term of the logarithmic expression. When they become zeros, R also equals zero.

The dimensionless part of the cover impedance is $Z_2 = 0$ at

$$\sin^2\Theta \approx \frac{f_k}{f}\left(1 + 0{,}25\frac{f_k}{f}k_c^2a_2^2\right) \approx \frac{f_k}{f},$$

or

$$\Theta \approx \arcsin\sqrt{f_k/f}.$$

Those are the known coincidence angles when the length of the transverse wave is $\lambda_0 = \lambda_c$, the latter being the wavelength in air.

The values of $\sin\Delta = 0$lie at $\Delta = m\pi$, where $m = 1, 2, ...$ is an integer number. In order to clarify the physical nature of this condition, let us analyze the asymptotic of the Hankel function at $k_ca_1\cos\Theta >> 1$. Then $\Delta = k_cd\cos\Theta = m\pi$ or $\frac{m\lambda_c}{2\cos\Theta} = d = a_2 - a_1$. In the latter expression $\lambda_c/\cos\Theta = \lambda_r$ is the trace of a wave which is propagating at the angle Θ along the axis $r(\lambda_r) = 2\pi/\mu$. The value of $\Delta\sin$ will equal zero when a whole number of the wave trace half-waves would fit into the clearance between the cover and the pipeline. This is the condition of sound wave resonance in a layer with thickness d. It should be noted that the first resonance arises at m = 1, when $\Delta = k_c\cos\Theta(a_2 - a_1) = \pi$. Then $k_c\cos\Theta a_1 > \pi$ and $k_ca_2\cos\Theta > \pi$, since it almost never occurs in practice that the radius of the cover is twice as big as the radius of the pipeline. Usually $a_2/a_1 < 2$. In that case it is possible to use the asymptotic formulas for the Hankel function to achieve a level of accuracy sufficient for practical purposes, and to compute the sound insulation of the cover according to the formula

$$R = 20\lg\left|1 - \frac{\omega M \cos\Theta}{\rho c \pi a_2} Z_{20} \cdot e^{ik_c d \cos\Theta} \sin(k_c d \cos\Theta)\right|, \quad (12)$$

when $k_c a_1 \cos\Theta > \pi$.

If the resonance and coincidence areas are eliminated, then the magnitude of the second term under the logarithm sign in Eq. 12 is much greater than one. Expression (11) can thus be simplified

$$R = 20\lg\left|\frac{\omega M k_c \cos\Theta}{2\rho c} Z_{20} \cdot C'^2_1 (k_c a_2 \cos\Theta) \sin\Delta\right| \quad (13)$$

or at $k_c a_1 \cos\Theta > \pi$

$$R \approx 20\lg\left|\frac{\omega M \cos\Theta}{\rho c \pi a_2} Z_{20} \sin(k_c d \cos\Theta)\right|. \quad (14)$$

Validity of these formulas can be tested: they are not valid if R is sufficiently close to zero or assumes a negative value. For practical purposes a sufficiently accurate value is computed to be $R > 10$ dB.

Results

An analysis of the obtained results leads to the following conclusions. Analysis of the roots of the dispersion equation at $n = 1$ shows that for low frequencies there is a real root which describes beam-type transverse vibration up to frequencies approximately equal to $f \approx 0{,}2 f_\Pi$, where $f_\Pi = c_\Pi / 2\pi a$ is the frequency of the longitudinal resonance. For higher frequencies more precise values of the cover impedance must be used. In principle, they are not difficult to determine.

Analysis shows that the obtained expressions are valid for all frequencies if in stead of the impedance Z_2 we substitute the exact value of the impedance $Z_{1\delta,}$ (Eq. 5) for n=1 multiplied by πa_2, i.e., $Z_2 = Z_{1\delta}\ \pi a_2$. In the expression for $Z_{1\delta}$ k is just replaced by $k_c \sin\Theta$. The expression turns out to be more cumbersome, but this difficulty can be easily overcome with the use of computer technology.

Attention should be given to the fact that computation of R at n=1 can be done in accordance with the methodology of evaluating effectiveness of the cover at n=0 [5]. To that end magnitudes and phases C_1 and δ_1 should be replaced by magnitudes and phases of the first order derivative of the Hankel function C_1' and δ_1'.

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Gyvenamųjų namų lifto kabinų žemųjų dažnių garso izoliacija

Reziumė

Šiuolaikinėje namų statyboje plačiai naudojami liftai žmonėms ir kroviniams kelti. Nors dabartiniai modernūs liftai, palyginti su kitais triukšmo šaltiniais, veikia tyliai, jų judėjimas šachtose sukelia žemųjų dažnių virpesius ir per stačias konstrukcijas išspinduliuoja atitinkamo dažnio triukšmą.

Straipsnyje pateikiama teorija, kuria remiantis galima prognozuoti žemųjų dažnių virpesių ir triukšmo sumažinimą taikant atitinkamus cilindrinius gaubtus (šachtas).

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